



High-order correlations in the time blocking approach

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Outline

- **Motivation:** to build a consistent and predictive approach based on fundamental NN-interaction to describe the entire nuclear chart
- **Challenges:** the nuclear hierarchy problem, complexity of NN-interaction
- **Solution:** Relativistic Nuclear Field Theory (RNFT). Emerged as a synthesis of the extended Migdal Fermi-liquid theory, Copenhagen-Milano NFT and Quantum Hadrodynamics
- **One-fermion self-energy** in nuclear medium
- **Nuclear response theory beyond RPA:** time blocking approximation. Neutral (non-isospin-flip) channels. Isospin-flip response: see the talk of Caroline Robin
- **High-order correlations**
- **Conclusions and perspectives**

Hierarchy problem

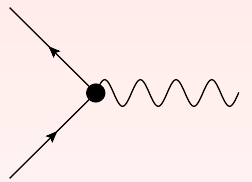
- Nuclear scales: Hierarchy problem

$$H = K + V$$

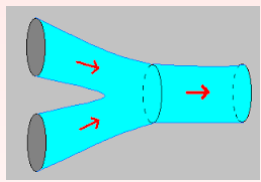
↑
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center of mass
*internal DOF's:
next energy scale*

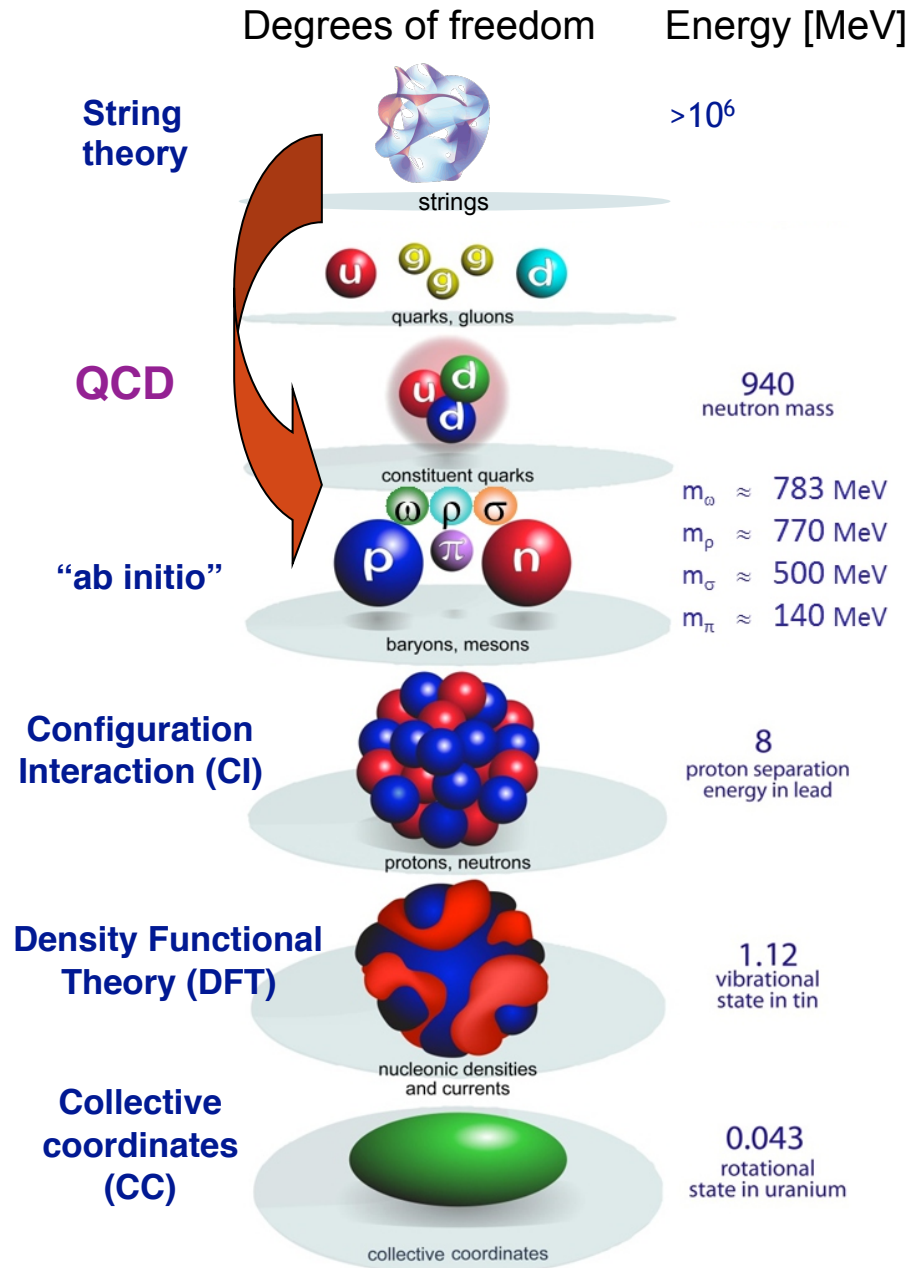
QFT:
interaction



String theory:
merging strings



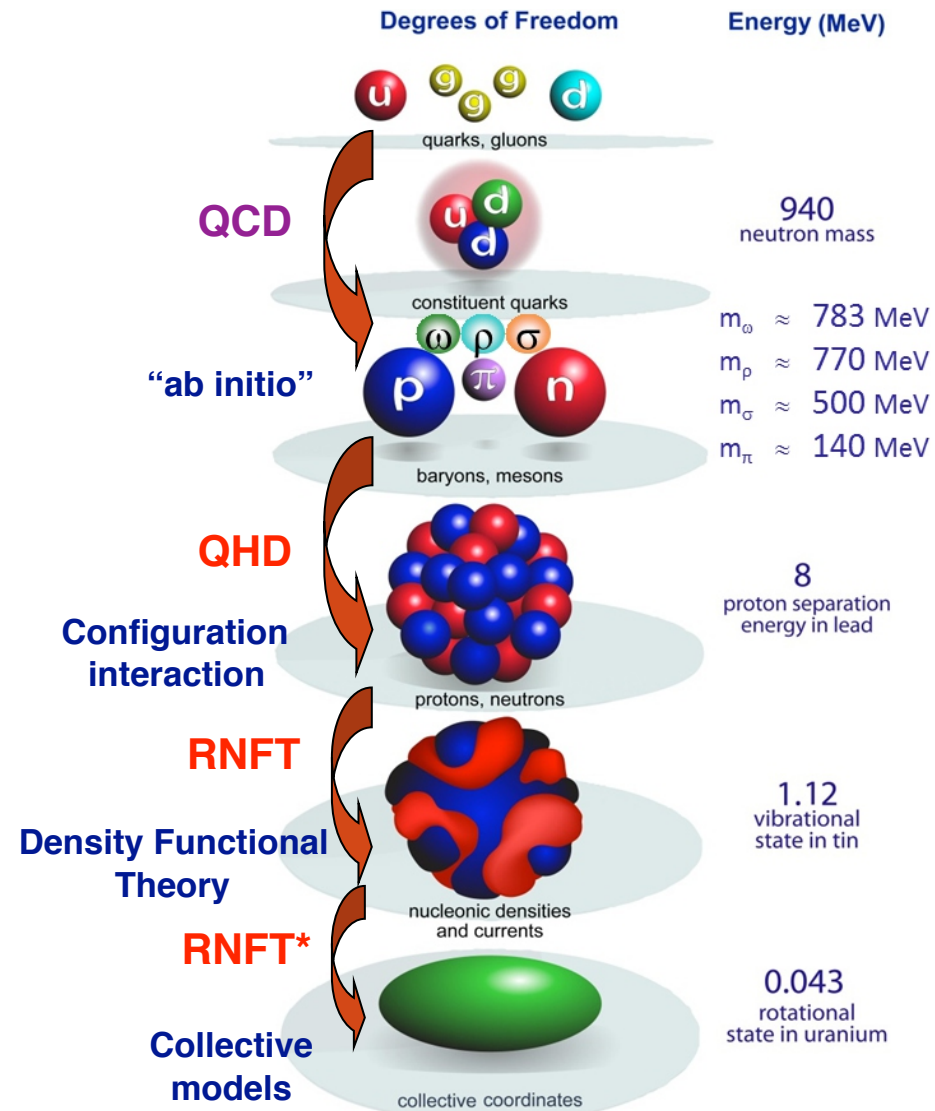
- *No connection between the scales in the traditional NS models*
- *Effective theories (most often) lose the energy dependence of the interaction*



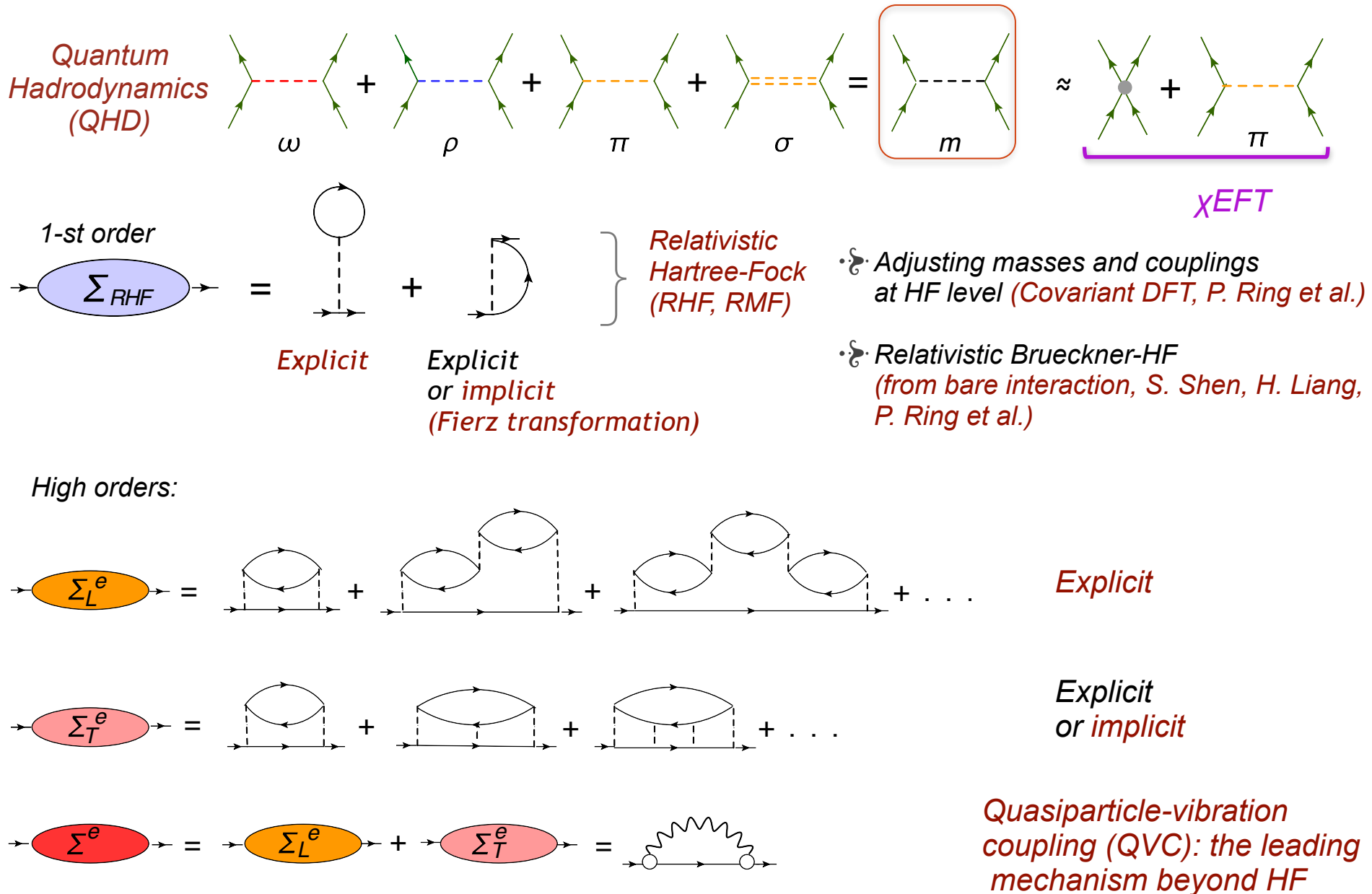
Relativistic Nuclear Field Theory (RNFT):

- RNFT as a solution: microscopic, universal, connecting scales from Quantum Hadrodynamics to emergent collective phenomena
- Lagrangian for mesons and nucleons constrained by QCD symmetries and sum rules
- **Lorentz covariance:** ~5-10% accuracy at the excitation energy of interest (grows with energy)
- Spin-orbit and tensor “forces” are naturally included
- Fewer parameters; hidden correlations minimized (4-10 universal parameters)
- Natural extension to the inclusion of the delta isobar, to higher excitation energy ~200-300 MeV and to hypernuclei
- **Non-perturbative self-consistent response theory with high-order NN correlations**

Nuclear scales



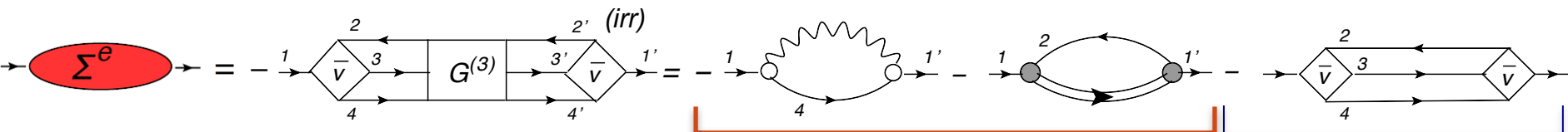
Systematic expansion in the RNFT: single-nucleon self-energy



From a strongly coupled to a weakly coupled theory: emergent collective DOF

The **exact** t -dependent one-nucleon self-energy (P. Schuck, J. Dukelsky, S. Adachi et al.):

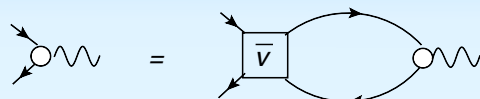
Neglecting three-body forces (P. Schuck, N. Vinh Mau et al.) (well justified in the relativistic theory):



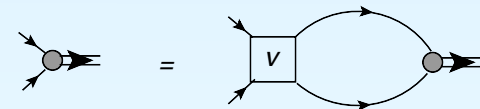
QVC: leading for strong coupling

SRPA (up to a factor $-1/2$): leading for weak coupling

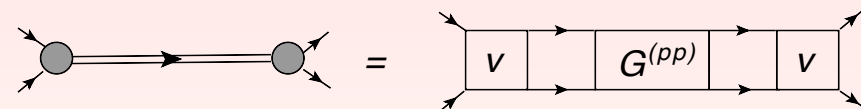
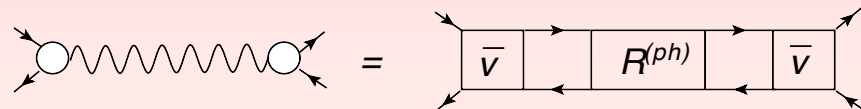
Particle-hole phonons



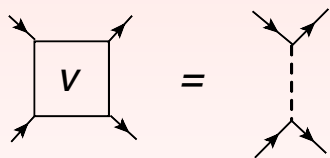
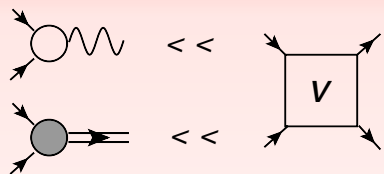
Particle-particle phonons



• The most important phonons are well described in the (relativistic) random phase approximation (RRPA)



• The phonons act as mediators of an additional boson-exchange interaction



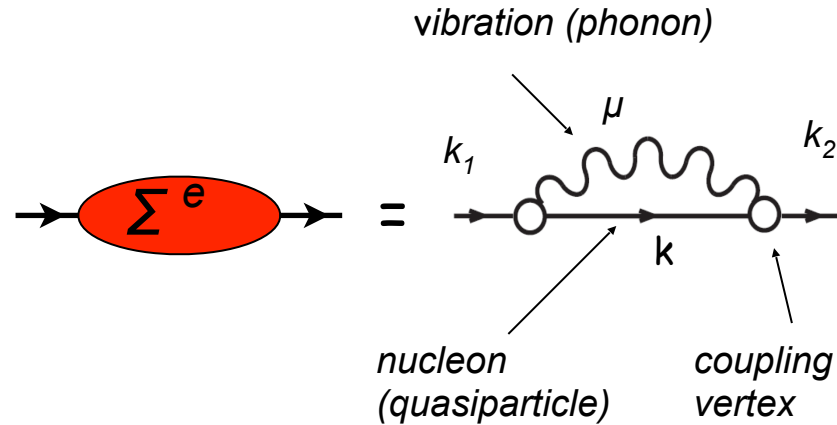
• Phonon vertices contain smallness: QVC is a reduction to a weakly coupled theory

• Phonons mediate the interaction of the order of nuclear size: long-range compared to short- and intermediate-range meson-exchange

• QVC takes into account the dynamical (retardation) effects while the meson exchange is considered instantaneous (static)

Beyond Hartree-Fock: quasiparticle-vibration coupling (QVC)

Additional "potential"
 = "self-energy" =
 = "mass operator"
 with **energy dependence**



One-body propagator G : Dyson equation for Gor'kov Green function

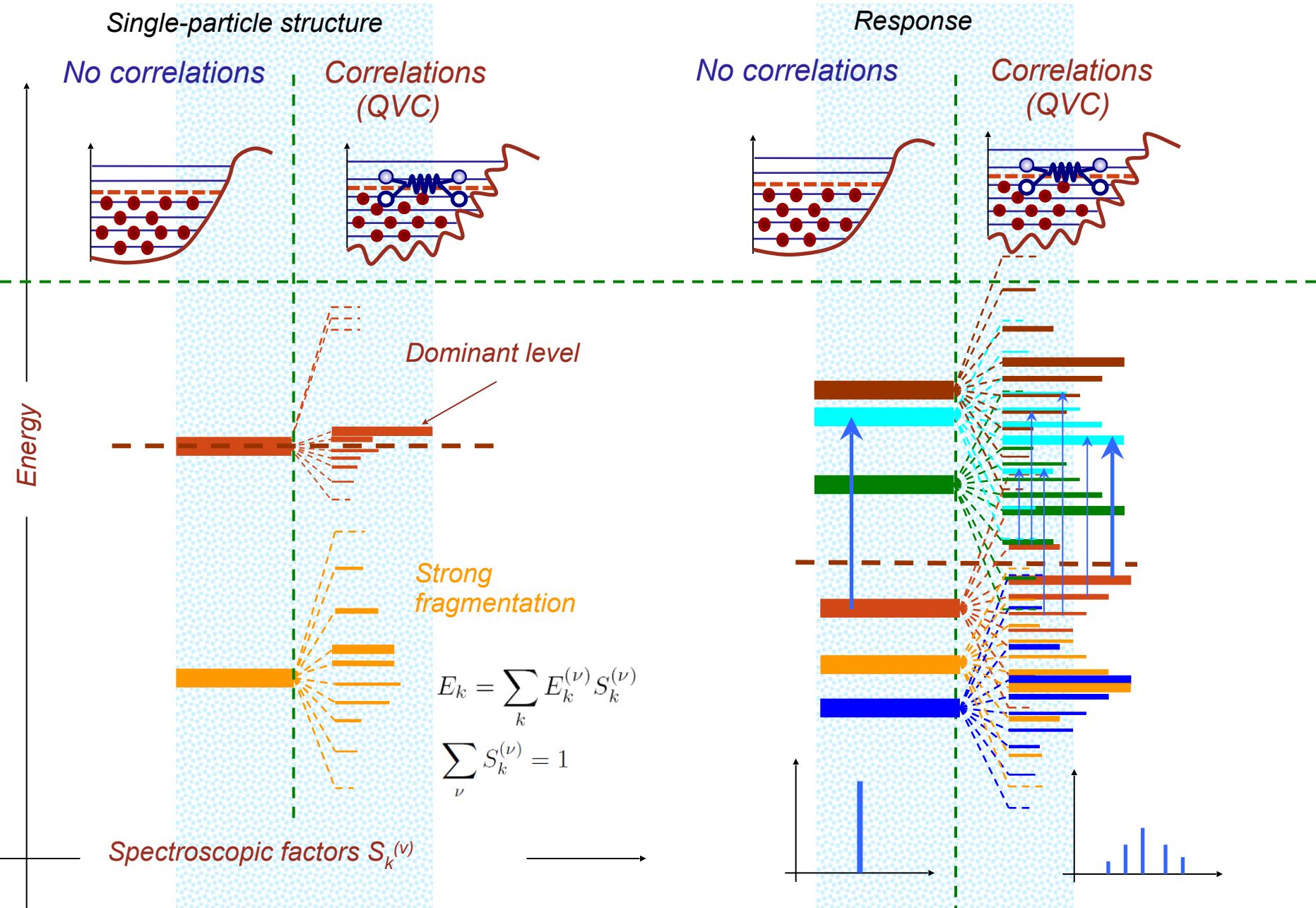
$$\begin{array}{c}
 \begin{array}{c} k \quad k' \\ \longrightarrow \quad \longrightarrow \\ G(\varepsilon) \end{array} = \begin{array}{c} k \quad k' \\ \longrightarrow \quad \longrightarrow \\ G_0(\varepsilon) \end{array} + \begin{array}{c} k \quad k_1 \quad k_2 \quad k' \\ \longrightarrow \quad \text{[} \Sigma^{RHF} \text{]} \quad \longrightarrow \quad \text{[} \Sigma^e \text{]} \quad \longrightarrow \\ G_0(\varepsilon) [\Sigma^{RHF} + \Sigma^e(\varepsilon)] G(\varepsilon) \end{array}
 \end{array}$$

$$\Sigma_{k_1 k_2}^{(e)\eta_1 \eta_2}(\varepsilon) = \sum_{\eta=\pm 1} \sum_{k, \mu} \frac{\gamma_{\mu; k_1 k}^{\eta; \eta_1 \eta} \gamma_{\mu; k_2 k}^{\eta; \eta_2 \eta^*}}{\varepsilon - \eta(E_k + \Omega_\mu - i\delta)}$$

$$\eta = \pm 1$$

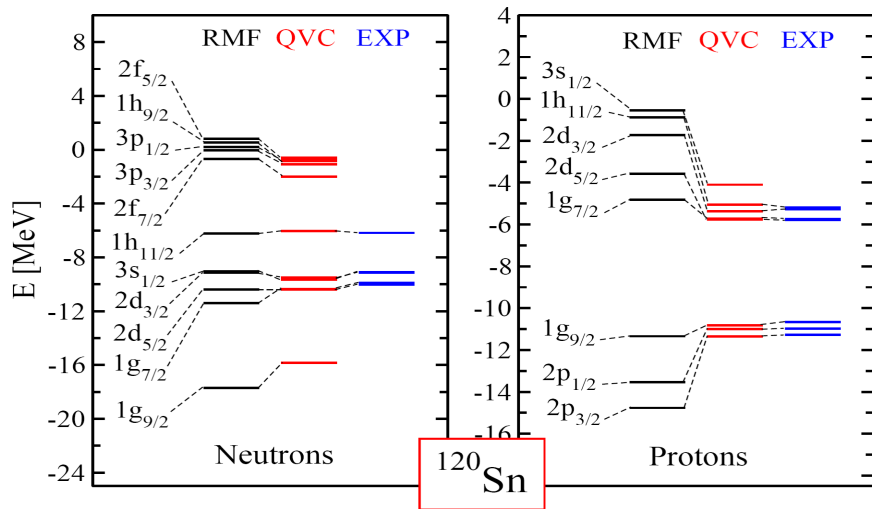
forward / backward components
 in Nambu space

Fragmentation of single-particle states and particle-hole excitations



(Quasi)particle-vibration coupling (QVC, PVC): Pairing correlations of the superfluid type + coupling to phonons

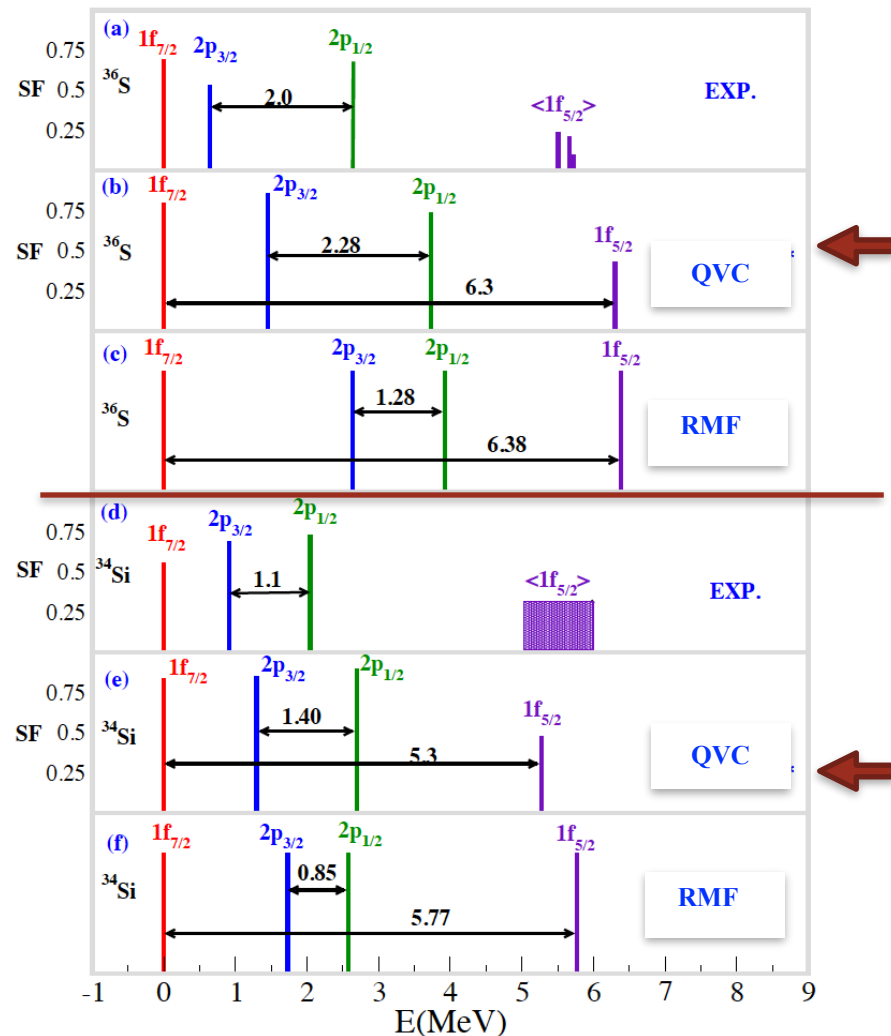
Dominant states and spectroscopic factors in ^{120}Sn :



(nlj) v	S^{th}	S^{exp}
$2d_{5/2}$	0.32	0.43
$1g_{7/2}$	0.40	0.60
$2d_{3/2}$	0.53	0.45
$3s_{1/2}$	0.43	0.32
$1h_{11/2}$	0.58	0.49
$2f_{7/2}$	0.31	0.35
$3p_{3/2}$	0.58	0.54

E. L., P. Ring, PRC 73, 044328 (2006)
E.L., PRC 85, 021303(R) (2012)

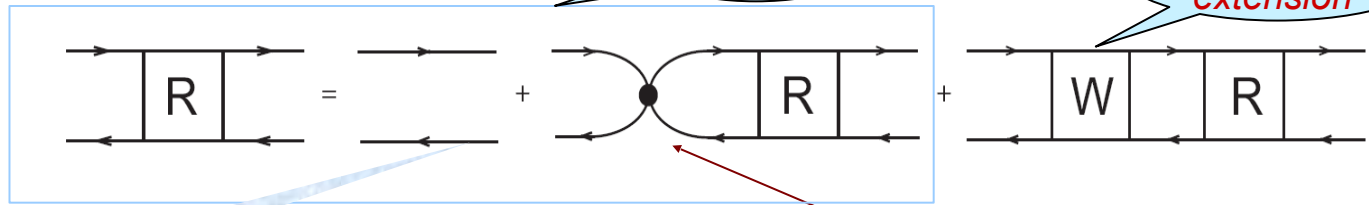
Spin-orbit splittings in ^{36}S
vs a bubble nucleus ^{34}Si ; neutron states:



Exp: Burgunder et al., PRL 112, 042502 (2014)
Th: K. Karakatsanis et al., PRC 95, 034901 (2017)

Excited states: nuclear response function

Bethe-Salpeter Equation (BSE):



E. L., P. Ring, and V. Tselyaev,
Phys. Rev. C 78, 014312 (2008)

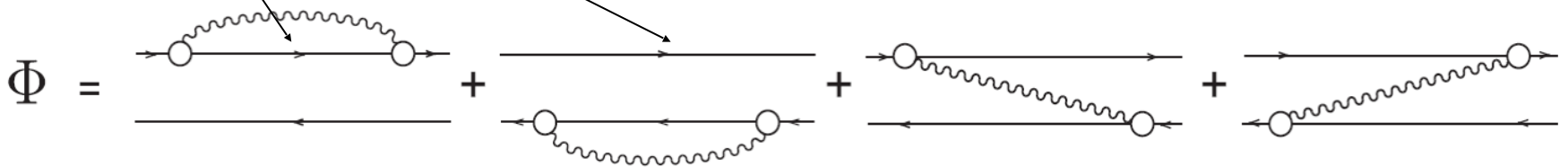
$$\text{---} = \left(\begin{array}{cc} \text{---} & \text{---} \\ \text{---} & \text{---} \end{array} \right)$$

$$R(\omega) = A(\omega) + A(\omega) [V + W(\omega)] R(\omega)$$

$$V = \frac{\delta \Sigma^{RHF}}{\delta \rho}$$

$$W(\omega) = \Phi(\omega) - \Phi(0)$$

Self-consistency

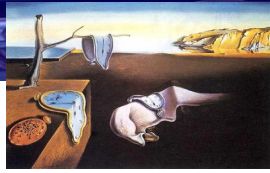


$$i \frac{\delta}{\delta G} \text{---} \times \text{---} = i \frac{\delta \Sigma^e}{\delta G} = \text{---}$$

$$U^e = i \frac{\delta \Sigma^e}{\delta G}$$

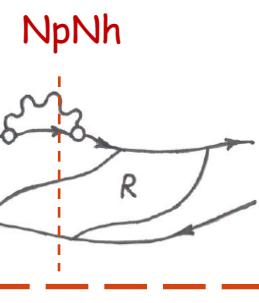
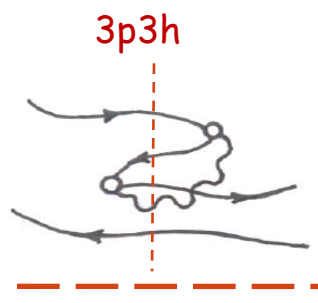
Consistency on 2p2h-level

Time blocking technique

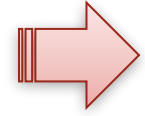


Problem:

'Zigzag' diagrams



Perturbative schemes:



Unphysical result:
negative cross sections

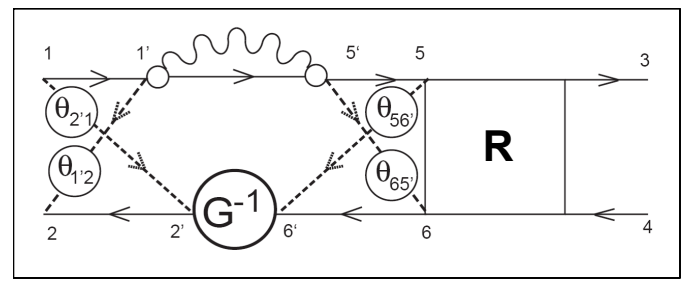


Solution:

Time-projection operator:

$$\delta_{\sigma_1 - \sigma_2} \theta(\sigma_1 t_{2'1}) = 1 \rightarrow \theta_{2'1}$$

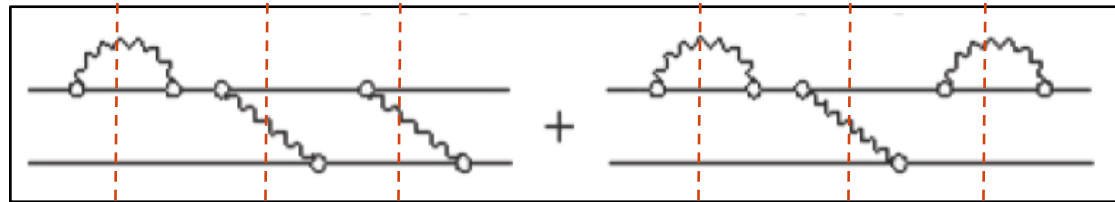
$$\delta_{\sigma_2 - \sigma_1} \theta(\sigma_1 t_{1'2}) = 2 \leftarrow \theta_{1'2}$$



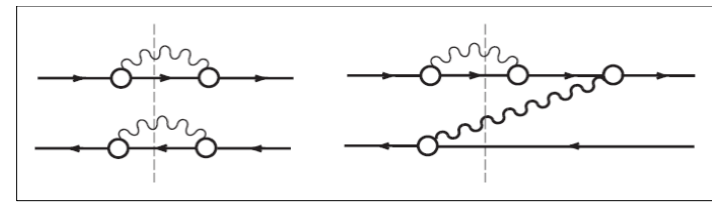
Partially fixed

V.I. Tselyaev,
Yad. Fiz. 50,1252 (1989)

Allowed terms: 1p1h, 2p2h



Blocked terms: 3p3h, 4p4h, ...



Time

- Separation of the integrations in the BSE kernel
- R has a simple-pole structure (spectral representation)
- »» Strength function is positive definite!

Included on the next step

Response function in the neutral channel (leading order in QVC): relativistic quasiparticle time blocking approximation (RQTBA)

Response

$$R(\omega) = \tilde{R}^0(\omega) + \tilde{R}^0(\omega)W(\omega)R(\omega)$$

Interaction

$$W(\omega) = V_\sigma + V_\omega + V_\rho + V_e + \underbrace{\Phi(\omega) - \Phi(0)}_{\text{Subtraction to avoid double counting (if CDFT-based)}}$$

Static:
RQRPA

$$\left\{ \begin{aligned} v_\sigma(1, 2) &= -g_\sigma^2 \gamma_1^0 D_\sigma(1, 2) \gamma_2^0 \\ v_\omega(1, 2) &= +g_\omega^2 (\gamma^0 \gamma_\mu)_1 D_\omega^{\mu\nu}(1, 2) (\gamma^0 \gamma_\nu)_2 \\ v_\rho^V(1, 2) &= +g_\rho^2 (\gamma^0 \gamma_\mu \vec{\tau})_1 \vec{\tau}_1 \cdot \vec{\tau}_2 D_\rho^{\mu\nu}(1, 2) (\gamma^0 \gamma_\nu \vec{\tau})_2 \end{aligned} \right.$$

Subtraction
to avoid double
counting (if CDFT-based)

Dynamic
(retardation):

Quasiparticle-
vibration
coupling

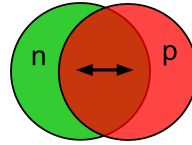
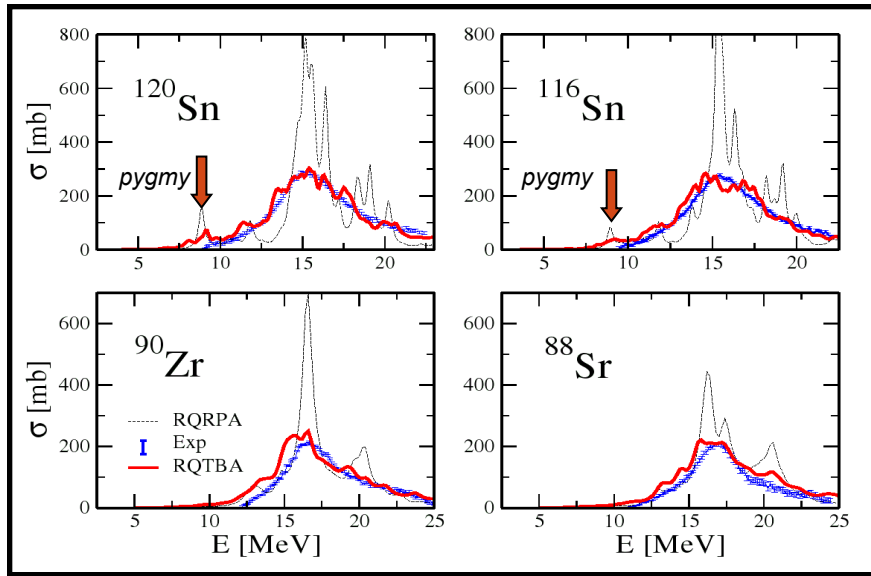
in time blocking
approximation

$$\begin{aligned} & \Phi_{k_1 k_4, k_2 k_3}^\eta(\omega) = \\ &= \sum_{\mu\xi} \delta_{\eta\xi} \left[\delta_{k_1 k_3} \sum_{k_6} \frac{\gamma_{\mu; k_6 k_2}^{\eta; -\xi} \gamma_{\mu; k_6 k_4}^{\eta; -\xi*}}{\eta\omega - E_{k_1} - E_{k_6} - \Omega_\mu} + \delta_{k_2 k_4} \sum_{k_5} \frac{\gamma_{\mu; k_1 k_5}^{\eta; \xi} \gamma_{\mu; k_3 k_5}^{\eta; \xi*}}{\eta\omega - E_{k_5} - E_{k_2} - \Omega_\mu} \right. \\ & \quad \left. - \left(\frac{\gamma_{\mu; k_1 k_3}^{\eta; \xi} \gamma_{\mu; k_2 k_4}^{\eta; -\xi*}}{\eta\omega - E_{k_3} - E_{k_2} - \Omega_\mu} + \frac{\gamma_{\mu; k_3 k_1}^{\eta; \xi*} \gamma_{\mu; k_4 k_2}^{\eta; -\xi}}{\eta\omega - E_{k_1} - E_{k_4} - \Omega_\mu} \right) \right] \end{aligned}$$

E. L., P. Ring, and V. Tselyaev, *Phys. Rev. C* 78, 014312 (2008)
Alternative: Equation of Motion method (P. Schuck et al.)

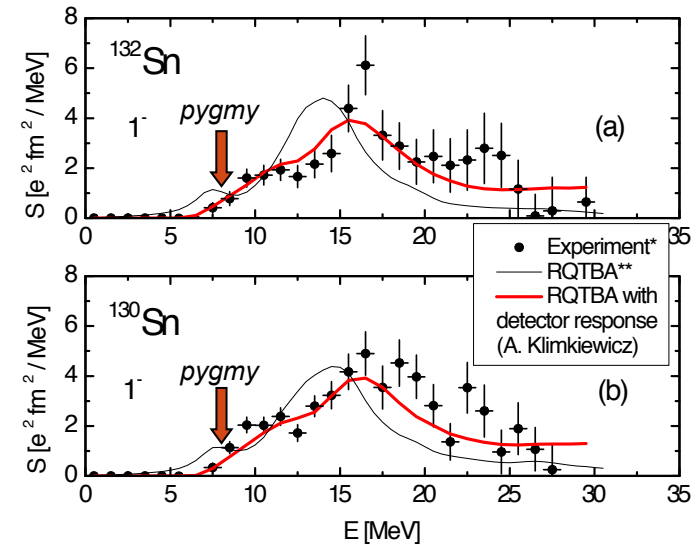
Dipole response in medium-mass and heavy nuclei within Relativistic Quasiparticle Time Blocking Approximation (RQTBA)

Giant dipole resonance (GDR) in stable nuclei



Giant & pygmy dipole resonances

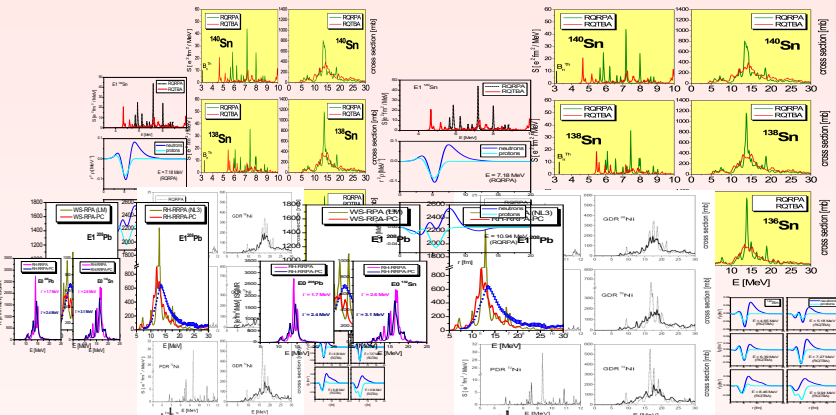
Neutron-rich Sn



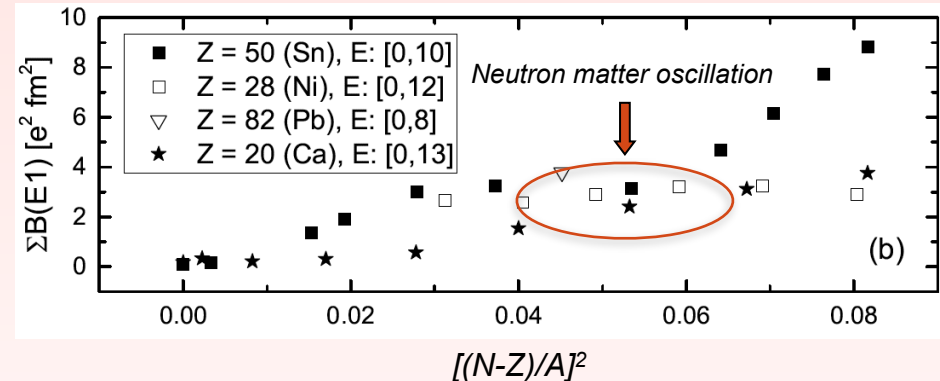
* P. Adrich et al.,
PRL 95, 132501 (2005)

** E. L., P. Ring, and V. Tselyaev,
Phys. Rev. C 78, 014312 (2008)

Systematic GMR calculations (various multipoles) systematic improvement compared to RQRPA and other microscopic approaches

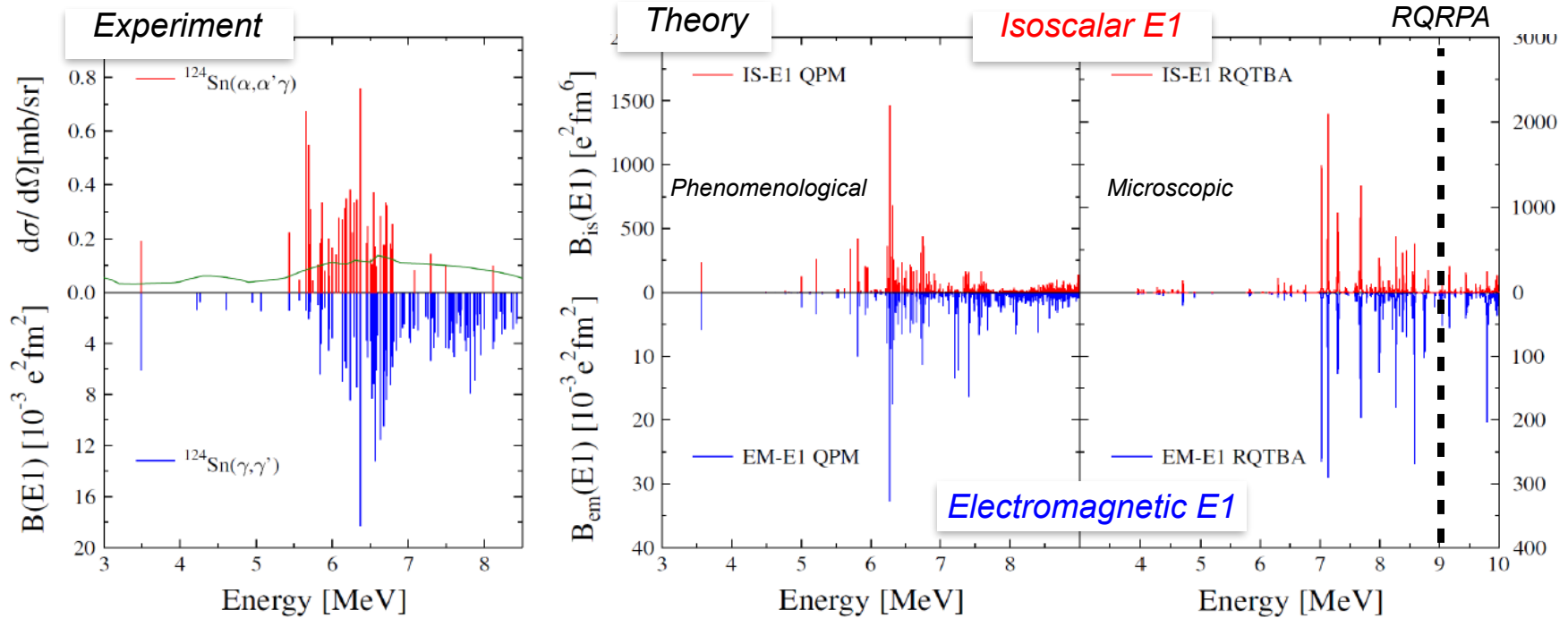


Pygmy dipole strength systematics (important for EOS and astrophysics)



I.A. Egorova, E. L., Phys. Rev. C 94, 034322 (2016)

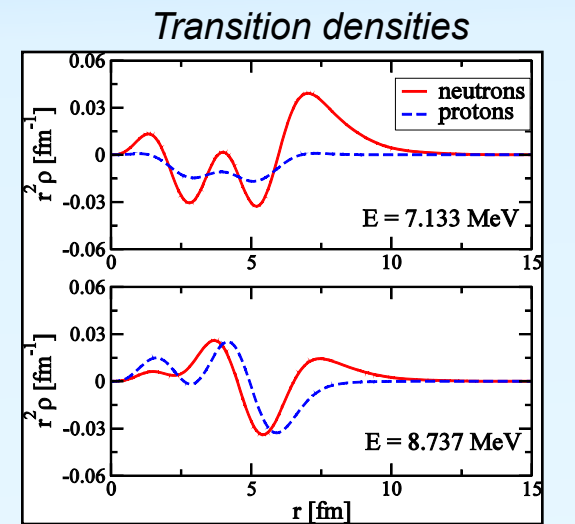
Isospin splitting of the pygmy dipole resonance (PDR)



- Pygmy dipole resonance has been successfully described by RQTBA in a fully microscopic and self-consistent way
- PDR below 9 MeV forms solely due to QVC (absent in RQRPA)
- The suppression of the dipole strength at higher energies has been explained by a careful analysis of RQTBA transition densities and changing their character toward the GDR pattern
- **Still higher-order correlations are needed**

J. Endres, E.L., D. Savran et al., PRL 105, 212503 (2010)

E. Lanza, A. Vitturi, E.L., D. Savran, PRC 89, 041601(R) (2014)



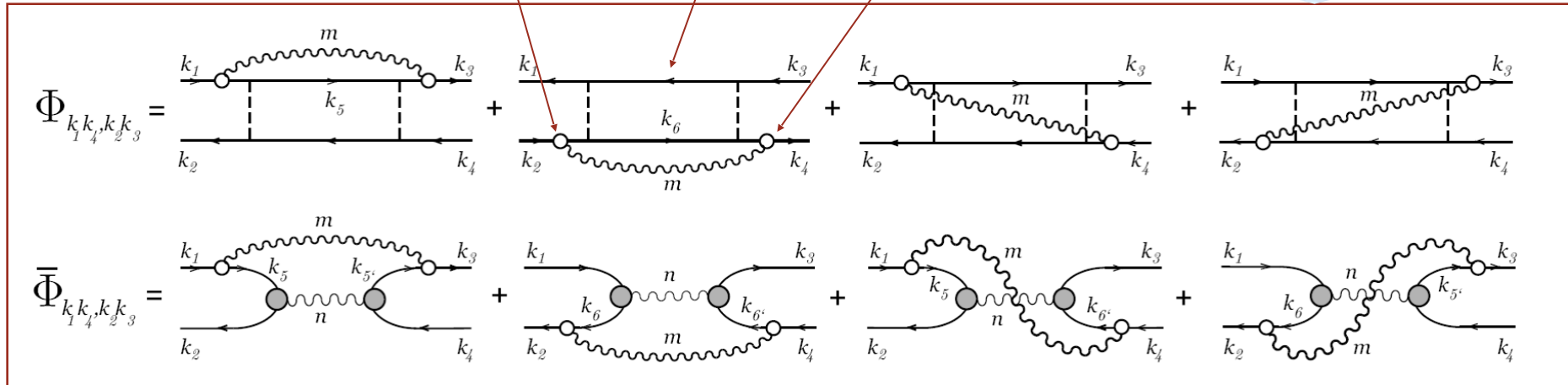
Fine structure of spectra: next-order correlations from "2q+phonon" to "2 phonons"

P. Schuck, Z. Phys. A 279, 31 (1976)
V.I. Tselyaev, PRC 75, 024306 (2007)

& Mode Coupling Theory (EOM)
Time Blocking

$$\Phi_{12,34}(\omega) = - \sum_{5678,\eta,m} \gamma_{12}^{m56(\eta)} A_{56,78}^{(\eta)}(\omega - \eta \omega_m) \gamma_{34}^{m78(\eta)*}$$

Replacement of the uncorrelated propagator inside the Φ amplitude by a correlated one

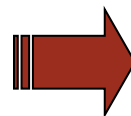


Nuclear response: $R = A + A (V + \bar{\Phi} - \bar{\Phi}_0) R$

Poles may appear at lower energies:

'2q+phonon' response:

$$\Phi_{ijj'}(\omega) \sim \sum_{\mu k} \alpha_{ijk\mu} / (\omega - E_{i'} - E_k - \Omega_\mu)$$

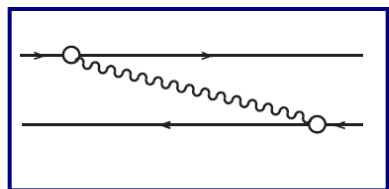


'2 phonon' response:

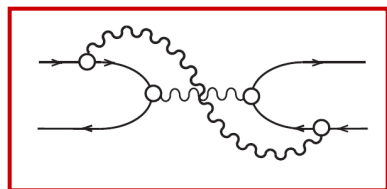
$$\Phi_{ijj'}(\omega) \sim \sum_{\mu\nu} \alpha_{ijj'\nu} / (\omega - \Omega_\nu - \Omega_\mu)$$

Fine features of dipole spectra: two-phonon effects

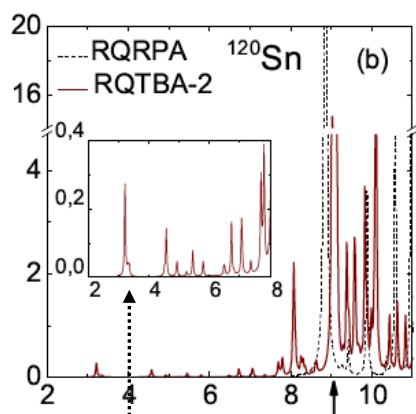
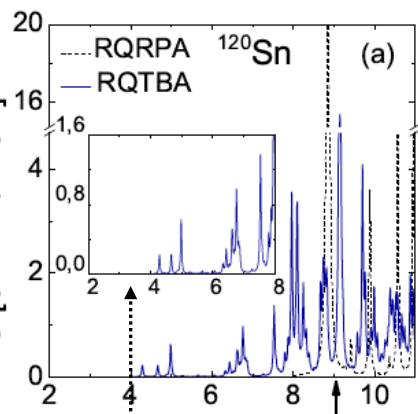
2q+phonon



2 phonon



^{120}Sn



2-phonon state does not exist

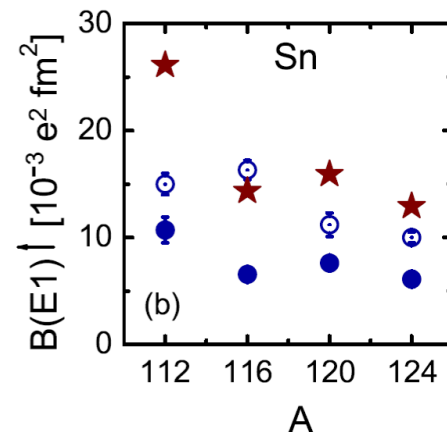
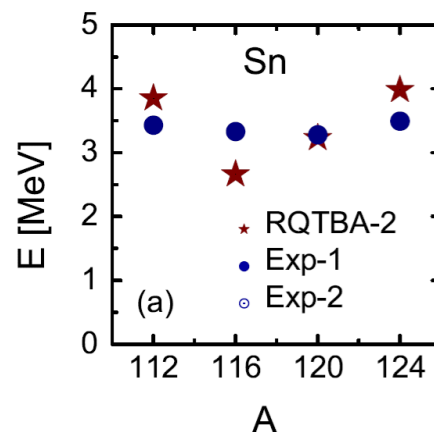
$3^- \otimes 2^+$

E.L., P.Ring, V.Tselyaev, *PRL* 105, 02252 (2010)
PRC 88, 044320 (2013)

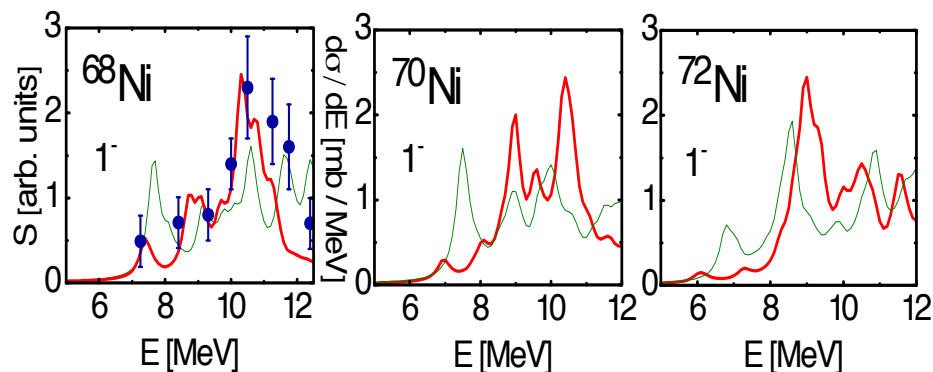
First two-phonon state 1^-_1 : $3^- \otimes 2^+$

$E(1^-_1)$

$B(E1) \uparrow$



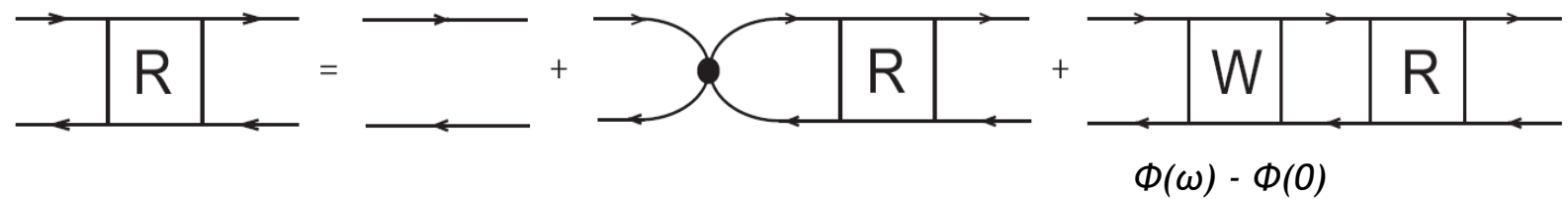
Pygmy dipole resonance in neutron-rich Ni:
2q+phonon vs 2 phonon



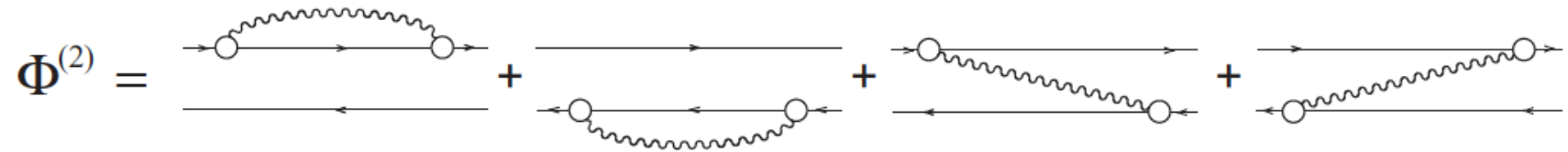
Data: O. Wieland et al., *PRL* 102, 092502 (2009)

Higher orders: toward a unified description of high-frequency oscillations and low-energy spectroscopy

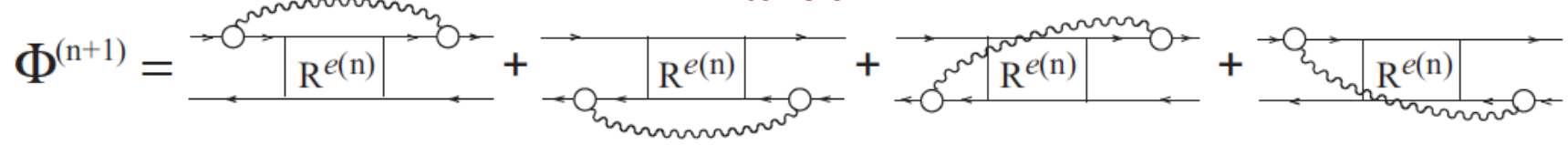
Bethe-Salpeter Equation:



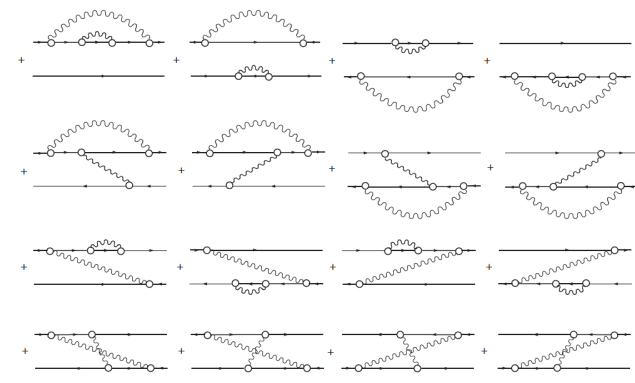
“Conventional” NFT:



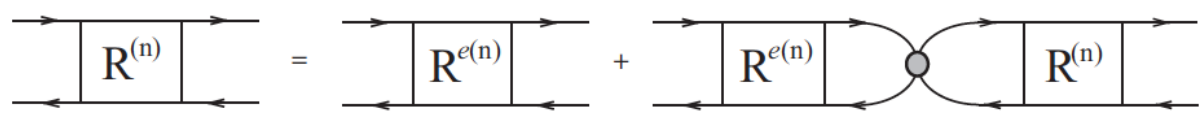
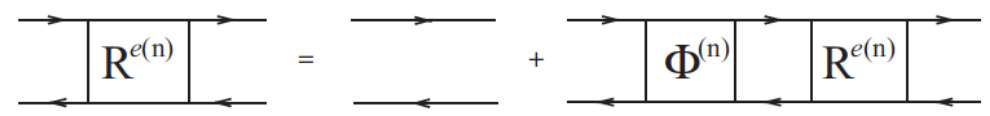
Extension:



$n = 3: 3p3h$ (“3body”)



n -th order correlated propagator:



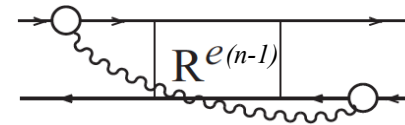
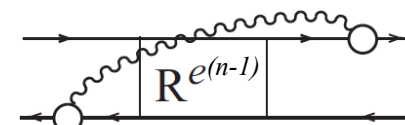
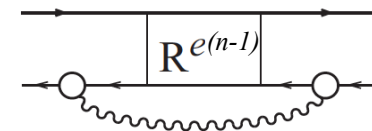
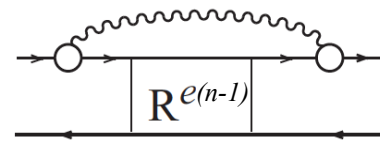
$V - \Phi^n(0)$

Convergence: geometrical factors

Amplitude $\Phi(\omega)$ in a coupled form (spherical basis):

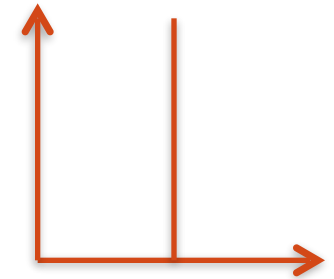
$$\Phi_{(k_1 k_4, k_2 k_3)}^{(n)J, \eta}(\omega) = \frac{(-1)^{j_1 + j_2 + j_3 + j_4}}{2J + 1} \sum_{(\mu) J_e} \times$$

$$\left[\sum_{(k_1' k_3')} \gamma_{(\mu; k_1 k_1')}^\eta R_{(k_1' k_2, k_3' k_4)}^{e(n-1)J_e, \eta}(\omega - \eta\omega_\mu) \gamma_{(\mu; k_3 k_3')}^{\eta*} \times \begin{Bmatrix} J & J_\mu & J_e \\ j_1' & j_2 & j_1 \end{Bmatrix} \begin{Bmatrix} J & J_\mu & J_e \\ j_3' & j_4 & j_3 \end{Bmatrix} + \sum_{(k_2' k_4')} \gamma_{(\mu; k_2' k_2)}^\eta R_{(k_1 k_2', k_3 k_4')}^{e(n-1)J_e, \eta}(\omega - \eta\omega_\mu) \gamma_{(\mu; k_4' k_4)}^{\eta*} \times \begin{Bmatrix} J & J_\mu & J_e \\ j_2' & j_1 & j_2 \end{Bmatrix} \begin{Bmatrix} J & J_\mu & J_e \\ j_4' & j_3 & j_4 \end{Bmatrix} - \sum_{(k_1' k_4')} \gamma_{(\mu; k_1 k_1')}^\eta R_{(k_1' k_2, k_3 k_4')}^{e(n-1)J_e, \eta}(\omega - \eta\omega_\mu) \gamma_{(\mu; k_4' k_4)}^{\eta*} \times \begin{Bmatrix} J & J_\mu & J_e \\ j_1' & j_2 & j_1 \end{Bmatrix} \begin{Bmatrix} J & J_\mu & J_e \\ j_4' & j_3 & j_4 \end{Bmatrix} - \sum_{(k_2' k_3')} \gamma_{(\mu; k_2' k_2)}^\eta R_{(k_1 k_2', k_3' k_4)}^{e(n-1)J_e, \eta}(\omega - \eta\omega_\mu) \gamma_{(\mu; k_3 k_3')}^{\eta*} \times \begin{Bmatrix} J & J_\mu & J_e \\ j_2' & j_1 & j_2 \end{Bmatrix} \begin{Bmatrix} J & J_\mu & J_e \\ j_3' & j_4 & j_3 \end{Bmatrix} \right]$$

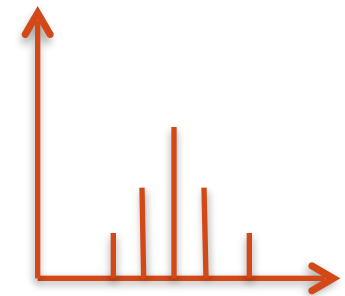


Fragmentation:

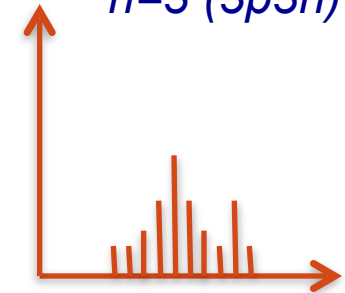
$n=1$ (1p1h)



$n=2$ (2p2h)



$n=3$ (3p3h)



...



Outlook

Summary:

- *Relativistic NFT offers a powerful framework for a high-precision solution of the nuclear many-body problem*
- *A non-perturbative self-consistent response theory based on QHD and including high-order correlations is available for a large class of nuclear excited states*
- *RNFT allows for a wide range of applications to nuclear structure and astrophysics*

Current and future developments:

- *Dynamical like-particle and proton-neutron pairing*
- *Higher-order and complex ground-state correlations (see talk of Caroline Robin)*
- *Covariant response theory for deformed nuclear systems*
- *Toward a completely ab initio description: realization of the energy-dependent G-matrix approach based on the relativistic meson-exchange potential*



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