Consistent Quasiparticle Random Phase Approximation for deformed systems

D.S. Delion
Horia Hulubei National Institute of Physics and Nuclear Engineering (IFIN-HH)
Bucharest-Măgurele, Romania

Extreme Light Infrastucture - Nuclear Physics (ELI-NP) Bucharest-Măgurele

Outline

I. Intrinsic versus laboratory systems of coordinates

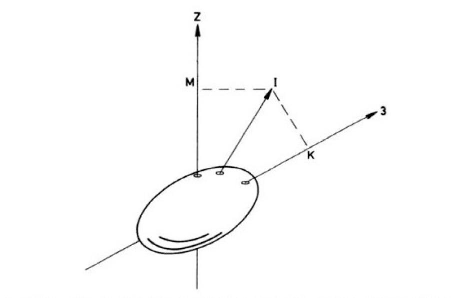
II. Projected equations of motion

III. Electromagnetic transitions in even-even nuclei

IV. Conclusions

I. Intrinsic versus laboratory systems of coordinates

The intrinsic system of coordinates is connected
to the symmetry axis (3) of an axially deformed nucleus.
I is the angular momentum of the rotating nucleus.
K is the the projection of **I** in the intrinsic system.
M is the the projection of **I** in the laboratory system.



Quantum rotation of an axially symmetric top

is described by the sum of normalized Wigner rotation functions with opposite intrinsic projections

$$\Phi^{I}_{MK}(\Omega) = \frac{1}{\sqrt{2(1+\delta_{K0})}} \left[\mathcal{D}^{I*}_{MK}(\Omega) + (-)^{I+K} \mathcal{D}^{I*}_{M-K}(\Omega) \right] . \qquad (1)$$

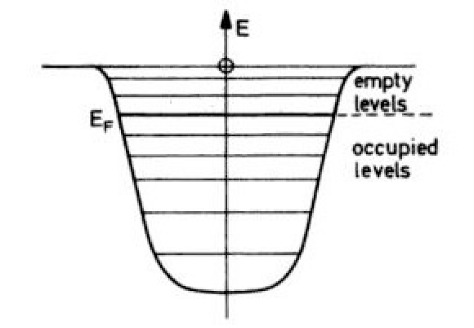
$K = 0$ corresponds to the **ground rotational band**.

In a spherical nuclear mean field

protons and neutrons occupy

sigle particle orbitals $c^{\dagger}_{njm}$,

n labels the level, $j = l + s$ total spin and m its projection.



In a deformed nuclear mean field

the intrinsic nucleon wave function with a given projection K
(**called intrinsic Nilsson wave functions**)
is a superposition of spherical orbitals
with the same total spin projection K

$$b^{\dagger}_{kK} = \sum_{nj} \mathbf{d}^{nj}_{kK} c^{\dagger}_{njK} \ , \qquad (2)$$

where k denotes the deformed eigenvalue index.

One can use the short-hand notations
$(nj) \equiv j \ , \quad (kK) \equiv K.$

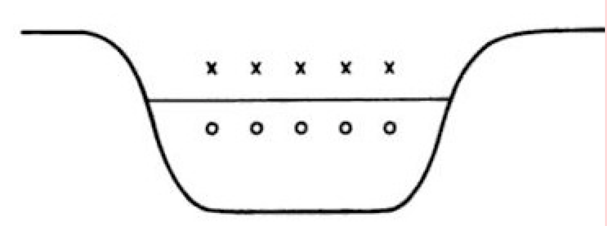
Standard description
of collective excitations in the intrinsic system
is given by the deformed RPA phonon operator
which is a coherent suporposition

$$\Gamma^{\dagger}_{K} \;=\; \sum_{ph} \left(X_{ph}\,\delta\Gamma^{\dagger}_{ph} - Y_{ph}\,\delta\Gamma_{ph} \right) \;, \qquad (3)$$

of particle-hole (p-h) excitations

$$\delta\Gamma^{\dagger}_{ph} \;\equiv\; b^{\dagger}_{p} b_{h} \;. \qquad (4)$$

Here, p is a particle state (x), h is a hole state (o) and
$K = K_p + K_h$ is the intrinsic pair projection.



The measured state has a given angular momentum

$$\Psi_{IMK} = \int d\Omega \mathcal{D}^{I*}_{MK} \Gamma^\dagger_K |0\rangle \ , \tag{5}$$

where $\mathcal{D}^{I}_{MK}(\Omega)$ is the normalized Wigner function depending on the Euler angles between laboratory and intrinsic system of coordinates.

In all calculations it is considered only the projection of the collective excitation $\Gamma^\dagger$ (because it is easy to extract the projected pairs $[a^\dagger_{I_p} \otimes a_{I_h}]_I$) **while the deformed vacuum $|0\rangle$ is approximated as spherical.**

Moreover, this a ”projection after variation” procedure.

We propose a new ”variation after projection procedure”

by using a single particle basis with good angular momentum and a consistent RPA vacuum state.

Total wave wave function in the intrinsic system

is a symmetrised product between rotational function with spin *I* and single particle (Nilsson) wave function

$$a^{\dagger}_{IMK}(\Omega) = \frac{1}{\sqrt{2(1+\delta_{K0})}} \left[\mathcal{D}^{I*}_{MK}(\Omega) b^{\dagger}_{K} + (-)^{I+K} \mathcal{D}^{I*}_{M-K}(\Omega) b^{\dagger}_{-K}\right] , \qquad (6)$$

In the configuration space the Nilsson wave function

$$\phi_K(\mathbf{r}') = \langle \mathbf{r}' | b^{\dagger}_{K} \rangle , \qquad (7)$$

depends upon the intrinsic coordinate $\mathbf{r}'$.

By rotating $b_K^\dagger$ to the laboratory system

one obtains a single particle operator with good angular momentum as a superposion of spherical orbitals "dressed" by deformation. For $K = 0$ one gets

$$a_{IM}^\dagger(\Omega) \rightarrow \sum_{J=even} \sum_{j \geq I} \mathcal{X}_I^{Jj} \left[\mathcal{D}_{.0}^{J*}(\Omega) \otimes c_j^\dagger \right]_{IM} , \qquad (8)$$

where by dot we denoted angular momentum projection coupling. The coefficient is proportional to the intrinsic Nilsson coefficient

$$\mathcal{X}_I^{Jj} = \sqrt{2} \langle II; j - I | J0 \rangle \mathbf{d}_I^j . \qquad (9)$$

Therefore the spin I plays role of the "spin projection" in lab. system.

We build a deformed many-body system

by using these single particle operators
satisfying the anticommutation rule
obtained by integrating over Euler angles

$$\int d\Omega \left\{ a_{IM}(\Omega), a^{\dagger}_{IM}(\Omega) \right\} = \delta_{II'} \delta_{MM'} \ . \qquad (10)$$

**Any product of deformed operators $a^{\dagger}_{IM}$ becomes
a superposition of products between spherical orbitals $c^{\dagger}_{jm}$
because the product of Wigner functions
becomes a superposition of Wigner functions.**

Spherical operator with multipolarity λ
in the deformed basis is given by

$$Q_{\lambda\mu} = \sum_{I_1 I_2} \frac{(I_1||Q_\lambda||I_2)}{\sqrt{2\lambda+1}} \left[a^\dagger_{I_1} \otimes \tilde{a}_{I_2}\right]_{\lambda\mu} . \tag{11}$$

One obtains the following rule expressing
the deformed reduced matrix element integrated over Euler angles
in terms of standard spherical reduced matrix element

$$(I_1||Q_\lambda||I_2) = \sum_{Jj_1j_2} \mathcal{X}^{Jj_1j_2}_{I_1I_2} \langle j_1||Q_\lambda||j_2\rangle , \tag{12}$$

where the two-particle expansion coefficient contains
the product of single particle coefficients $\mathcal{X}^{Jj_1}_{I_1}\mathcal{X}^{Jj_2}_{I_2}$.

In particular, by considering the leading $J = 0$ term, one obtains in the laboratory system of coordinates for the number of particles operator

$$\mathcal{N}_l = \frac{2\left(\mathbf{d}_l^l\right)^2}{2l+1} \sum_M a_{lM}^\dagger a_{lM} \equiv S_l N_l \ , \tag{13}$$

and the pair operator

$$\mathcal{P}_l^\dagger = \frac{2\left(\mathbf{d}_l^l\right)^2}{2l+1} \sum_M a_{lM}^\dagger a_{l-M}^\dagger (-)^{l-M} \equiv S_l P_l^\dagger \ . \tag{14}$$

S_l are called statistical factors:
two particles ocuppy $(2l+1)/\left(\mathbf{d}_l^l\right)^2 \approx 2l+1$
projections in the lab. system.

II. Projected RPA equations of motion

Deformed phonon is defined by

$$\Gamma^{\dagger}_{\nu} = \sum_{l_1 l_2} \left(X^{\nu}_{l_1 l_2} \delta\Gamma^{\dagger}_{l_1 l_2} - Y^{\nu}_{l_1 l_2} \delta\Gamma_{l_1 l_2} \right) \ , \tag{15}$$

Basis building blocks can be considered for

I. Non-coupled case
of p-h excitations with a given multipolarity

$$\delta\Gamma^{\dagger}_{l_1 l_2} = \left[a^{\dagger}_{l_1} \otimes \tilde{a}_{l_2} \right]_{\lambda\mu} \ . \tag{16}$$

II. Coupled case
of p-h excitations with the core motion

$$\delta\Gamma^{\dagger}_{l_1 l_2} = \sum_{J_1 J_2} \left\{ \mathcal{D}^{J_1 *}_{.0} \otimes \left[a^{\dagger}_{l_1} \otimes \tilde{a}_{l_2} \right]_{J_2} \right\}_{\lambda\mu} \tag{17}$$

Standard equation of motion

$$[H, \Gamma_\nu^\dagger] = \omega_\nu \Gamma_\nu^\dagger \ , \tag{18}$$

leads to **projected equations**
by considering double commutators with basis operators
and by integrating over Euler angles

$$\begin{aligned} \int d\Omega[\delta\Gamma_{I_1 I_2}, [H, \Gamma_\nu^\dagger]] &= \omega_\nu \int d\Omega[\delta\Gamma_{I_1 I_2}, \Gamma_\nu^\dagger] \\ \int d\Omega[\delta\Gamma_{I_1 I_2}^\dagger, [H, \Gamma_\nu^\dagger]] &= \omega_\nu \int d\Omega[\delta\Gamma_{I_1 I_2}^\dagger, \Gamma_\nu^\dagger] \ . \end{aligned} \tag{19}$$

dRPA equations

have a similar to the spherical case form

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B}^* & -\mathcal{A}^* \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} . \qquad (20)$$

dRPA matrices

are given by the relations

$$\begin{aligned} \mathcal{A}_{l_1 l_2, l_1' l_2'} &= \int d\Omega \left[\delta\Gamma_{l_1 l_2}, \left[H, \delta\Gamma^\dagger_{l_1' l_2'} \right] \right] \\ \mathcal{B}_{l_1 l_2, l_1' l_2'} &= -\int d\Omega \left[\delta\Gamma^\dagger_{l_1 l_2}, \left[H, \delta\Gamma^\dagger_{l_1' l_2'} \right] \right] . \end{aligned} \qquad (21)$$

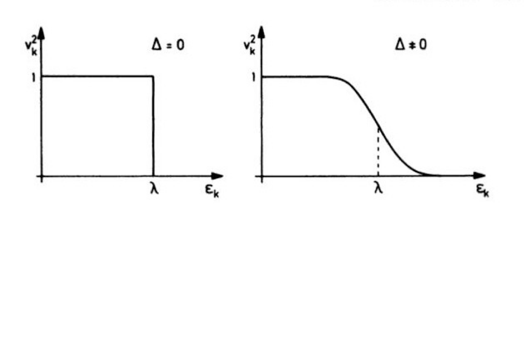
Deformed quasiparticle RPA (dQRPA)

considers pairing correlations

$$\begin{aligned} \alpha^\dagger_{lM} &= u_l a^\dagger_{lM} - v_l a_{l-M} (-)^{l-M} \\ &= u_l a^\dagger_{lM} + v_l \tilde{a}_{lM} \ . \end{aligned} \tag{22}$$

Occupation probability

for normal and superfluid systems



We can consider two versions of the dQRPA basis building blocks:

I. Non-coupled case

of p-h quasiparticle excitations with a given multipolarity

$$\delta\Gamma^{\dagger}_{l_1 l_2} = \left[\alpha^{\dagger}_{l_1} \otimes \alpha^{\dagger}_{l_2}\right]_{\lambda\mu} \to u_{l_1} v_{l_2} \left[a^{\dagger}_{l_1} \otimes \tilde{a}_{l_2}\right]_{\lambda\mu} + v_{l_1} u_{l_2} \left[\tilde{a}_{l_1} \otimes a^{\dagger}_{l_2}\right]_{\lambda\mu} \ . \quad (23)$$

II. Coupled case

of p-h quasiparticle excitations with the core motion

$$\delta\Gamma^{\dagger}_{l_1 l_2} = \sum_{J_1 J_2} \left\{ \mathcal{D}^{J_1 *}_{.0} \otimes \left[\alpha^{\dagger}_{l_1} \otimes \alpha^{\dagger}_{l_2}\right]_{J_2} \right\}_{\lambda\mu} \ . \quad (24)$$

III. Electromagnetic transitions in even-even nuclei

Hamiltonian
contains single particle + pairing
for protons (π) and neutrons (ν)
+ quadrupole-quadrupole (QQ) interaction
for $\pi - \pi, \ \nu - \nu, \ \pi - \nu$ systems

$$H = \sum_{\tau=\pi\nu} \left[\sum_{I} (\epsilon_I(\tau) - \lambda_\tau) \mathcal{N}_I(\tau) - \frac{G_\tau}{4} \sum_{I_1 I_2} \mathcal{P}^\dagger_{I_1}(\tau) \mathcal{P}_{I_2}(\tau) \right] - \frac{1}{2} \sum_{\tau\tau'} F_{\tau\tau'} \sqrt{5} [Q_2(\tau) \otimes Q_2(\tau')]_0 \ . \quad (25)$$

We considered the non-coupled case in the dQRPA basis

$$\delta\Gamma^\dagger_{2\mu} = \left[a^\dagger_{I_1} \otimes a^\dagger_{I_2} \right]_{2\mu} \ . \quad (26)$$

Parameters

Single particle spectrum $\epsilon_I(\tau)$ is generated
by a deformed Woods-Saxon mean field
with universal parametrisation

λ_τ are determined by number of protons and neutrons conservation

G_τ are determined by experimental pairing gaps

$F_{\pi\pi} = F_{\nu\nu} = F_{\pi\nu}$ are determined by
experimental E_2 energies.

**Thus, B(E2)-values (reduced transition probabilities)
with a common polarisation parameter $\chi = 0.2$ are predicted
in all even-even superfluid nuclei with $Z > 50$.**

Strength function versus excitation energy

D.S. Delion and J. Suhonen, Phys. Rev. C **87**, 024309 (2013).

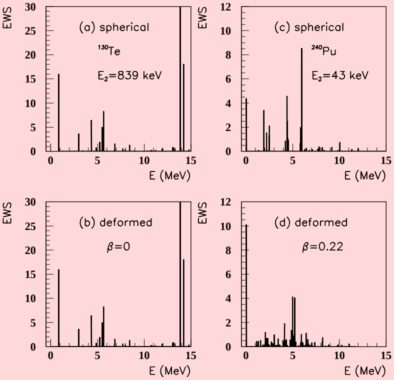
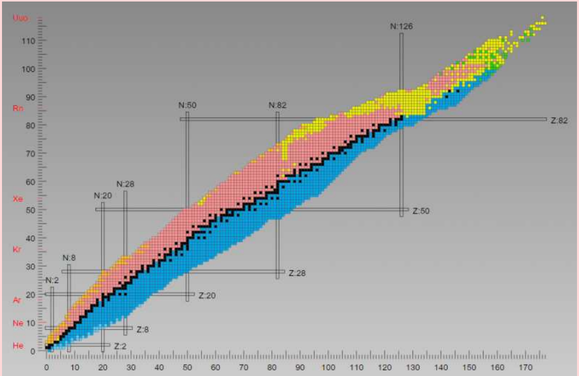


Figure : 1.

We investigated $E:2^+ \rightarrow 0^+$ transitions
in superfluid even even nuclei for two regions
$50 < Z < 82$ (open circles)
$Z > 82$ (dark circles)

Nuclear chart with magic numbers



B(E2) values versus excitation energy

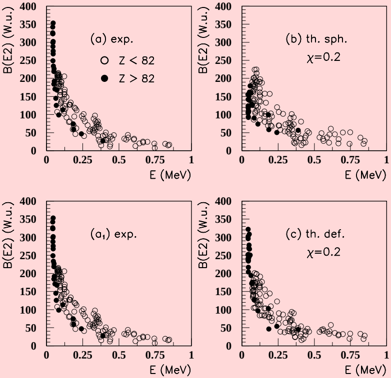


Figure : 2.

B(E2) values versus quadrupole deformation

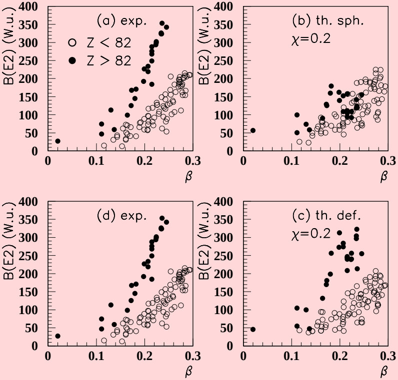


Figure : 3.

Exp./Theor. values versus excitation energy

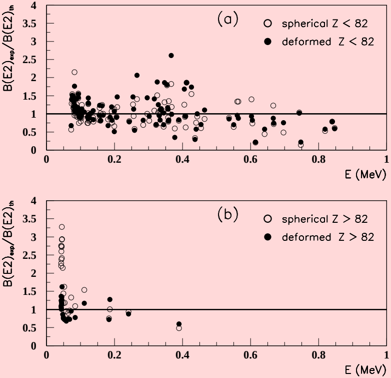


Figure : 4.

QQ coupling strengh versus mass number

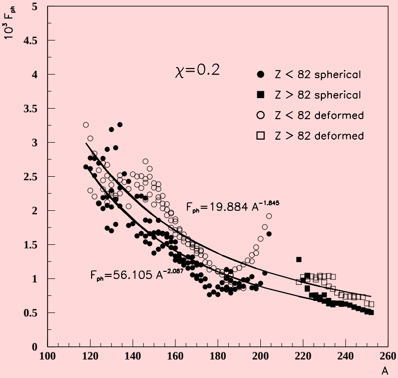


Figure : 5.

IV. Conclusions

1) Many-body deformed systems can be described in terms of single particle states with good angular momentum in the laboratory system of coordinates.

2) dRPA equations have a similar to the spherical case form.

3) Electromagnetic transitions in all superfluid even-even nuclei are described within dQRPA with a reasonable accuracy
D.S. Delion and J. Suhonen, Phys. Rev. C **87**, 024309 (2013).

4) dQRPA was extended to proton-neutron excitations (pn-dQRPA) to analyze Gamow-Teller resonances and $2\nu\beta\beta$ decays
D.S. Delion and J. Suhonen, Phys. Rev. C **91**, 054329 (2015);
Phys. Rev. C **95**, 034334 (2017).

THANK YOU !