$\mathcal{H}$ $\quad \dim \mathcal{H} = m$

$\varphi_1, \ldots, \varphi_m$

P_m set of all subsets $\{1, \ldots, m\}$

$P_{m,n}$ " " " n-elements " "

$I \in P_{m,n}$, $I = \{i_1, \ldots, i_n\}$ $\quad i_1 < \cdots < i_n$

$$\Phi_I = a^\dagger(\varphi_{i_1}) \cdots a^\dagger(\varphi_{i_n})|0\rangle \in F_n$$

$$= \varphi_{i_1} \wedge \cdots \wedge \varphi_{i_n} \in \Lambda^n \mathcal{H}$$

$$\Psi = \sum_{I \in P_{m,n}} \lambda_I \Phi_I$$

$$a^\dagger(\Psi) = \sum_{I \in P_{m,n}} \lambda_I \, a^\dagger(\varphi_{i_1}) \cdots a^\dagger(\varphi_{i_n})$$

$$\mathcal{H} = I(\Psi) \oplus E(\Psi)$$

$E(\Psi) = \mathrm{Ker}(\mathrm{1RDM})$

external space

$I(\Psi)$

internal space

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A "new" Math techique to simplify Group functions

Ultimate goal

$$\Psi = \Psi_1 \wedge \cdots \wedge \Psi_k$$

$\in \bigwedge^{n_1} \mathcal{H}$ $\quad\quad$ $\in \bigwedge^{n_k} \mathcal{H}$

$$n_1 + \cdots + n_k = n$$

$$S = \left\{ (\Psi_1', \ldots, \Psi_k') \in \bigwedge^{n_1} \mathcal{H} \times \cdots \times \bigwedge^{n_k} \mathcal{H} \text{ such that } \Psi_1' \wedge \cdots \wedge \Psi_k' = \Psi \right\}$$

$$S \neq \emptyset$$

Ex1 $n_1 = 1$ $k = n$

$$\Psi = \psi_1 \wedge \dots \wedge \psi_n$$

$$S = \left\{ \begin{array}{l} (u(\psi_1), \dots, u(\psi_n)) \\ u \text{ linear} \quad \det u = 1 \end{array} \right\}$$

$\dim \mathcal{I}(\Psi_i) = 2n$

$\dim \mathcal{I}(\Psi_i') = 2$

$i \neq j$ $\mathcal{I}(\Psi_i) = \mathcal{I}(\Psi_j)$

$\mathcal{I}(\Psi_i) \perp \mathcal{I}(\Psi_j)$

Ex2 $n_1 = 2$ AGP

$$g = \sum_{i=1}^{n} \frac{\psi_{2i-1} \wedge \psi_{2i}}{(n!)^{\frac{1}{n}}}$$

$$\Psi_i = g$$

$$\Psi = \underbrace{g \wedge \dots \wedge g}_{n \text{ times}}$$

$$\Psi_i' = u(\psi_{2i-1}) \wedge u(\psi_{2i})$$

...fy Group functions

$\Rightarrow \quad S_i = \{ (\Psi_1, \dots, \Psi_{i-1}, \Psi_i', \Psi_{i+1}, \dots, \Psi_k) \}$

$\Psi_1' \wedge \Psi_i' \wedge \Psi_k'' = \Psi \qquad (E)$

$i \quad \Psi_i^{(0)} \longrightarrow \Psi_i^{(1)} \quad , \; i \to i+1$

(E) $\Psi_1 \wedge \cdot \wedge \Psi_{i-1} \wedge (\Psi_i - \Psi_i') \wedge \Psi_{i+1} \wedge \cdot \wedge \Psi_k = 0$

$i=1 \quad A_1 = \{ \Psi_1'' , \Psi_1'' \wedge \Psi_2 \wedge \quad \wedge \Psi_k = 0 \}$

$A \, (\Psi_2 \wedge \quad \wedge \Psi_k = 0 \}$

Ψ₁ = Σ_{I∈P_{m,n₁}} λ_I Φ_I = Σ_{i=1}^{N} λ_i Φ_i
(m n₁)
∧Ψ_k = Σ_{k∈P_{m,n}} Φ_k Σ_{I,J; I∪J=K} e_{I,J} μ_J λ_I
= 0
e_{I,K\I} μ_{K\I}
= 0
Ultimat
Ψ =
S = {
S ≠

...fy Group functions

$$\Psi = R_1(\Psi) + Q_1(R_2(\Psi)) + \cdots + Q_{1 \cdots k}(R_{k+1}) + \cdots + Q_{1 \cdots N}(\Psi)$$

$\ldots_1, \ldots, R_N$

$\forall \Psi$
$$\begin{cases} Q_i(R_i) = 0 \\ R_i - Q_j(R_i) = R_j - Q_i(R_j) \end{cases}$$
Structure eq

$$F \wedge \Psi = 0 = \sum_{i=1}^{N} \underbrace{\lambda_i R_i(F)}_{\nu_i} \wedge \Phi_i$$

$$0 = \sum_{i=1} \nu_i \wedge \Phi_i$$

$$\Psi_1 = \sum_{I \in P_{m,n_1}} \lambda_I \Phi_I = \sum_{i=1}^{N} \lambda_i \Phi_i$$

$$\Psi = Q_i(\Psi) + R_i(\Psi)$$

$$R_1 = \tilde{a}(\Phi_i) a^{\dagger}(\Phi_i)$$

$$Q_j(Q_i(\Psi)) = Q_i(Q_j(\Psi)) = Q_{ij}(\Psi)$$

$$Q_{i_1 \; i_2}$$

Simplify Group functions

$$\Psi = R_1(\Psi) + Q_1(R_2(\Psi)) + \dots + Q_{\dots}(R_{n+1}) + \dots$$

$$Q_{1\dots N}(\Psi)$$

$R_1, \dots, R_N$

$\exists \Psi$

$$\begin{cases} Q_i(R_i) = 0 \\ R_i - Q_j(R_i) = R_j - Q_i(R_j) \end{cases}$$

Structure eq

$\wedge^{n_e} \mathcal{H}$

Ψ

$$F \wedge \Psi = 0 = \sum_{i=1}^{N} \lambda_i R_i(F) \wedge \Phi_i$$

$$0 = \sum_{i=1} \nu_i \wedge \Phi_i$$

$$\Psi_1 = \sum_{I \in P_{m,n_1}} \lambda_I \, \Phi_I = \sum_{i=1}^{N} \lambda_i \, \Phi_i$$

$$g^n = \underbrace{g \wedge \cdots \wedge g}_{n-1} \wedge g' \ldots$$

$$g' = \left(\sum \varphi_{2i}\right) \wedge \left(\sum \varphi_{2i+1}\right)$$

$$g' = \varphi_1 \wedge \varphi_2$$