Generalized Seniority and conicity

$\mathcal{H} = V_1 \oplus V_2 \oplus \cdots \oplus V_k$ $\quad k$ "Shells

$2d_1 + 2d_2 + \cdots + 2d_k = 2m$

$\chi_{1,1} \; \chi_{1,2d_1} \qquad \chi_{k,1} \; \chi_{k,2d_k}$

Ex: $d_i = 1 \quad k = m \quad \chi_{i,1} = \bar{\chi}_{i,2}$ "primitive shell"

$\Phi_i = \chi_{i,1} \wedge \cdots \wedge \chi_{i,2d_i} = a^+(\chi_{i,1}) \cdots a^+(\chi_{i,2d_i}) |0\rangle$

$= a^+(\Phi_i) |0\rangle$

$$\forall i \quad \mathrm{Id}\,\Psi = Q_i(\Psi) + R_i(\Psi)$$

$$Q_i(\Psi) = \mathring{Q}_i(\Psi) + a^\dagger(\Phi_i)\,\tilde{a}(\Phi_i)$$

$$\forall i \quad \mathring{Q}_i = \mathrm{Id} - \tilde{a}(\Phi_i)\,a^\dagger(\Phi_i) - a^\dagger(\Phi_i)\,\tilde{a}(\Phi_i)$$

0 fully occupied or unoccupied

1 partially occupied

$$\hat{\Omega} = \sum_i \mathring{Q}_i$$ generalized seniority number operator

$$\hat{N} = \sum_i a^\dagger(\Phi_i)\,\hat{a}(\Phi_i)$$ " ion