

Multi-reference Energy Density Functional calculations for nuclei

Constraints from consistency requirements

T. Duguet^{1,2}

¹DSM/Irfu/SPhN, CEA Saclay, France

²National Superconducting Cyclotron Laboratory,
Department of Physics and Astronomy, Michigan State University, USA

Multidisciplinary workshop on RPA, Paris, January 26th-29th, 2010



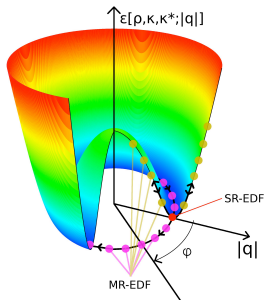
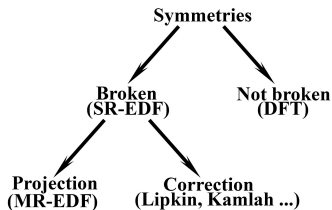
Outline

- 1 Introduction
 - Main ingredients of the nuclear EDF method
 - Ground-state correlations et collective excitations
 - Link with QRPA
- 2 Constraining the MR-EDF method to avoid pathologies
 - Basic consistency requirements
 - Unexpected pathologies
 - Regularization method
- 3 Conclusions
- 4 Bibliography

Talk in one slide

Context

- 1 Two-step nuclear EDF method (i) single-reference (ii) multi-reference
- 2 Built by analogy with wave-function based methods (no existence theorem)
- 3 SR-EDF has both similarities and differences with (standard) DFT
- 4 Strongly relies on spontaneous symmetry breaking and restoration



Talk in one slide

Context

- 1 Two-step nuclear EDF method (i) single-reference (ii) multi-reference
- 2 Built by analogy with wave-function based methods (no existence theorem)
- 3 SR-EDF has both similarities and differences with (standard) DFT
- 4 Strongly relies on spontaneous symmetry breaking and restoration

Take-away message

- 1 MR-EDF tackles long-range correlations and accesses collective excitations
- 2 QRPA is recovered from the harmonic limit of MR-EDF
- 3 MR-EDF calculations must be constrained through consistency requirements

Outline

- 1 Introduction
 - Main ingredients of the nuclear EDF method
 - Ground-state correlations et collective excitations
 - Link with QRPA
- 2 Constraining the MR-EDF method to avoid pathologies
 - Basic consistency requirements
 - Unexpected pathologies
 - Regularization method
- 3 Conclusions
- 4 Bibliography

Outline

- 1 Introduction
 - Main ingredients of the nuclear EDF method
 - Ground-state correlations et collective excitations
 - Link with QRPA
- 2 Constraining the MR-EDF method to avoid pathologies
 - Basic consistency requirements
 - Unexpected pathologies
 - Regularization method
- 3 Conclusions
- 4 Bibliography

Ingredients of the nuclear EDF method

Two-level variational wave-function method

1 st level: HFB	2 nd level: projected HFB + GCM
Trial WF: $ \Phi_q\rangle = \prod_{\mu} \beta_{\mu}^q 0\rangle$	Trial WF: $ \Psi\rangle = \sum_q f_q \Phi_q\rangle = \sum_{ q } g_{ q } P_X \Phi_{ q }\rangle$
Sym. break. $q = q e^{i\varphi} \neq 0$	Sym. restor. (\sum_{φ}) and zero-point fluct. ($\sum_{ q }$)
$[\mathbf{X}, \mathbf{H}] = \mathbf{0}$ for $\{\mathbf{X}\} = \{\mathbf{N}, \mathbf{Z}, \mathbf{P}, \mathbf{J}^2, \mathbf{J}_z, \mathbf{T}^2, \mathbf{T}_z, \mathbf{T}^2\}$	
<i>Static</i> collective correlations	<i>Dynamical</i> collective correlations
$E_{ q }^{1\text{st}} = \langle \Phi_q H \Phi_q \rangle$ \Downarrow Standard Wick Theorem \Downarrow $\langle \Phi_q H \Phi_q \rangle = E[\rho^{qq}, \kappa^{qq}, \kappa^{qq*}]$ $\mathbf{E}_{ q }^{1\text{st}}$ is a functional of <i>diagonal density matrices</i> ρ^{qq}, κ^{qq} and κ^{qq*}	$E_X^{2\text{nd}} = \langle \Psi H \Psi \rangle = \sum_{qq'} f_q^* f_{q'} \langle \Phi_q H \Phi_{q'} \rangle$ \Downarrow Generalized Wick Theorem \Downarrow $\langle \Phi_q H \Phi_{q'} \rangle = E[\rho^{qq'}, \kappa^{qq'}, \kappa^{q'q*}] \langle \Phi_q \Phi_{q'} \rangle$ $\mathbf{E}_X^{2\text{nd}}$ invokes the SAME functional of <i>(all) transition density matrices</i> $\rho^{qq'}, \kappa^{qq'}$ and $\kappa^{q'q*}$

Ingredients of the nuclear EDF method

Two-level energy density functional method

1st level: single-reference

$$\text{Trial state } |\Phi_q\rangle = \prod_{\mu} \beta_{\mu}^q |0\rangle$$

$$\text{Sym. break. } q = |q| e^{i\varphi} \neq 0$$

$$[\mathbf{X}, \mathbf{H}] = 0 \text{ for } \{\mathbf{X}\} = \{\mathbf{N}, \mathbf{Z}, \mathbf{P}, \mathbf{J}^2, \mathbf{J}_z, \mathbf{T}^2, \mathbf{T}_z, \mathcal{T}^2\}$$

Static collective correlations

$$\mathcal{E}_{|q|}^{\text{SR}} \equiv \mathcal{E}[\Phi_q; \Phi_q]$$

$$\delta \left[\mathcal{E}^{\text{SR}} - \lambda \text{Tr}\{\rho\} - \lambda^{|\mathbf{q}|} \text{Tr}\{\rho \mathbf{Q}\} \right] = 0$$

↓
HFB-like equations

2nd level: multi-reference

$$\text{Trial set of states } \{|\Phi_q\rangle\} \neq |\Psi\rangle$$

$$\text{Sym. restor. } (\sum_{\varphi}) \text{ and zero-point fluct. } (\sum_{|q|})$$

Dynamical collective correlations

$$\mathcal{E}_X^{\text{MR}} \equiv \sum_{qq'} f_q^* f_{q'} \mathcal{E}[\Phi_q; \Phi_{q'}] \langle \Phi_q | \Phi_{q'} \rangle \neq \langle \Psi | H | \Psi \rangle$$

$$\text{Bulk of correlations resummed into } \mathcal{E}[\Phi_q; \Phi_{q'}] \equiv \mathcal{E}[\rho^{qq'}, \kappa^{qq'}, \kappa^{q'q*}]$$

$$\delta \mathcal{E}_X^{\text{MR}} / \delta \mathbf{f}_q^* = 0$$

↓
Hill-Wheeler-Griffin-like equations

Ingredients of the nuclear EDF method

Two-level energy density functional method

1st level: single-reference

$$\text{Trial state } |\Phi_q\rangle = \prod_{\mu} \beta_{\mu}^q |0\rangle$$

$$\text{Sym. break. } q = |q| e^{i\varphi} \neq 0$$

2nd level: multi-reference

$$\text{Trial set of states } \{|\Phi_q\rangle\} \neq |\Psi\rangle$$

$$\text{Sym. restor. } (\sum_{\varphi}) \text{ and zero-point fluct. } (\sum_{|q|})$$

Relevant questions

- ① Is the WF→EDF mapping efficient? Is it safe? How is it constrained?
- ② Is the GWT-inspired mapping $\mathcal{E}[\Phi_q; \Phi_{q'}] \equiv \mathcal{E}[\rho^{qq'}, \kappa^{qq'}, \kappa^{q'q*}]$ appropriate?
- ③ What is the connection of all that to QRPA?
- ④ What is the link to (Ensemble) DFT?

Bulk of correlations resummed into $\mathcal{E}[\Phi_q; \Phi_{q'}] \equiv \mathcal{E}[\rho^{qq'}, \kappa^{qq'}, \kappa^{q'q*}]$

$$\delta \left[\mathcal{E}^{\text{SR}} - \lambda \text{Tr}\{\rho\} - \lambda |q| \text{Tr}\{\rho \mathbf{Q}\} \right] = 0$$

↓
HFB-like equations

$$\delta \mathcal{E}_{\mathbf{X}}^{\text{MR}} / \delta \mathbf{f}_{\mathbf{q}}^* = 0$$

↓
Hill-Wheeler-Griffin-like equations

Outline

- 1 Introduction
 - Main ingredients of the nuclear EDF method
 - **Ground-state correlations et collective excitations**
 - Link with QRPA
- 2 Constraining the MR-EDF method to avoid pathologies
 - Basic consistency requirements
 - Unexpected pathologies
 - Regularization method
- 3 Conclusions
- 4 Bibliography

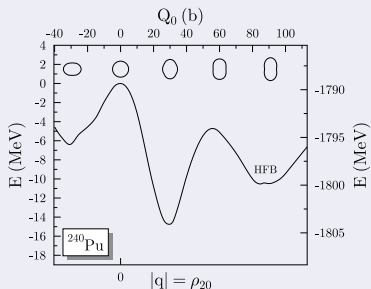
Ground-state correlation energy associated with $X = J^2$

Collective coordinates = "Order parameter"

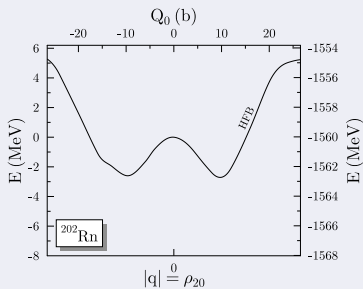
- 1 $|q|$ = multipole moments of $\rho(\vec{r})$
- 2 φ = Euler angles (α, β, γ)

From static deformation

$$\Delta\mathcal{E}_{|q|_{\min}}^{\text{SR}} = \text{Min}_{|q|} \left\{ \mathcal{E}_{|q|}^{\text{SR}} \right\} - \mathcal{E}_0^{\text{SR}}$$

Well-deformed nucleus for $|q| = \rho_{20}$ 

[M. Bender, private communication]

Transitional nucleus for $|q| = \rho_{20}$ 

[M. Bender, private communication]

Ground-state correlation energy associated with $X = J^2$

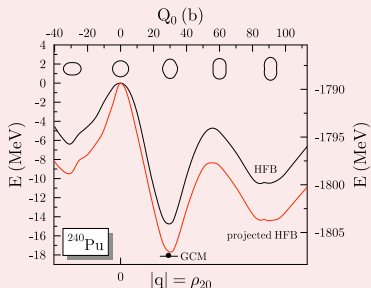
Collective coordinates = "Order parameter"

- 1 $|q|$ = multipole moments of $\rho(\vec{r})$
- 2 φ = Euler angles (α, β, γ)

From dynamical fluctuations

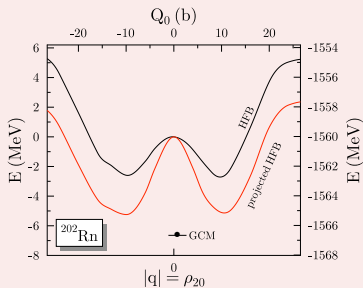
$$\Delta\mathcal{E}_{J=0}^{\text{MR}} = \mathcal{E}_{J=0}^{\text{MR}} - \text{Min}_{|q|} \left\{ \mathcal{E}_{|q|}^{\text{SR}} \right\}$$

Stiff nucleus for $|q| = \rho_{20}$



[M. Bender, private communication]

Soft nucleus for $|q| = \rho_{20}$

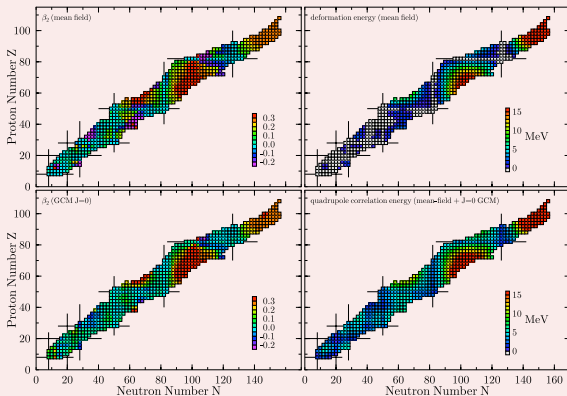


[M. Bender, private communication]

Ground-state correlation energy associated with $X = J^2$

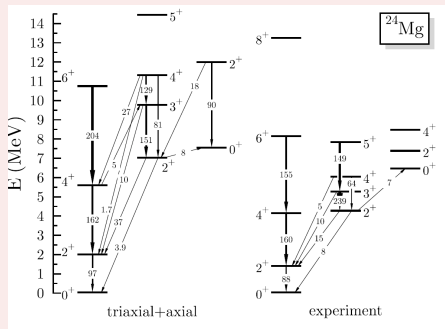
Systematic of quadrupole correlations: $\rho_{20} \neq 0 + J = 0 + \Delta\rho_{20}$

[M. Bender, G.-F. Bertsch, P.-H. Heenen, PRC73 (2006) 034322]



- ① Deformation/fluctuations dominant in heavy/light nuclei
- ② Right balance between open- and closed-shell nuclei from single EDF
- ③ Improve ground-state observables systematically, e.g. $\sigma_{2149}^{\text{mass}} = 800\text{keV}$

Collective excitations

Restoration of $N, Z, J^2, M + \Delta\rho_{20} + \Delta\rho_{22}$ 

[M. Bender, P.-H. Heenen, PRC78 (2008) 024309]

Energy spectrum

- Vibrational + rotational states
- Nicely aligned with experiment
- Too spread out spectrum

Electromagnetic transitions

- Restoration of (J^2, J_z) essential
- Selection rules recovered
- Good in- and out-band $B(E2)$

Outline

- 1 Introduction
 - Main ingredients of the nuclear EDF method
 - Ground-state correlations et collective excitations
 - **Link with QRPA**
- 2 Constraining the MR-EDF method to avoid pathologies
 - Basic consistency requirements
 - Unexpected pathologies
 - Regularization method
- 3 Conclusions
- 4 Bibliography

Link with QRPA

QRPA from Time-Dependent SR-EDF calculations

- ➊ **Adiabatic-type scheme** (omitting anomalous densities κ)

$$A_{minj} = (\epsilon_m - \epsilon_i) \delta_{mn} \delta_{ij} + \frac{\partial^2 \mathcal{E}[\rho]}{\partial \rho_{im} \partial \rho_{nj}} \quad ; \quad B_{minj} = \frac{\partial^2 \mathcal{E}[\rho]}{\partial \rho_{im} \partial \rho_{jn}}$$

- ➋ Extensions (e.g. second RPA...) needed to access *spreading width*

QRPA from Projection+GCM in WF method

- QRPA (WF)

$$A_{minj} = (\epsilon_m - \epsilon_i) \delta_{mn} \delta_{ij} + \bar{v}_{mjn} \quad ; \quad B_{minj} = \bar{v}_{mni}$$

- QRPA (WF) is recovered from the harmonic limit of GCM (WF)

[B. Jancovici, D. H. Schiff, NP58 (1964) 678]

- ➊ Use Thouless parameterization $|\Phi_q\rangle = \exp[\sum_{im} z_{mi}^{q*} a_m^\dagger a_i] |\Phi_0\rangle$
- ➋ Expand $\langle \Phi_q | H | \Phi_{q'} \rangle / \langle \Phi_q | \Phi_{q'} \rangle$ to second order in $\mathbf{z}^q / \mathbf{z}^{q'*}$
- ➌ Assume gaussian overlap $\langle \Phi_q | \Phi_{q'} \rangle \propto \exp[\text{Tr}(\mathbf{z}^q \mathbf{z}^{q'\dagger})]$

- HFB + GCM can tackle large amplitude *anharmonic* collective motions

Link with QRPA

QRPA from Time-Dependent SR-EDF calculations

- 1 **Adiabatic-type scheme** (omitting anomalous densities κ)

$$A_{minj} = (\epsilon_m - \epsilon_i) \delta_{mn} \delta_{ij} + \frac{\partial^2 \mathcal{E}[\rho]}{\partial \rho_{im} \partial \rho_{nj}} \quad ; \quad B_{minj} = \frac{\partial^2 \mathcal{E}[\rho]}{\partial \rho_{im} \partial \rho_{jn}}$$

- 2 Extensions (e.g. second RPA...) needed to access *spreading width*

QRPA from Projection+GCM in WF method

- QRPA (WF)

$$A_{minj} = (\epsilon_m - \epsilon_i) \delta_{mn} \delta_{ij} + \bar{v}_{mjn} \quad ; \quad B_{minj} = \bar{v}_{mni}$$

- QRPA (WF) is recovered from the harmonic limit of GCM (WF)

[B. Jancovici, D. H. Schiff, NP58 (1964) 678]

- 1 Use Thouless parameterization $|\Phi_q\rangle = \exp[\sum_{im} z_{mi}^{q*} a_m^\dagger a_i] |\Phi_0\rangle$
- 2 Expand $\langle \Phi_q | H | \Phi_{q'} \rangle / \langle \Phi_q | \Phi_{q'} \rangle$ to second order in $\mathbf{z}^q / \mathbf{z}^{q'*}$
- 3 Assume gaussian overlap $\langle \Phi_q | \Phi_{q'} \rangle \propto \exp[\text{Tr}(\mathbf{z}^q \mathbf{z}^{q'\dagger})]$

- HFB + GCM can tackle large amplitude *anharmonic* collective motions

Outline

- 1 Introduction
 - Main ingredients of the nuclear EDF method
 - Ground-state correlations et collective excitations
 - Link with QRPA
- 2 **Constraining the MR-EDF method to avoid pathologies**
 - **Basic consistency requirements**
 - **Unexpected pathologies**
 - **Regularization method**
- 3 Conclusions
- 4 Bibliography

Outline

- 1 Introduction
 - Main ingredients of the nuclear EDF method
 - Ground-state correlations et collective excitations
 - Link with QRPA
- 2 Constraining the MR-EDF method to avoid pathologies
 - **Basic consistency requirements**
 - Unexpected pathologies
 - Regularization method
- 3 Conclusions
- 4 Bibliography

Set of constraints from consistency requirements

SR-EDF [J. Dobaczewski, J. Dudek, APP B27 (1996) 45]

- $\mathcal{E}_{|q|}^{SR}$ must be real and a scalar under all $\mathcal{R}(g) \in$ symmetry group \mathcal{G}
- Rules to build local Skyrme EDF to 2^{nd} order in σ_ν and ∇

$$\mathcal{E}[\rho, \kappa, \kappa^*] \equiv \int d\vec{r} \mathcal{E}[\rho_T(\vec{r}), \tau_T(\vec{r}), J_{T,\mu\nu}(\vec{r}), \vec{s}_T(\vec{r}), \vec{j}_T(\vec{r}), \vec{T}_T(\vec{r}), \vec{F}_T(\vec{r}), \tilde{\rho}_T(\vec{r}); \nabla]$$

MR-EDF [L. Robledo, IJMP E16 (2007) 337; JPG: Nucl. Part. Phys. 37 (2010) in press]

- ① \mathcal{E}_X^{MR} must be real and a scalar under all $\mathcal{R}(g) \in$ symmetry group \mathcal{G}
- ② Consistency of MR and SR schemes
 - ① $\mathcal{E}_X^{MR} = \mathcal{E}_{|q|}^{SR}$ when $\{|\Phi_q\rangle\} \longrightarrow |\Phi_q\rangle$
 - ② Chemical potential λ must be recovered from Kamlah expansion of \mathcal{E}_N^{MR}
 - ③ QRPA must be recovered through harmonic limit of \mathcal{E}_X^{MR}

Extension to MR kernel must implicate *transition densities* only

Diagonal SR kernel

$$\mathcal{E}[\rho^{qq}, \kappa^{qq}, \kappa^{qq*}]$$

GWT-inspired connection
(only) viable option?

Off-diagonal MR kernel

$$\mathcal{E}[\rho^{qq'}, \kappa^{qq'}, \kappa^{q'q*}]$$

Set of constraints from consistency requirements

SR-EDF [J. Dobaczewski, J. Dudek, APP B27 (1996) 45]

- $\mathcal{E}_{|q|}^{SR}$ must be real and a scalar under all $\mathcal{R}(g) \in$ symmetry group \mathcal{G}
- Rules to build local Skyrme EDF to 2^{nd} order in σ_ν and ∇

$$\mathcal{E}[\rho, \kappa, \kappa^*] \equiv \int d\vec{r} \mathcal{E}[\rho_T(\vec{r}), \tau_T(\vec{r}), J_{T,\mu\nu}(\vec{r}), \vec{s}_T(\vec{r}), \vec{j}_T(\vec{r}), \vec{T}_T(\vec{r}), \vec{F}_T(\vec{r}), \tilde{\rho}_T(\vec{r}); \nabla]$$

MR-EDF [L. Robledo, IJMP E16 (2007) 337; JPG: Nucl. Part. Phys. 37 (2010) in press]

- ① \mathcal{E}_X^{MR} must be real and a scalar under all $\mathcal{R}(g) \in$ symmetry group \mathcal{G}
- ② Consistency of MR and SR schemes
 - ① $\mathcal{E}_X^{MR} = \mathcal{E}_{|q|}^{SR}$ when $\{|\Phi_q\rangle\} \longrightarrow |\Phi_q\rangle$
 - ② Chemical potential λ must be recovered from Kamlah expansion of \mathcal{E}_N^{MR}
 - ③ QRPA must be recovered through harmonic limit of \mathcal{E}_X^{MR}

Extension to MR kernel must implicate *transition densities* only

Diagonal SR kernel

$$\mathcal{E}[\rho^{q\bar{q}}, \kappa^{q\bar{q}}, \kappa^{q\bar{q}*}]$$

GWT-inspired connection
(only) viable option?

Off-diagonal MR kernel

$$\mathcal{E}[\rho^{q\bar{q}'}, \kappa^{q\bar{q}'}, \kappa^{q'\bar{q}*}]$$

Outline

- 1 Introduction
 - Main ingredients of the nuclear EDF method
 - Ground-state correlations et collective excitations
 - Link with QRPA
- 2 Constraining the MR-EDF method to avoid pathologies
 - Basic consistency requirements
 - **Unexpected pathologies**
 - Regularization method
- 3 Conclusions
- 4 Bibliography

Unexpected pathologies

The example of particle number restoration (PNR)

- MR set $\{|\Phi_\varphi\rangle \equiv e^{i\hat{N}\varphi}|\Phi_0\rangle; \varphi \in [0, 2\pi]\}$
- Real, scalar, particle-number-restored MR energy reads

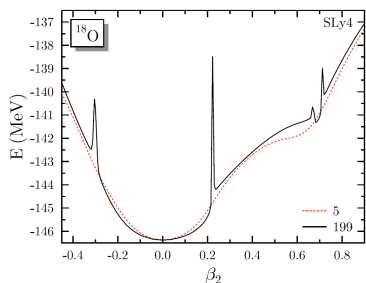
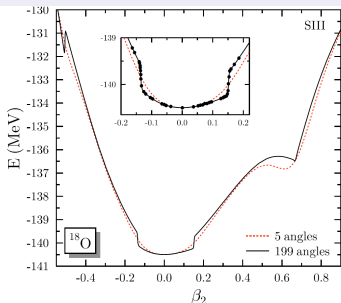
$$\mathcal{E}_N^{MR} \equiv \int_0^{2\pi} d\varphi \frac{e^{-i\varphi N}}{2\pi c_N^2} \mathcal{E}[\rho^{0\varphi}, \kappa^{0\varphi}, \kappa^{\varphi 0*}] \langle \Phi_0 | \Phi_\varphi \rangle$$

that corresponds to the Fourier decomposition of MR kernel on $U(1)$ Irreps

$$\begin{aligned} \mathcal{E}[\rho^{0\varphi}, \kappa^{0\varphi}, \kappa^{\varphi 0*}] \langle \Phi_0 | \Phi_\varphi \rangle &= \sum_{N \in \mathbb{Z}} c_N^2 \mathcal{E}_N^{MR} e^{iN\varphi} \\ \langle \Phi_0 | \Phi_\varphi \rangle &= \sum_{N \in \mathbb{Z}} c_N^2 e^{iN\varphi} \end{aligned}$$

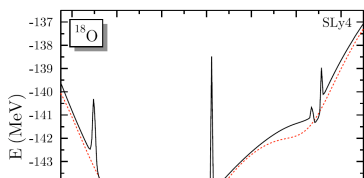
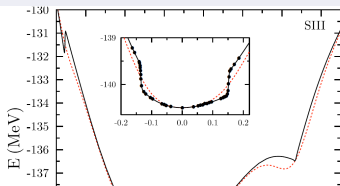
- So far so good...

Unexpected pathologies

 $\mathcal{E}_{Z=8, N=10}^{MR}$ vs ρ_{20} for $\mathcal{E}[\rho\rho\rho^{1/6}]$
[M. Bender *et al.*, PRC79 (2009) 044319]
 $\mathcal{E}_{Z=8, N=10}^{MR}$ vs ρ_{20} for $\mathcal{E}[\rho\rho\rho]$
[M. Bender *et al.*, PRC79 (2009) 044319]

- ➊ Divergencies and finite steps [J. Dobaczewski *et al.*, PRC76 (2007) 054315]
- ➋ Non-analyticity of $\mathcal{E}[\rho^{0\varphi}, \kappa^{0\varphi}, \kappa^{\varphi 0*}]$ over \mathbb{C} -plane with $e^{i\varphi} \equiv z$
- ➌ $\mathcal{E}_N^{MR} \neq 0$ for $N \leq 0$!! [M. Bender, T.D., D. Lacroix, PRC79 (2009) 044319]
- ➍ Similar problems for other MR modes, e.g. angular momentum restoration

Unexpected pathologies

 $\mathcal{E}_{Z=8, N=10}^{MR}$ vs $[\rho_{20}]$ for $\mathcal{E}[\rho\rho\rho^{1/6}]$
 β_2 [M. Bender *et al.*, PRC79 (2009) 044319]
 $\mathcal{E}_{Z=8, N=10}^{MR}$ vs $[\rho_{20}]$ for $\mathcal{E}[\rho\rho\rho]$
 β_2 [M. Bender *et al.*, PRC79 (2009) 044319]

Absent from wave-function method

- ➊ Divergencies and finite steps [J. Dobaczewski *et al.*, PRC76 (2007) 054315]
- ➋ Non-analyticity of $\mathcal{E}[\rho^{0\varphi}, \kappa^{0\varphi}, \kappa^{\varphi 0*}]$ over \mathbb{C}^* via $e^{i\varphi} \equiv z$
- ➌ $\mathcal{E}_N^{MR} \neq 0$ for $N \leq 0$!! [M. Bender, T.D., D. Lacroix, PRC79 (2009) 044319]
- ➍ Similar problems for other MR modes, e.g. angular momentum restoration

Outline

- 1 Introduction
 - Main ingredients of the nuclear EDF method
 - Ground-state correlations et collective excitations
 - Link with QRPA
- 2 Constraining the MR-EDF method to avoid pathologies
 - Basic consistency requirements
 - Unexpected pathologies
 - Regularization method
- 3 Conclusions
- 4 Bibliography

Regularization method

Origin of the problem and its solution

- Alternative method to define $\mathcal{E}[\Phi_q; \Phi_{q'}]$ that relies on
 - ① Considering Bogoliubov transformation connecting $|\Phi_q\rangle$ to $|\Phi_{q'}\rangle$
 - ② Using Bloch-Messiah-Zumino decomposition to reach BCS-like connection

$$|\Phi_{q'}\rangle = \tilde{C}_{qq'} \prod_{p>0} (\bar{A}_{pp}^* + \bar{B}_{p\bar{p}}^* \tilde{\alpha}_p^+ \tilde{\alpha}_{\bar{p}}^+) |\Phi_q\rangle$$

- ③ Using SWT to compute $\langle \Phi_q | H | \Phi_{q'} \rangle / \langle \Phi_q | \Phi_{q'} \rangle$
 - ④ Extending to general EDF disconnected from genuine operator H
- GWT-inspired $\mathcal{E}[\Phi_q; \Phi_{q'}] \equiv \mathcal{E}[\rho^{qq'}, \kappa^{qq'}, \kappa^{q'q*}]$ unsafe in EDF context
 - ① Provides dangerous weights to terms that are zero with SWT
 - ② Such terms cancel for WF method but not for more general EDF
 - ③ **Originates from self interaction and self pairing in the EDF kernel**

[D. Lacroix, T. D., M. Bender, PRC 79 (2009) 044318]

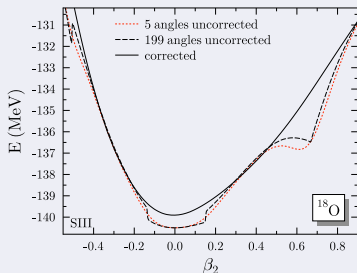
Regularization method

Regularized PNR calculations

$$\blacksquare \mathcal{E}_{REG}[\Phi_0; \Phi_\varphi] \equiv \mathcal{E}[\rho^{0\varphi}, \kappa^{0\varphi}, \kappa^{\varphi 0*}] - \mathcal{E}_C[\langle \Phi_0 |; | \Phi_\varphi \rangle]$$

① Analytical over \mathbb{C}^*

② \mathcal{E}_N^{MR} is free from divergencies/steps and zero for $N \leq 0$



■ The correction

① is crucial at critical points but also *away* from them

② depends on the quadrupole deformation ρ_{20}

③ is on the MeV scale = mass accuracy/spectroscopic scale

[M. Bender, T. D., D. Lacroix, PRC 79 (2009) 044319]

Outline

- 1 Introduction
 - Main ingredients of the nuclear EDF method
 - Ground-state correlations et collective excitations
 - Link with QRPA
- 2 Constraining the MR-EDF method to avoid pathologies
 - Basic consistency requirements
 - Unexpected pathologies
 - Regularization method
- 3 Conclusions
- 4 Bibliography

Summary

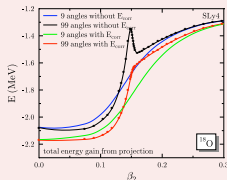
MR-EDF method

- 1 Efficient to tackle long-range correlations and access spectroscopy
- 2 Difficulties must be taken seriously

[J. Dobaczewski, W. Nazarewicz, P. G. Reinhard, M. V. Stoitsov, PRC76 (2007) 054315]

Regularization method

- 1 Valid for all MR modes; i.e. fluctuations of any $|q|$ and/or φ
- 2 Successful application to particle number restoration
- 3 Limited to EDF kernels of the form $\mathcal{E}[\rho^n, (\kappa^* \kappa)^m]$
- 4 Specific case of $\mathcal{E}[\rho^\gamma]$ with γ non integer



[T. D., M. Bender, K. Bennaceur, D. Lacroix, T. Lesinski, PRC 79 (2009) 044320]

Perspectives

To be done in the near future

- 1 Build correctable EDF of the form $\mathcal{E}[\rho^n, (\kappa^* \kappa)^m]$
[J. Sadoudi, T. D., M. Bender, K. Bennaceur, in progress]
- 2 Build SI- and SP-free EDF
- 3 Perform regularized MR calculations for AMR and $\Delta\rho_{20}$ mixing
[M. Bender, T.D., D. Lacroix, in progress]
- 4 Work out new constraints on $\mathcal{E}[\Phi_0; \Phi_\Omega]$ for various symmetry groups

$$\mathcal{E}[\Phi_0; \Phi_\Omega] \langle \Phi_0 | \Phi_\Omega \rangle = \sum_{\lambda ab} c_{\lambda a}^* c_{\lambda b} \mathcal{E}_\lambda S_{ab}^\lambda(\Omega)$$









[T. D., J. Sadoudi, J. Phys. G: Nucl. Part. Phys, in press]

- 5 Need a constructive framework for MR-EDF method

Outline

- 1 Introduction
 - Main ingredients of the nuclear EDF method
 - Ground-state correlations et collective excitations
 - Link with QRPA
- 2 Constraining the MR-EDF method to avoid pathologies
 - Basic consistency requirements
 - Unexpected pathologies
 - Regularization method
- 3 Conclusions
- 4 Bibliography

Selected bibliography

-  M. Bender, P.-H. Heenen, P.-G. Reinhard,
Rev. Mod. Phys. 75 (2003) 121
-  R. Balian, E. Brézin,
Nuevo Cimento 64 (1969) 37
-  B. Jancovici, D. H. Schiff,
Nucl. Phys. 58 (1964) 678
-  L. Robledo,
Int. Journ. Mod. Phys. E16 (2007) 337
-  J. Dobaczewski, J. Dudek,
Acta Phys. Polon. B27 (1996) 45
-  D. Lacroix, T. Duguet, M. Bender,
Phys. Rev. C 79 (2009) 044318
-  M. Bender, T. Duguet, D. Lacroix,
Phys. Rev. C 79 (2009) 044319
-  T. Duguet, M. Bender, K. Bennaceur, D. Lacroix, T. Lesinski,
Phys. Rev. C 79 (2009) 044320