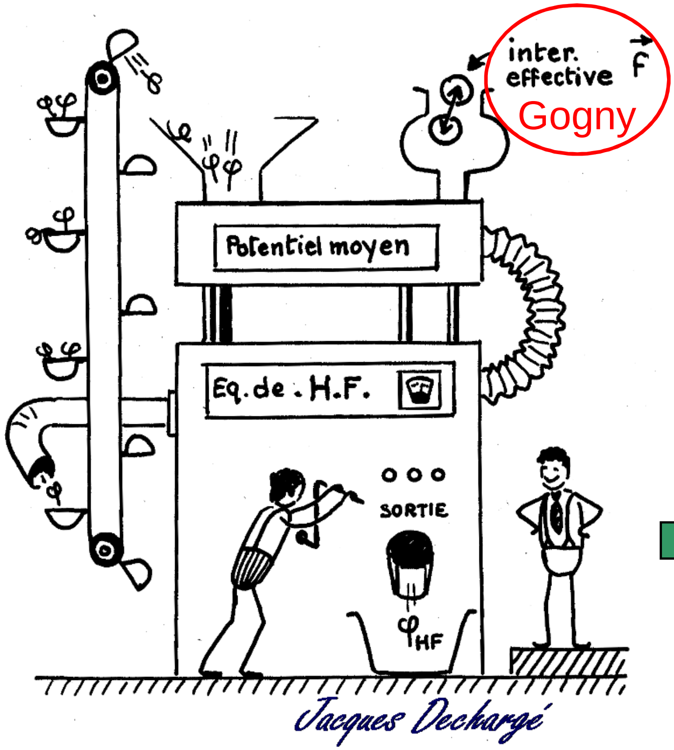
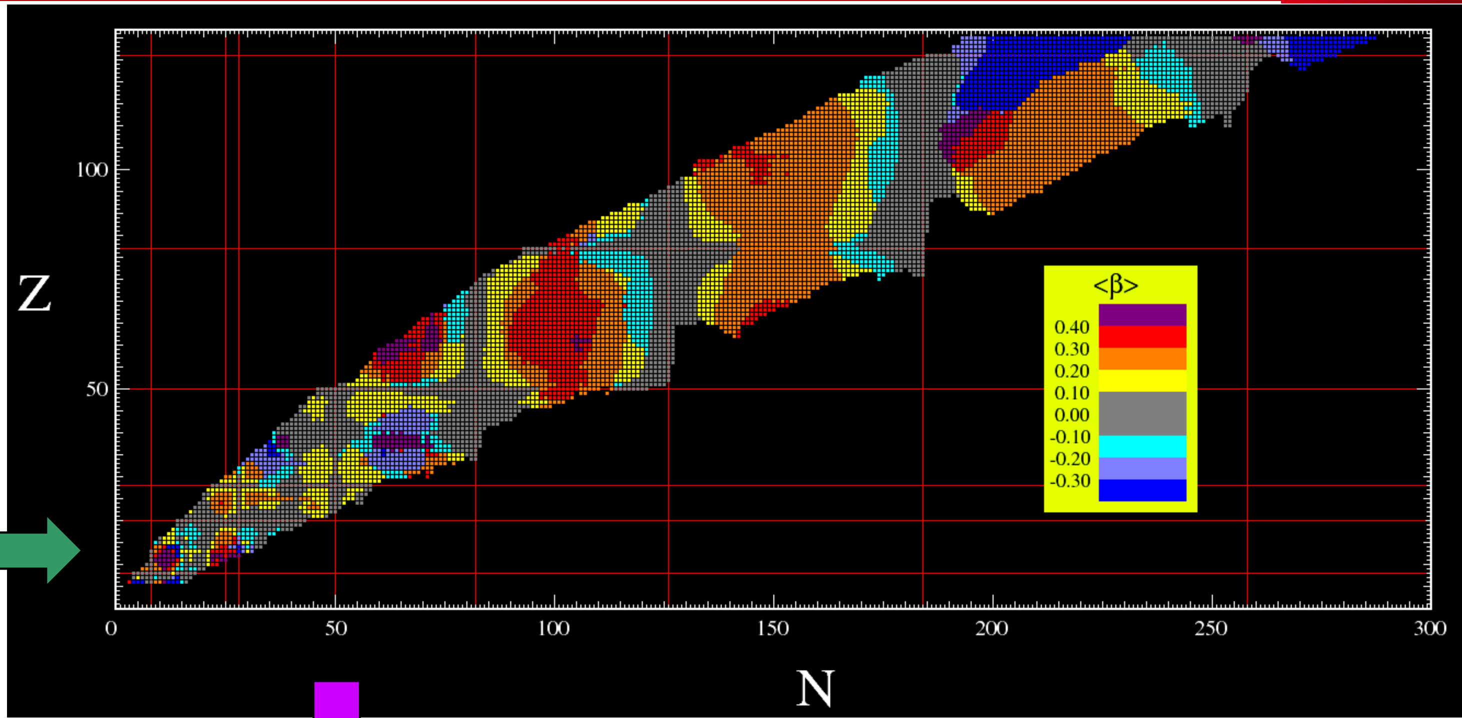
QRPA for axially-symmetric deformed nuclei
(or QRPA in cylindrical basis)

S. Péru (CEA,DAM,DIF)

Reminder





Static mean field (HFB)
for Ground State Properties :
- **Masses**
- **Deformation**
- **(Single particle levels)**

Amedee database :
http://www-phynu.cea.fr/HFB-Gogny_eng.htm
*S. Hilaire & M. Girod, EPJ **A33** (2007) 237*

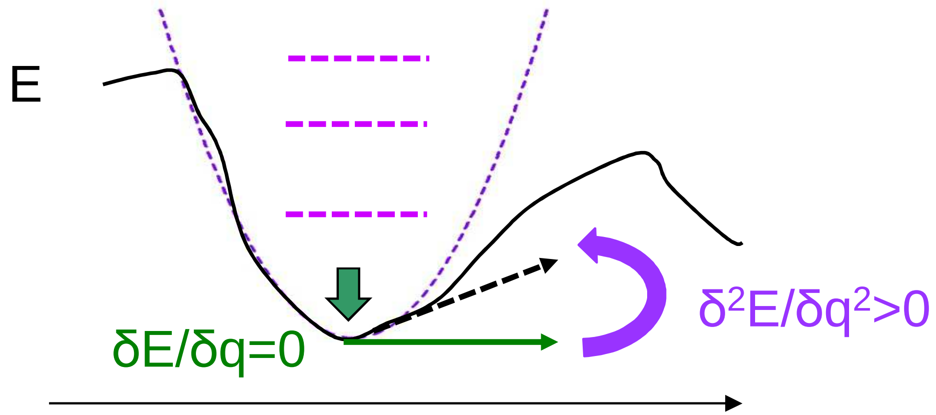
Beyond static mean field approximation (for exple QRPA)
for description of Excited State Properties
- **Low-energy collective levels**
- **Giant Resonances**
- **Beta decay**

Beyond mean field… with QRPA

RPA approaches describe

all multipolarties and all parities,
collective states and individual ones,
low energy and high energy states
with the same accuracy.

Within the small amplitude approximation, i.e. « harmonic » nuclei



Spherical RPA with Gogny force

J. Dechargé and L.Sips, Nucl. Phys. **A 407**,1 (1983)
J.P. Blaizot, J.F. Berger, J. Dechargé, M. Girod, Nucl. Phys. A 591, 435 (1995)
S. Péru, JF. Berger, PF. Bortignon, Eur. Phys. J. A **26**, 25-32, (2005)

Axially symetric deformed QRPA with Gogny force

S. Péru, H. Goutte, Phys. Rev. C **77**, 044313, (2008)
M. Martini, S. Péru and M. Dupuis, Phys. Rev. C **83**, 034309 (2011)
S. Péru *et al*, Phys. Rev. C **83**, 014314 (2011)
S. Péru and M. Martini, EPJA (2014) 50: 88

RPA approaches are well adapted for describing giant resonances

HFB formalism

$$F(\rho,\kappa) = \sum_{\alpha\beta} t_{\alpha\beta}\rho_{\beta\alpha} + \frac{1}{2}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|\mathcal{V}(\rho)|\widetilde{\gamma\delta}\rangle\rho_{\gamma\alpha}\rho_{\delta\beta} + \frac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|\mathcal{V}(\rho)|\widetilde{\gamma\delta}\rangle\kappa^*_{\beta\alpha}\kappa_{\gamma\delta}$$

$$\delta F = \sum_{\alpha\beta}\frac{\partial F}{\partial\rho_{\beta\alpha}}\delta\rho_{\alpha\beta} + \frac{1}{2}\sum_{\alpha\beta}\left(\frac{\partial F}{\partial\kappa_{\beta\alpha}}\delta\kappa_{\alpha\beta} + \frac{\partial F}{\partial\kappa^*_{\beta\alpha}}\delta\kappa^*_{\alpha\beta}\right)$$

$$H_B = \begin{pmatrix} e & \Delta \\ -\Delta^* & -e^* \end{pmatrix} \qquad e_{\alpha\beta} = \frac{\partial F}{\partial\rho_{\beta\alpha}} \qquad \Delta_{\alpha\beta} = \frac{\partial F}{\partial\kappa^*_{\alpha\beta}}$$

$$\mathcal{R} = \begin{pmatrix} \rho & \kappa \\ -\kappa^* & (1-\rho^*) \end{pmatrix} \qquad \boxed{[H_B,\mathcal{R}] = 0}$$

(Q)RPA formalism 1/3

$$F(\rho,\kappa) = \sum_{\alpha\beta} t_{\alpha\beta}\rho_{\beta\alpha} + \frac{1}{2}\sum_{\alpha\beta\gamma\delta} \langle\alpha\beta|\mathcal{V}(\rho)|\widetilde{\gamma\delta}\rangle \rho_{\gamma\alpha}\rho_{\delta\beta} + \frac{1}{4}\sum_{\alpha\beta\gamma\delta} \langle\alpha\beta|\mathcal{V}(\rho)|\widetilde{\gamma\delta}\rangle \kappa^*_{\beta\alpha}\kappa_{\gamma\delta}$$

$$\delta F_2 = \frac{1}{2}\sum_{\alpha\beta}\left[\delta\rho_{\alpha\beta}\sum_{\gamma\delta}\left(V^{CM}_{\beta\alpha,\delta\gamma}\delta\rho_{\gamma\delta} + V^{M}_{\beta\alpha,\delta\gamma}\delta\kappa_{\gamma\delta}\right) + \delta\kappa_{\alpha\beta}\sum_{\gamma\delta}\left(V^{M*}_{\beta\alpha,\delta\gamma}\delta\rho_{\gamma\delta} + V^{P}_{\beta\alpha,\delta\gamma}\delta\kappa_{\gamma\delta}\right)\right]$$

$$V^{CM}_{\beta\alpha,\gamma\delta} = \frac{1+\delta_{\alpha\beta}}{2}\frac{1+\delta_{\gamma\delta}}{2}\frac{\partial^2 F}{\partial\rho_{\alpha\beta}\partial\rho_{\gamma\delta}}$$

$$V^{M}_{\beta\alpha,\gamma\delta} = \frac{1+\delta_{\alpha\beta}}{2}\frac{1+\delta_{\gamma\delta}}{2}\frac{\partial^2 F}{\partial\rho_{\alpha\beta}\partial\kappa_{\gamma\delta}}$$

$$V^{M*}_{\beta\alpha,\gamma\delta} = \frac{1+\delta_{\alpha\beta}}{2}\frac{1+\delta_{\gamma\delta}}{2}\frac{\partial^2 F}{\partial\partial\kappa_{\alpha\beta}\rho_{\gamma\delta}}$$

$$V^{P}_{\beta\alpha,\gamma\delta} = \frac{1+\delta_{\alpha\beta}}{2}\frac{1+\delta_{\gamma\delta}}{2}\frac{\partial^2 F}{\partial\kappa_{\alpha\beta}\partial\kappa_{\gamma\delta}}$$

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{pp'}\delta_{hh'} + \frac{\partial^2 F}{\partial\rho_{hp}\partial\rho_{p'h'}}$$

$$B_{ph,p'h'} = \frac{\partial^2 F}{\partial\rho_{hp}\partial\rho_{h'p'}}$$

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix}\begin{pmatrix} X_n \\ Y_n \end{pmatrix} = \omega_n \begin{pmatrix} X_n \\ -Y_n \end{pmatrix}$$

(Q)RPA formalism 2/3

$$
\begin{aligned}
V^{CM}_{\alpha\beta,\gamma\delta} &= \frac{1+\delta_{\alpha\beta}}{2}\frac{1+\delta_{\gamma\delta}}{2}\frac{\partial^2 F}{\partial\rho_{\beta\alpha}\partial\rho_{\gamma\delta}} \\
&= \langle\alpha\gamma|\mathcal{V}|\widetilde{\beta\delta}\rangle \\
&\quad +\sum_{\gamma'\delta'}\langle\alpha\gamma'|\frac{\partial\mathcal{V}}{\partial\rho_{\delta\gamma}}|\widetilde{\beta\delta'}\rangle\rho_{\delta'\gamma'} \\
&\quad +\sum_{\gamma'\delta'}\langle\gamma\gamma'|\frac{\partial\mathcal{V}}{\partial\rho_{\alpha\beta}}|\widetilde{\delta\delta'}\rangle\rho_{\delta'\gamma'} \\
&\quad +\frac{1}{2}\sum_{\gamma'\delta'\gamma''\delta''}\langle\gamma'\gamma''|\frac{\partial^2\mathcal{V}}{\partial\rho_{\delta\gamma}\partial\rho_{\alpha\beta}}|\widetilde{\delta'\delta''}\rangle\rho_{\delta'\gamma'}\rho_{\delta''\gamma''} \\
&\quad +\frac{1}{2}\sum_{\gamma'\delta'\gamma''\delta''}\langle\gamma'\overline{\gamma''}|\frac{\partial^2\mathcal{V}}{\partial\rho_{\delta\gamma}\partial\rho_{\alpha\beta}}|\widetilde{\delta'\overline{\delta''}}\rangle\kappa_{\gamma''\gamma'}\kappa_{\delta'\delta''}. \quad (46)
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{\gamma'\delta'}\langle\alpha\gamma'|\frac{\partial\mathcal{V}}{\partial\rho_{\gamma\delta}}|\widetilde{\beta\delta'}\rangle\rho_{\delta'\gamma'} = \\
&\delta_{\sigma_\alpha\sigma_\beta}\delta_{\sigma_\gamma\sigma_\delta}\delta_{\tau_\alpha\tau_\beta}\delta_{\tau_\gamma\tau_\delta}t_0\alpha_0 \\
&\cdot\left\langle\alpha\gamma\middle|\delta(r_1-r_2)\rho^{\alpha_0-1}\left(\left(1+\frac{x_0}{2}\right)\rho\right.\right. \\
&\left.\left.-\left(x_0+\frac{1}{2}\right)\rho^{\tau_\alpha}\right)\middle|\beta\delta\right\rangle
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{\gamma'\delta'\gamma''\delta''}\langle\gamma'\gamma''|\frac{\partial^2\mathcal{V}}{\partial\rho_{\alpha\beta}\partial\rho_{\gamma\delta}}|\widetilde{\delta'\delta''}\rangle\rho_{\delta'\gamma'}\rho_{\delta''\gamma''} = \\
&\delta_{\sigma_\alpha\sigma_\beta}\delta_{\sigma_\gamma\sigma_\delta}\delta_{\tau_\alpha\tau_\beta}\delta_{\tau_\gamma\tau_\delta}t_0\alpha_0(\alpha_0-1) \\
&\cdot\left\langle\alpha\gamma\middle|\delta(r_1-r_2)\rho^{\alpha_0-2}\left(\left(1+\frac{x_0}{2}\right)\rho^2\right.\right. \\
&\left.\left.-\left(x_0+\frac{1}{2}\right)\sum_\tau\rho^{\tau_\alpha 2}\right)\middle|\beta\delta\right\rangle. \quad (49)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{A}_{ij,kl} &= (\epsilon_i+\epsilon_j)\delta_{ik}\delta_{jl} \\
&\quad +\frac{1}{2}\sum_{\alpha\beta\gamma\delta}(1+\delta_{\alpha\beta})(1+\delta_{\gamma\delta})\langle\alpha\beta|\mathcal{V}|\widetilde{\gamma\delta}\rangle \\
&\quad \left(\tilde{U}_{i\alpha}\tilde{V}_{j\gamma}U_{\delta k}V_{\beta l}-\tilde{U}_{i\alpha}\tilde{V}_{j\gamma}V_{\beta k}U_{\delta l}\right. \\
&\quad -\tilde{V}_{i\gamma}\tilde{U}_{j\alpha}U_{\delta k}V_{\beta l}+\tilde{V}_{i\gamma}\tilde{U}_{j\alpha}V_{\beta k}U_{\delta l} \\
&\quad \left.+\tilde{U}_{i\alpha}\tilde{U}_{j\beta}U_{\gamma k}U_{\delta l}+V_{\gamma i}V_{\delta j}\tilde{V}_{k\alpha}\tilde{V}_{l\beta}\right), \quad (50)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{B}_{ij,kl} &= \frac{1}{2}\sum_{\alpha\beta\gamma\delta}(1+\delta_{\alpha\beta})(1+\delta_{\gamma\delta})\langle\alpha\beta|\mathcal{V}|\widetilde{\gamma\delta}\rangle \\
&\quad \left(\tilde{U}_{i\alpha}\tilde{V}_{j\gamma}V_{\delta k}U_{\beta l}-\tilde{U}_{i\alpha}\tilde{V}_{j\gamma}U_{\beta k}V_{\delta l}\right. \\
&\quad -\tilde{V}_{i\gamma}\tilde{U}_{j\alpha}V_{\delta k}U_{\beta l}+\tilde{V}_{i\gamma}\tilde{U}_{j\alpha}U_{\beta k}V_{\delta l} \\
&\quad \left.+\tilde{U}_{i\alpha}\tilde{U}_{j\beta}V_{\delta k}V_{\gamma l}+\tilde{V}_{i\delta}\tilde{V}_{j\gamma}U_{\alpha k}U_{\beta l}\right), \quad (51)
\end{aligned}
$$

S. P, M. Martini, EPJA (2014) 50:88

(Q)RPA Formalism 3/3

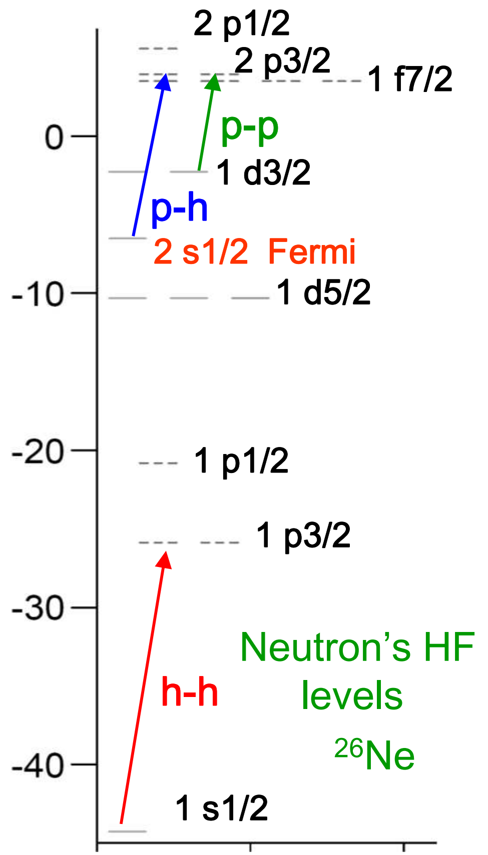
$$H|\nu\rangle = E_\nu|\nu\rangle \qquad Q^\dagger_\nu|0\rangle = |\nu\rangle \qquad Q_\nu|0\rangle = 0$$

Particle-hole excitations: RPA

$$Q^\dagger_\nu = \sum_{ph} X^\nu_{ph}\, a^\dagger_p a_h - Y^\nu_{ph}\, a^\dagger_h a_p$$

2 quasi-particles excitations: QRPA

$$Q^+_\nu = \sum_{ij} X^\nu_{ij}\eta^+_i\eta^+_j + Y^\nu_{ij}\eta_j\eta_i \qquad \eta^+_i = \sum_\alpha u_{i\alpha}a^+_\alpha - v_{i\alpha}a_\alpha$$



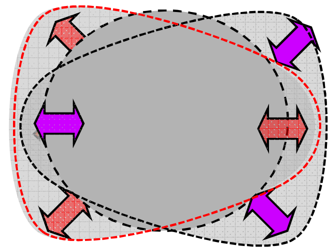
$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}\begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} X^\nu \\ -Y^\nu \end{pmatrix}$$

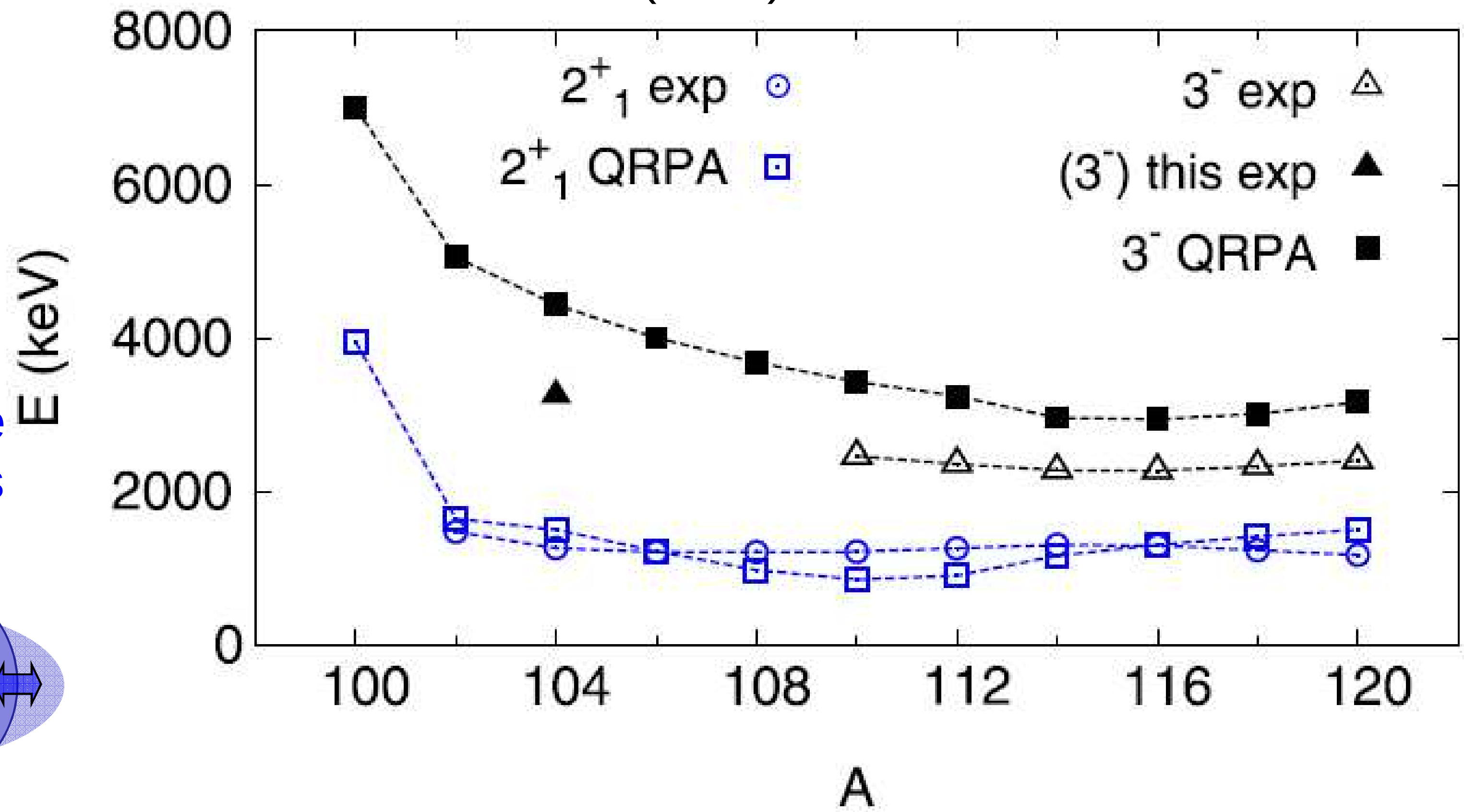
Hartree-Fock Bogoliubov: ε, u, v ⟶ **Ground state properties**

QRPA: ω, X, Y ⟶ **Excited states properties**

RPA approaches describe all multipolarties and all parities

A. Corsi et al PLB 743 (2015) 451-455





Quadrupole
oscillations

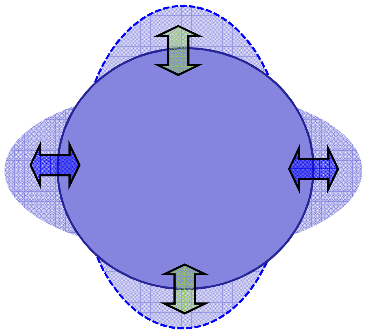
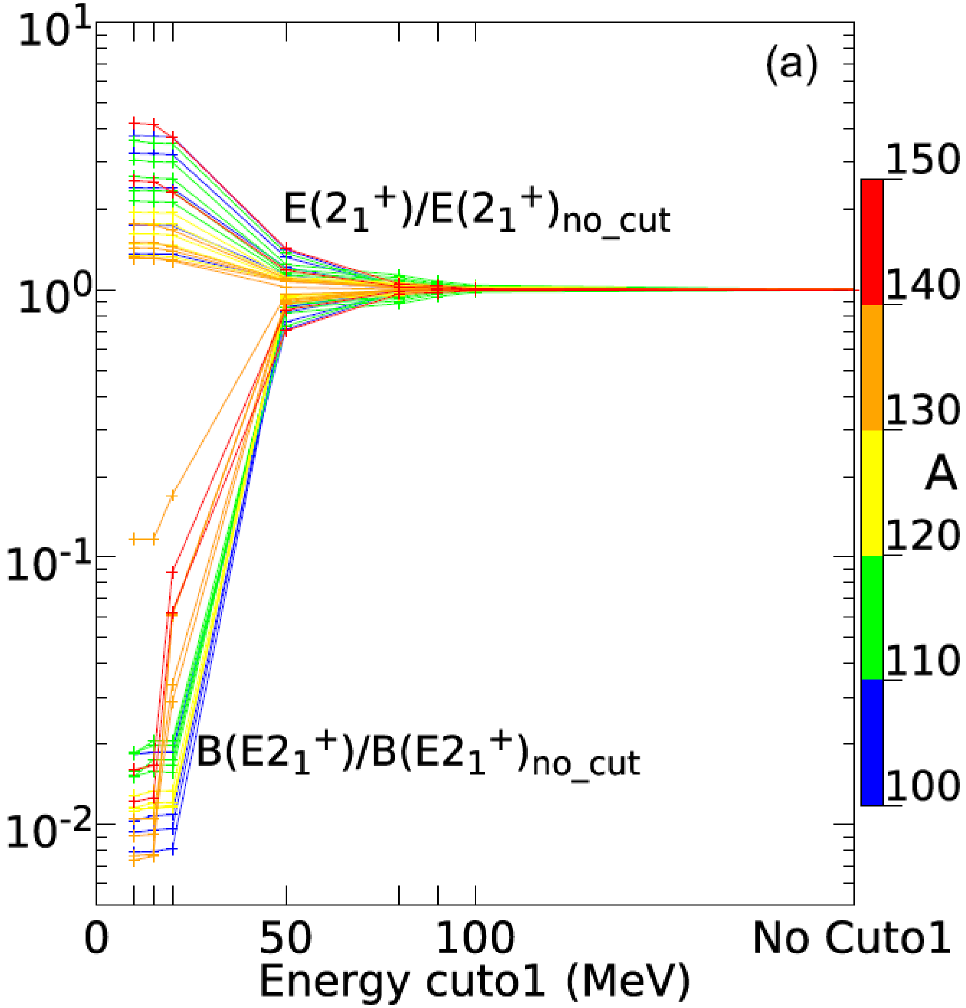
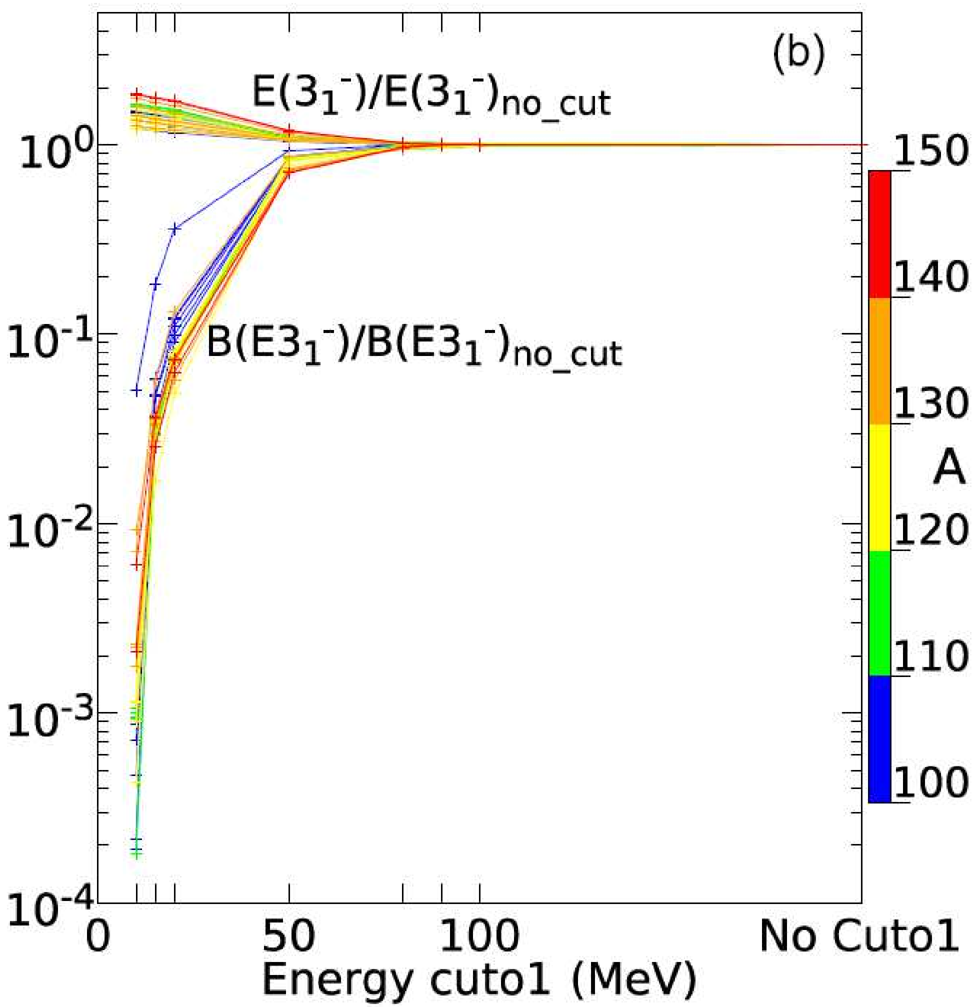


Fig. 3. (Color online.) Systematics of 2^+ and 3^- excitation energies in tin isotopes from experiment and HFB + QRPA calculations using the Gogny D1M interaction.

Impact of cutoff energy in 2qp excitation basis

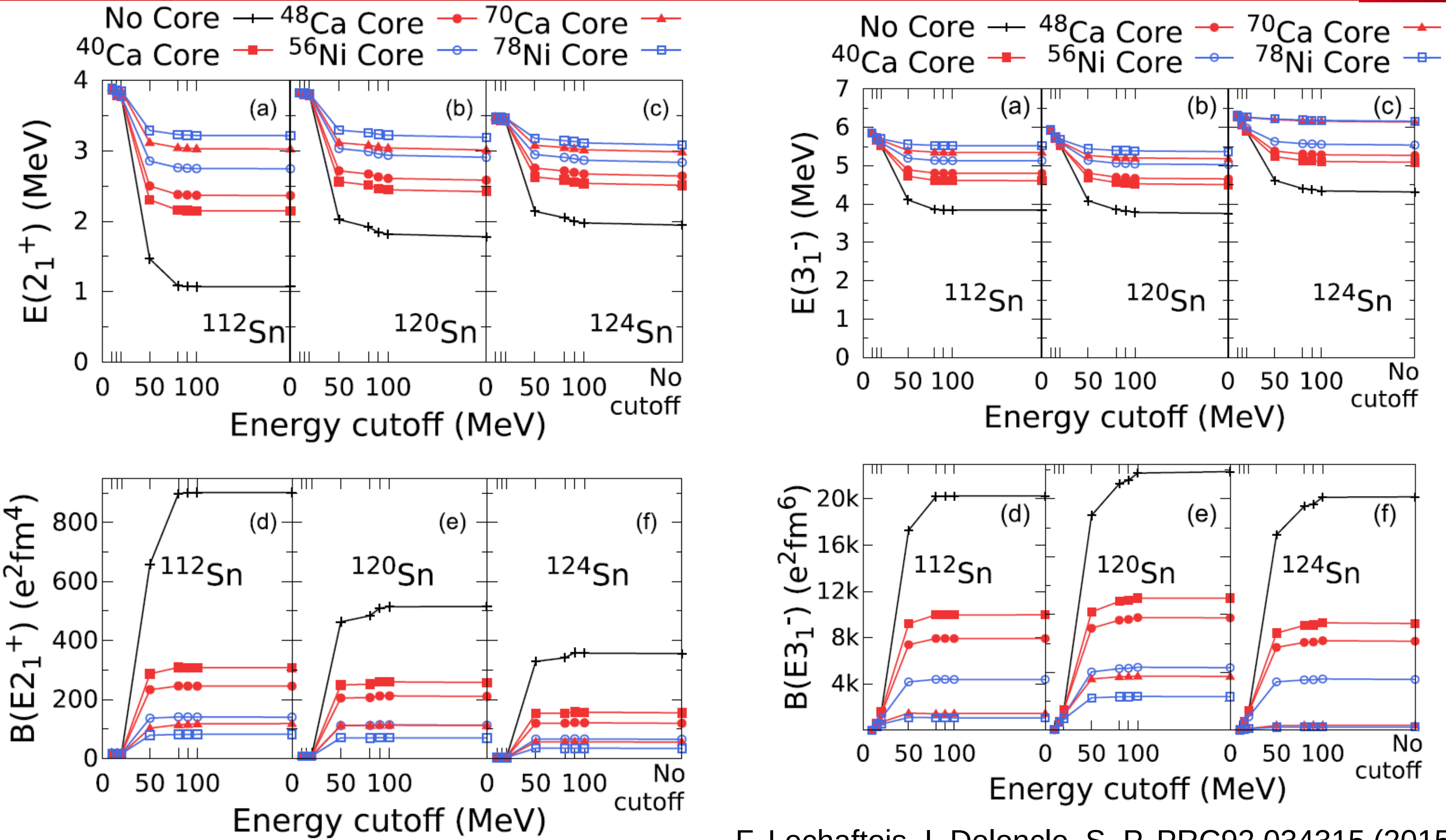
Sn isotopes





F. Lechaftois, I. Deloncle, S. P, PRC92,034315 (2015)

Impact of frozen core

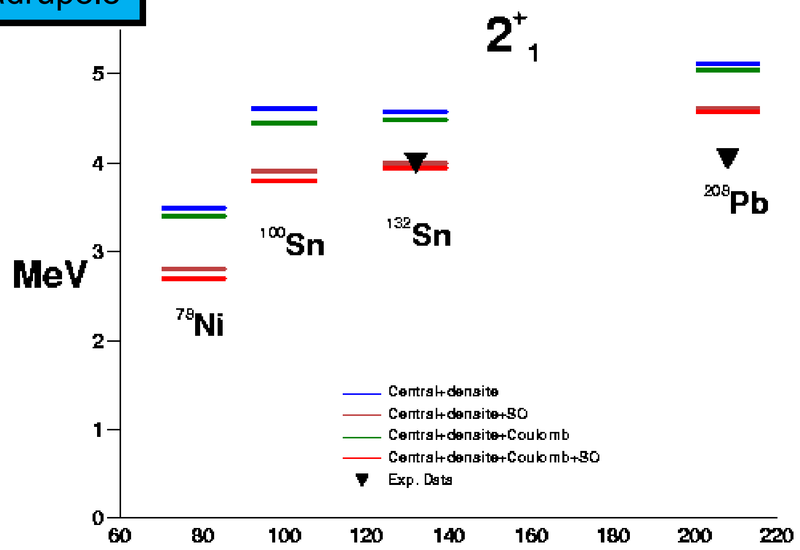


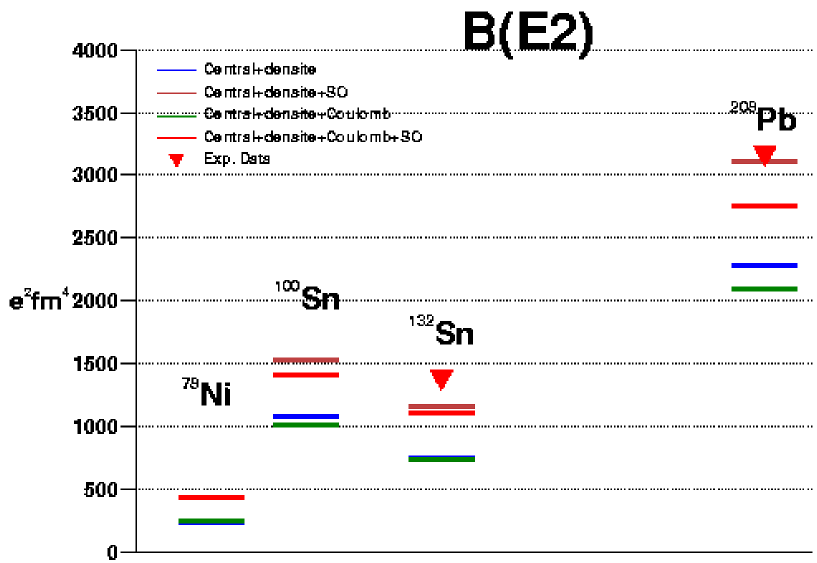
F. Lechaftois, I. Deloncle, S. P, PRC92,034315 (2015)

Role of the consistence between HF and RPA matrices

RPA in spherical symmetry

Quadrupole





S. Péru, J.F. Berger, and P.F. Bortignon, Eur. Phys. Jour. A 26, 25-32 (2005)

$$V(1,2) = \sum_{j=1,2} e^{-\frac{(\vec{r}_1-\vec{r}_2)^2}{\mu_j^2}} \left(W_j + B_j P_\sigma - H_j P_\tau - M_j P_\sigma P_\tau\right)$$ **central finite range**

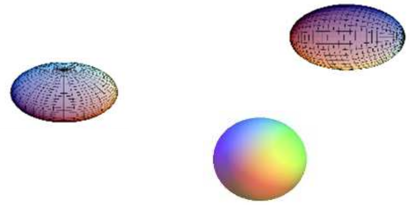
$$+t_0\left(1+x_0 P_\sigma\right)\delta\left(\vec{r}_1-\vec{r}_2\right)\left[\rho\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)\right]^\alpha$$ **density dependent**

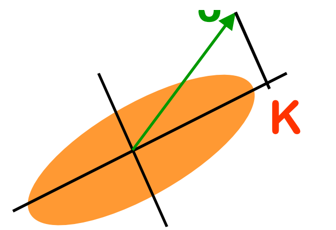
$$+i W_{ls} \overleftarrow{\nabla}_{12}\delta\left(\vec{r}_1-\vec{r}_2\right)\times\overrightarrow{\nabla}_{12}.\left(\vec{\sigma}_1+\vec{\sigma}_2\right)$$ **spin-orbit**

Axially-symmetric deformed QRPA

$$|\alpha, K\rangle = \theta^{+}_{\alpha,K}|0\rangle \qquad \theta^{+}_{n,K} = \sum_{i<j} X^{ij}_{n,K}\eta^{+}_{i,k_i}\eta^{+}_{j,k_j} - (-)^{K}\, Y^{ij}_{n,K}\eta_{j,-k_j}\eta_{i,-k_i}$$

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix}\begin{pmatrix} X_{\alpha,K} \\ Y_{\alpha,K} \end{pmatrix} = \omega_{\alpha,K}\begin{pmatrix} X_{\alpha,K} \\ -Y_{\alpha,K} \end{pmatrix}$$





- **Possibility to treat axially-symmetric deformed nuclei**

Restoration of rotational symmetry for deformed states

$$|JM(K)\rangle = \frac{\sqrt{2J+1}}{4\pi}\int d\Omega D^{J}_{MK}(\Omega)\mathrm{R}(\Omega)|\theta_K\rangle + (-)^{J-K} D^{J}_{M-K}(\Omega)\mathrm{R}(\Omega)|\overline{\theta}_K\rangle$$

to calculate: $\langle\tilde{0}|\hat{Q}_{\lambda\mu}|JM(K)\rangle$ **for all QRPA states (K ≤ J)**

$$\hat{Q}_{\lambda\mu} \propto \sum r^{\lambda}(Y_{\lambda\mu}) \qquad r^2Y_{\lambda\mu} = \sum_{\upsilon} D^{\lambda}_{\mu\upsilon} r^2 Y_{\lambda\upsilon}$$

In intrinsic frame

We use rotational approximation and relations for 3j symbols

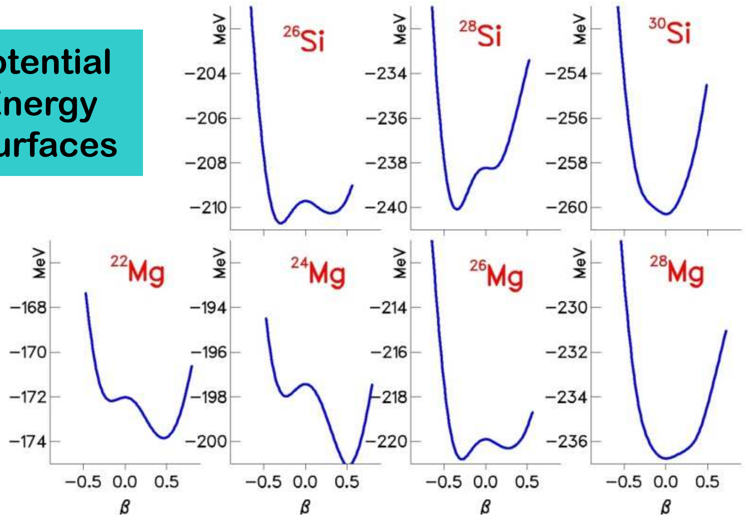
For example: $J^{\pi} = 2^{+}$

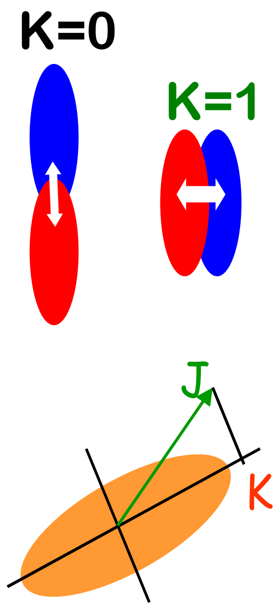
$$\langle\tilde{0}|\hat{Q}_{20}|JM(K)\rangle = \frac{1}{\sqrt{5}}\langle 0|\hat{Q}_{20}|\theta_K\rangle\delta_{K,0} + \frac{\sqrt{3}}{\sqrt{5}}\langle 0|\hat{Q}_{2-1}|\theta_K\rangle\delta_{K,\pm1} + \frac{\sqrt{3}}{\sqrt{5}}\langle 0|\hat{Q}_{22}|\theta_K\rangle\delta_{K,\pm2}$$

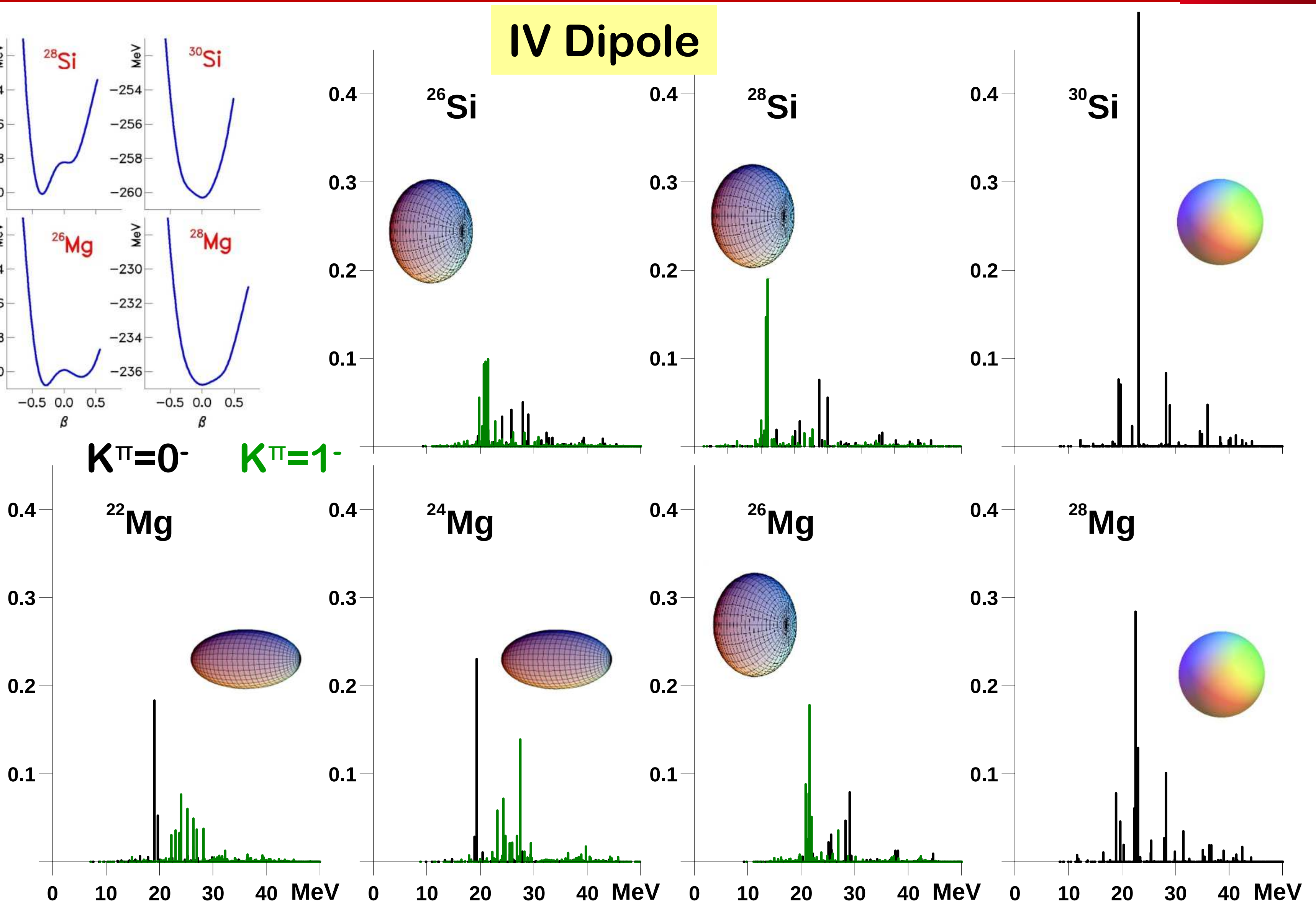
Using time reversal symmetry, three independent calculations **($K^{\pi} = 0^{+}, 1^{+}, 2^{+}$) are needed.**

First study with QRPA in axial symmetry

Potential Energy Surfaces

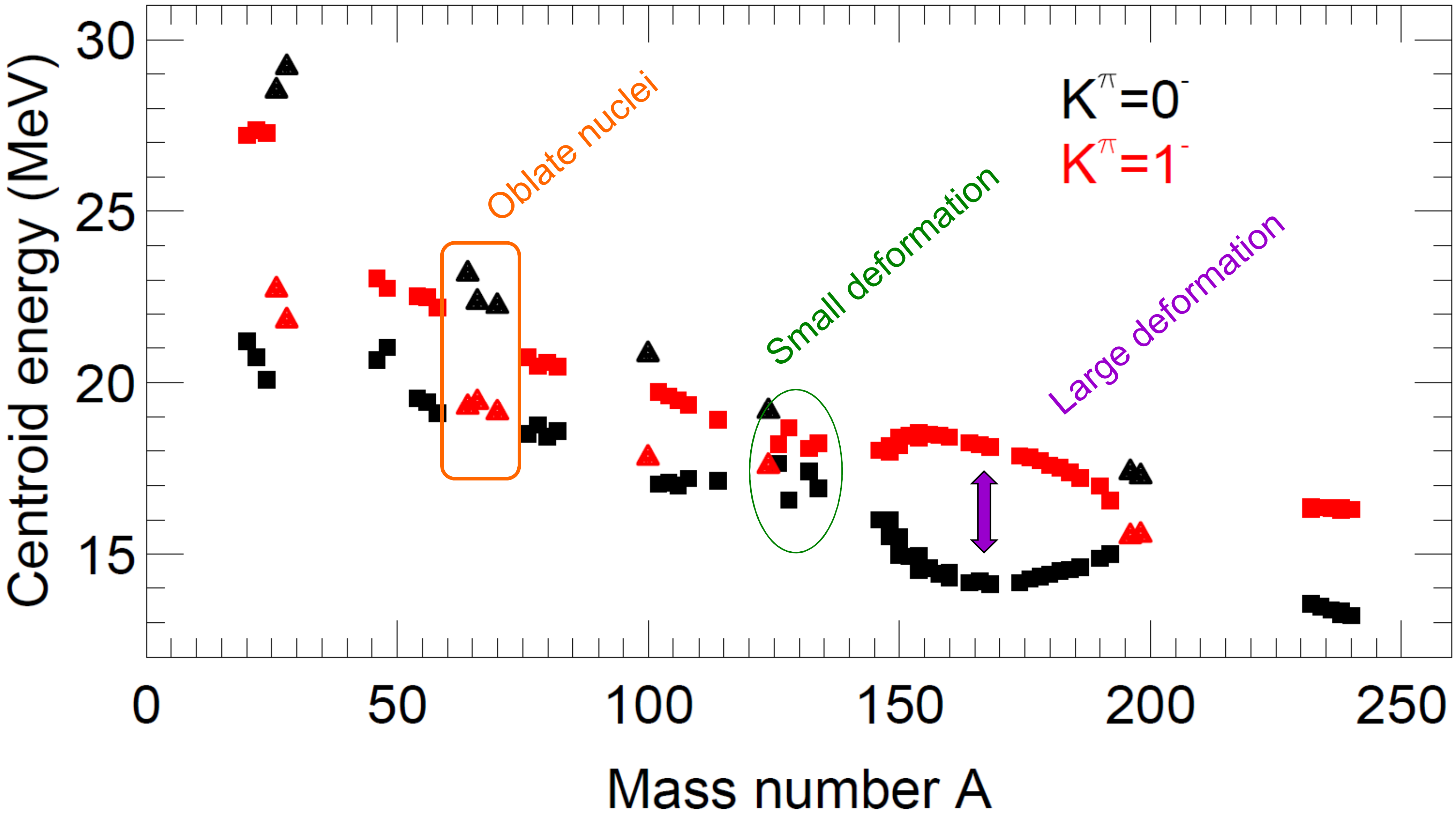






S. Péru and H. Goutte, Phys. Rev. C 77, 044313 (2008).

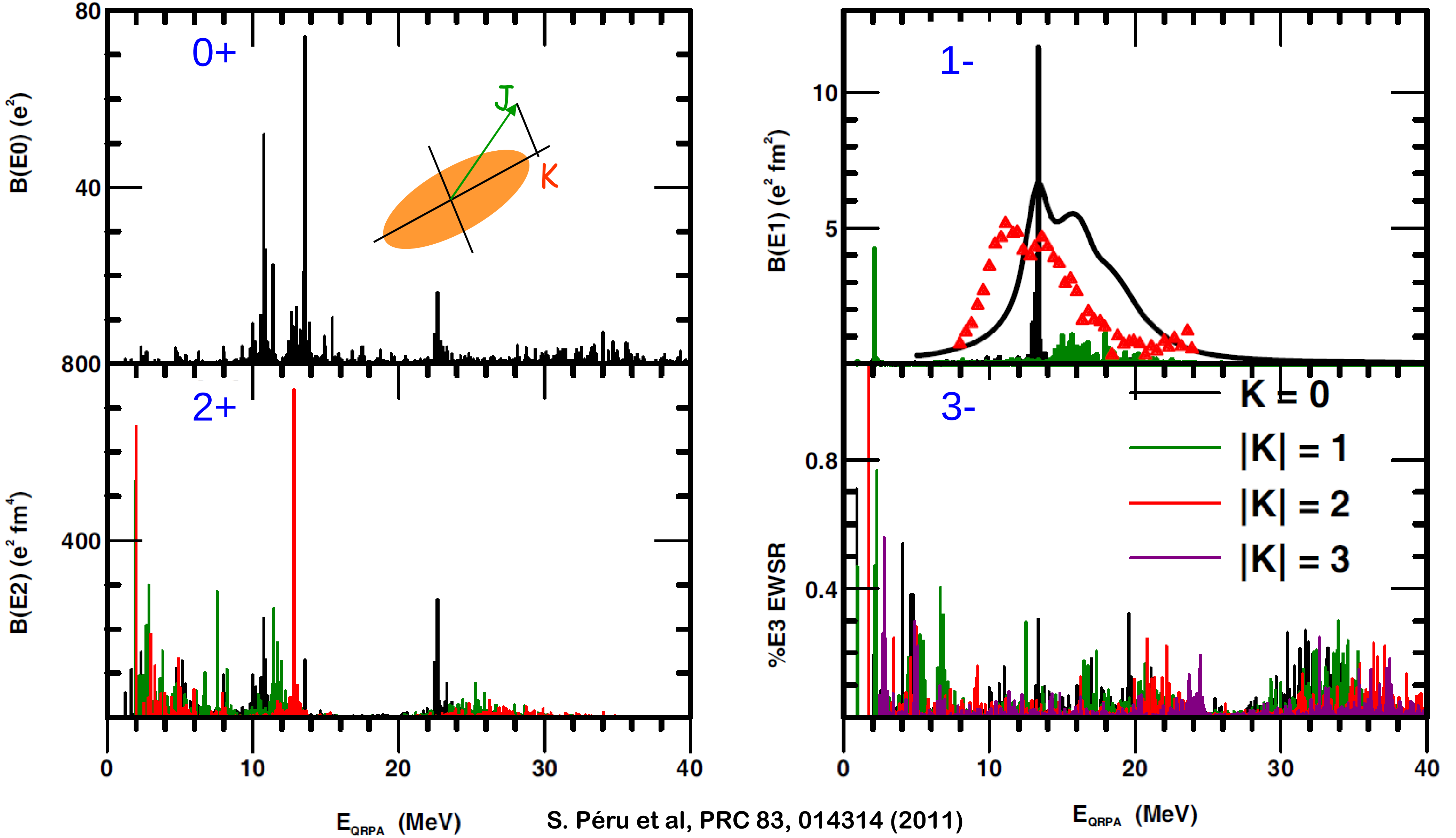
Impact of the deformation



M. Martini et al, PRC 94, 014304 (2016)

Multipolar responses for ^{238}U

Heavy deformed nucleus ⟶ massively parallel computation

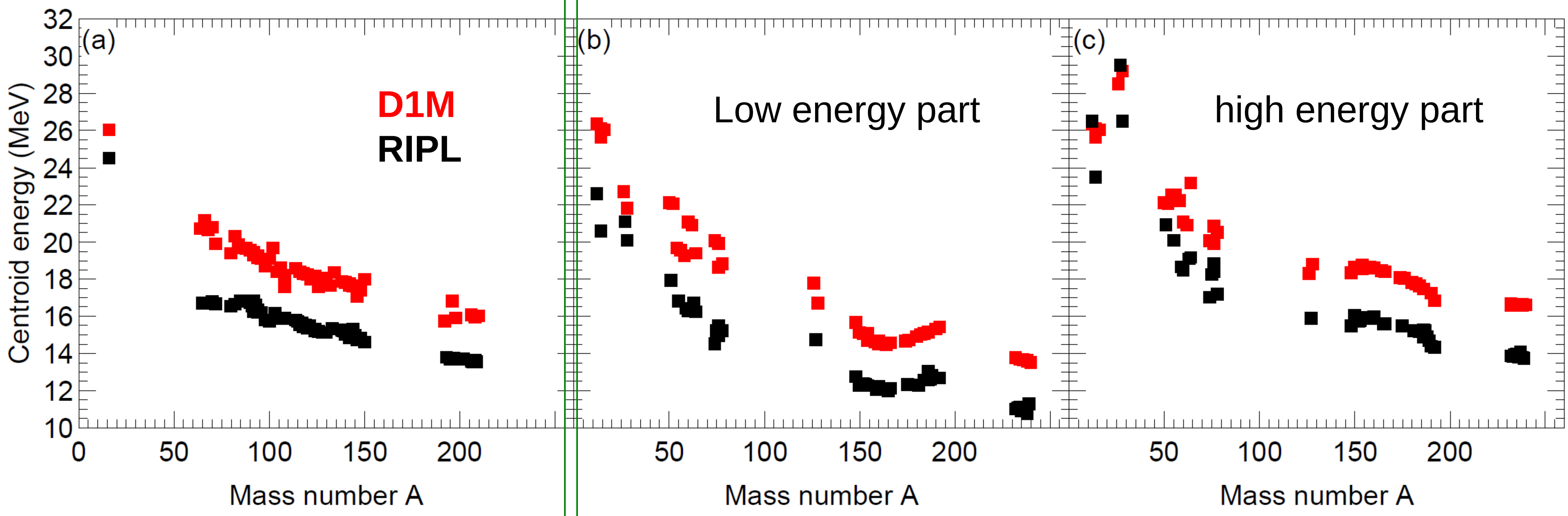


S. Péru et al, PRC 83, 014314 (2011)

Comparison with experimental data

One Lorentzian in RIPL

Two Lorentzians in RIPL



Systematic overestimation of the centroid energies :~ 2MeV

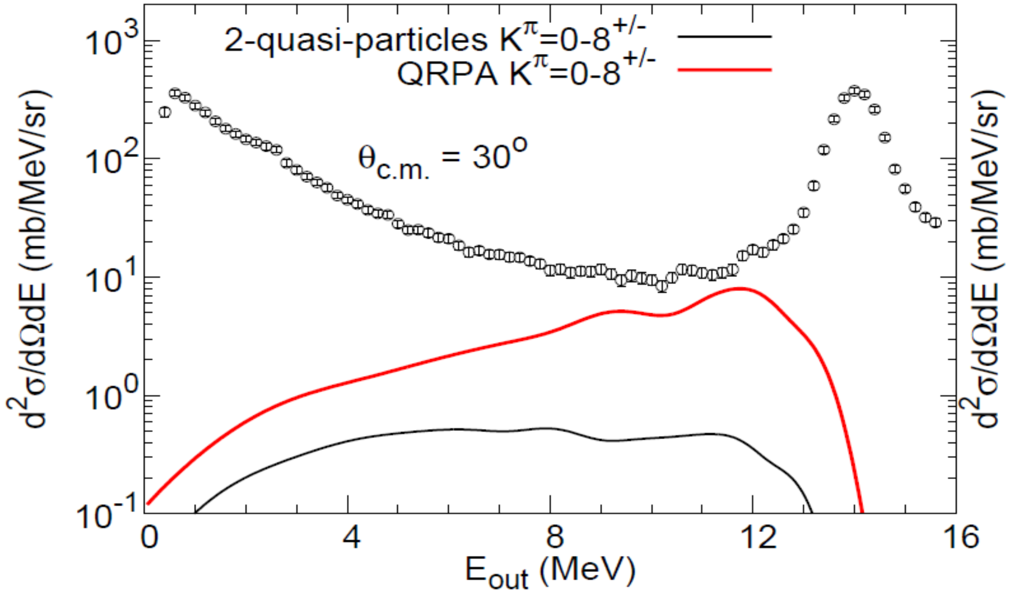
M. Martini et al, PRC 94, 014304 (2016)

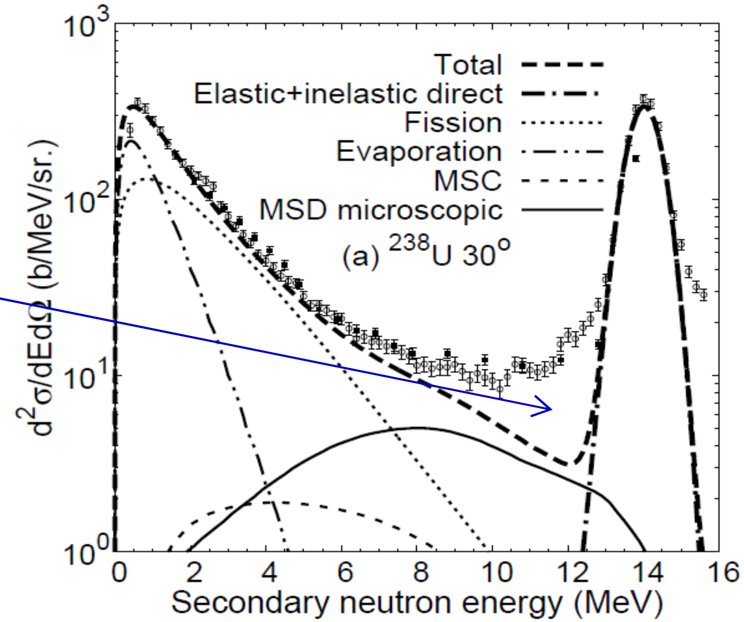
Beyond the nuclear structure

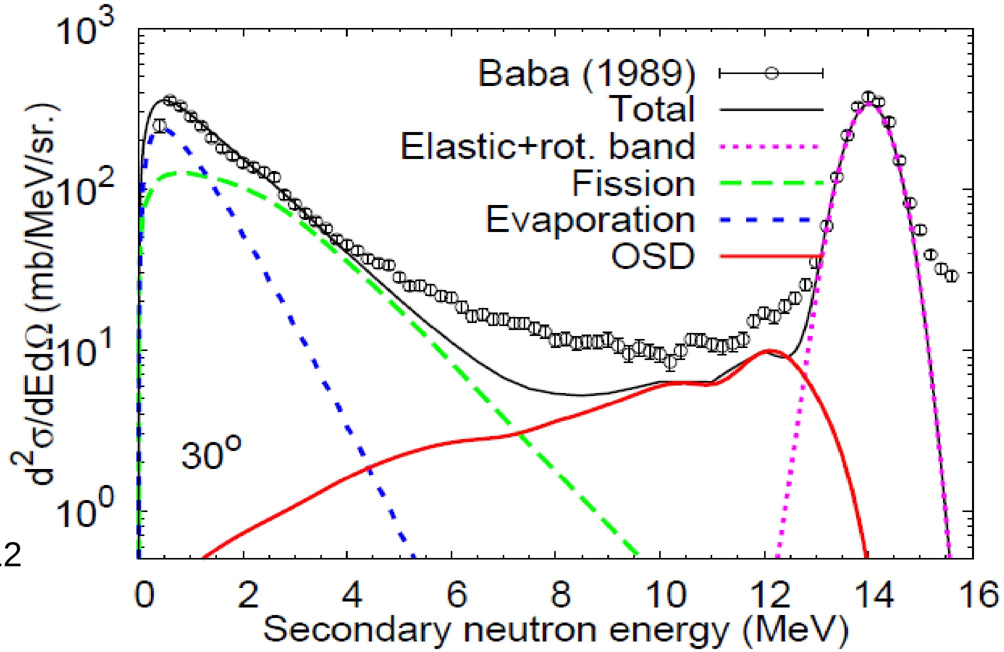
(n, *x* n) cross section on ^{238}U

Problem of underestimation of
n emission cross section at high energy

QRPA provides
enough collective contribution

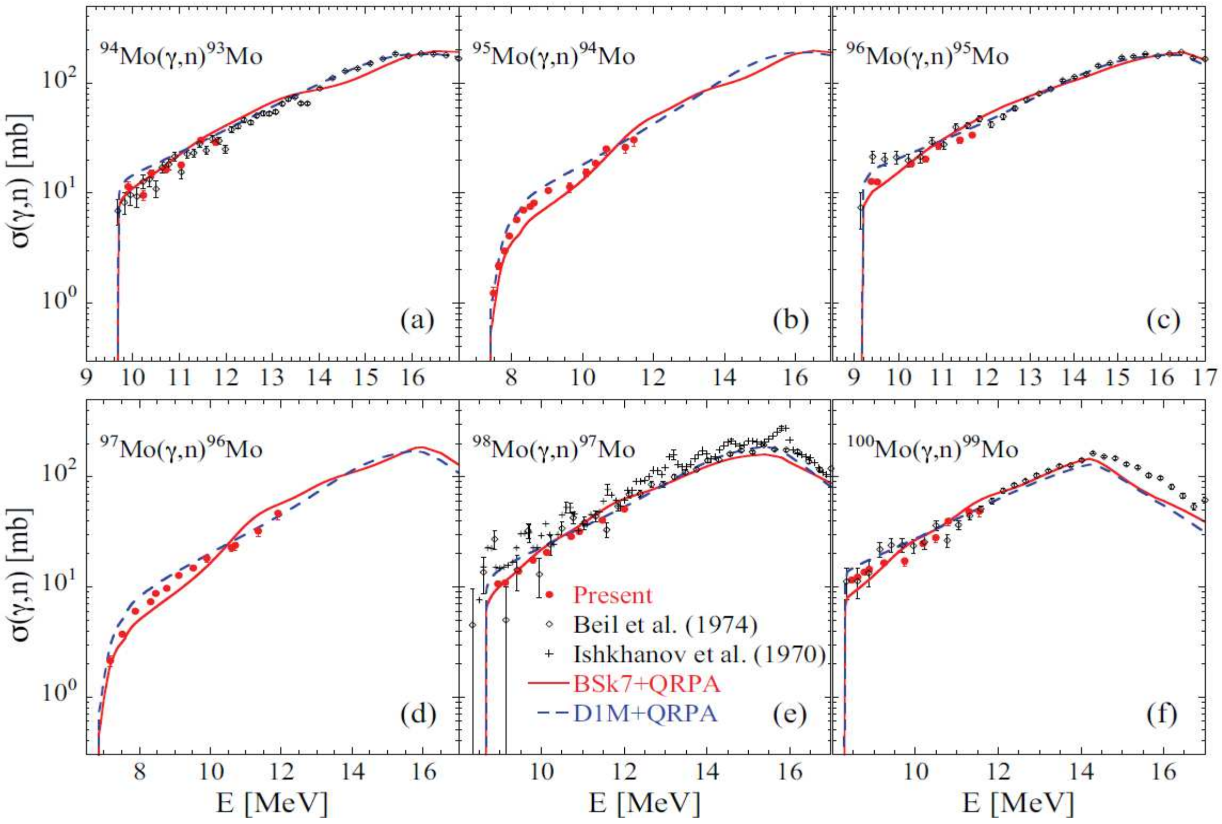


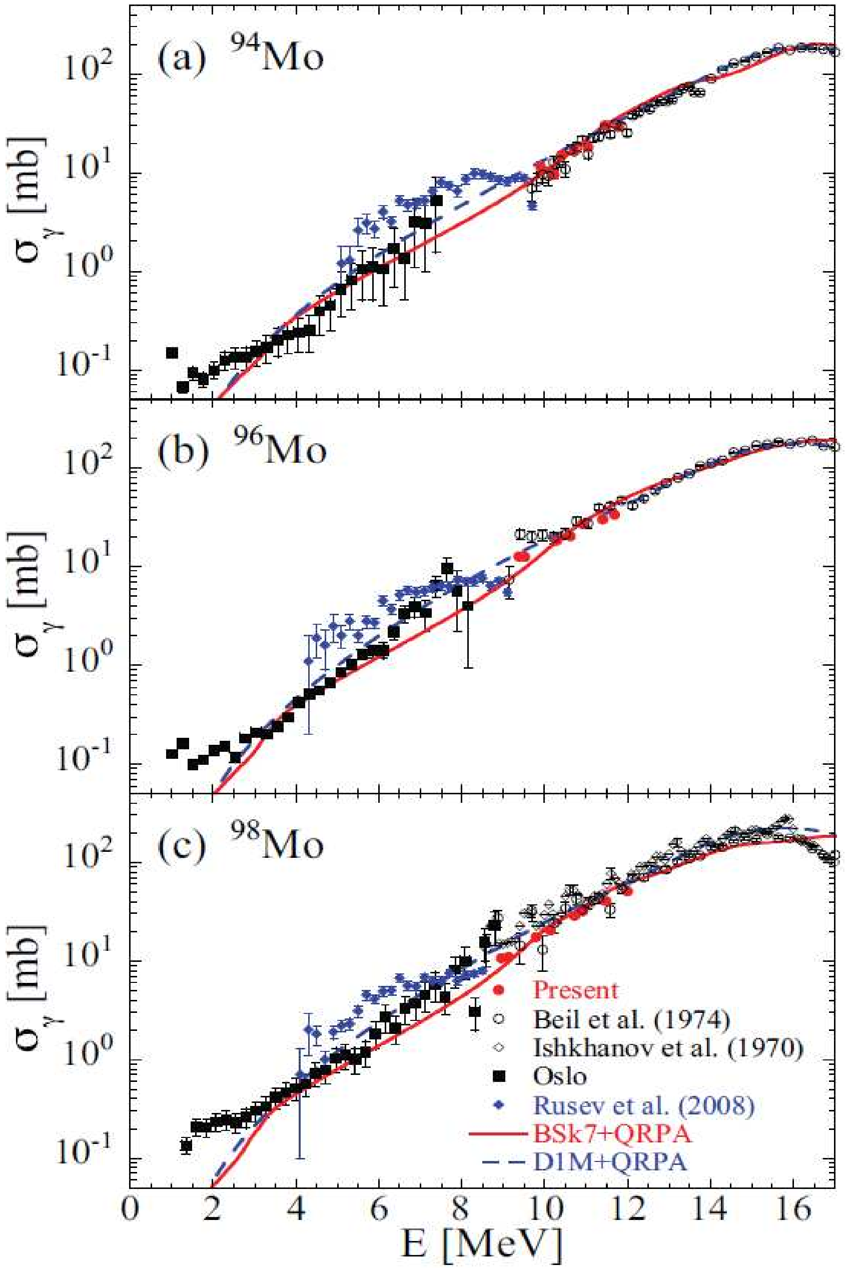




M. Dupuis, S.Péru, E. Bauge and T. Kawano,
13th International Conference on Nuclear Reaction Mechanisms, Varenna 2012
CERN-Proceedings-2012-002, p 95

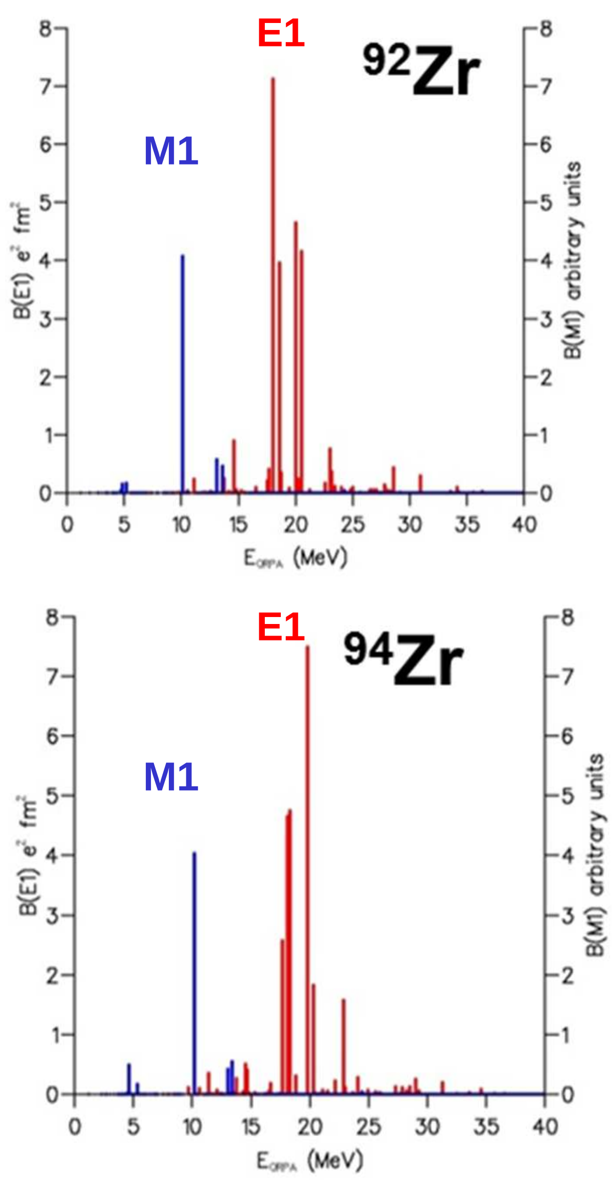
Photoneutron and Photo-absorption cross sections for Mo isotopes

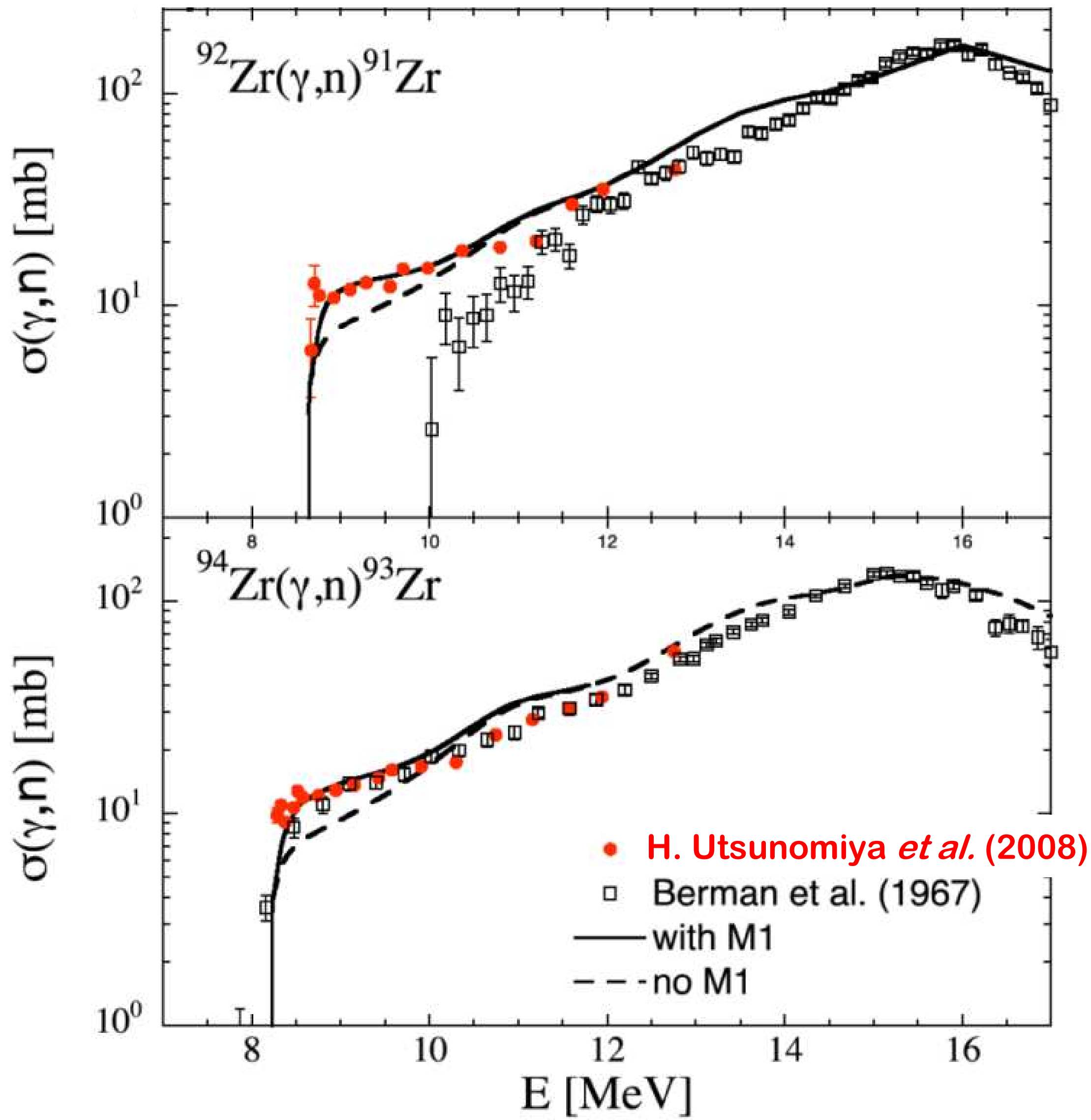




H. Utsunomiya et al, PRC88, 015805(2013)

Dipole electric and magnetic excitations for Zr isotopes

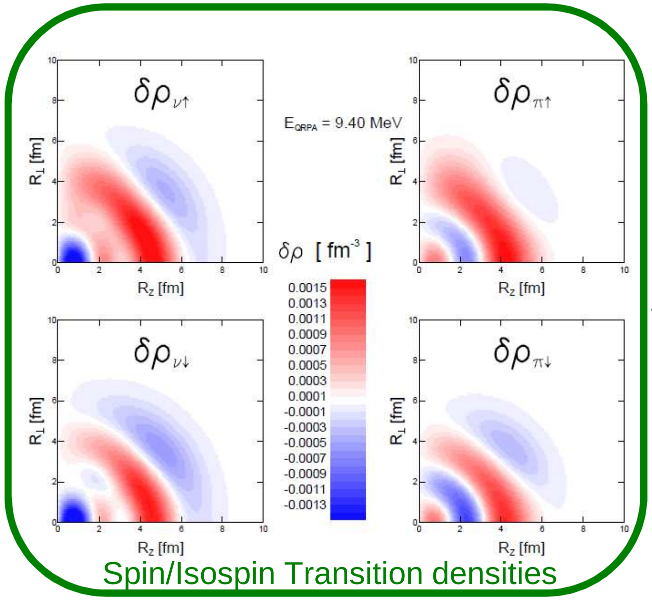


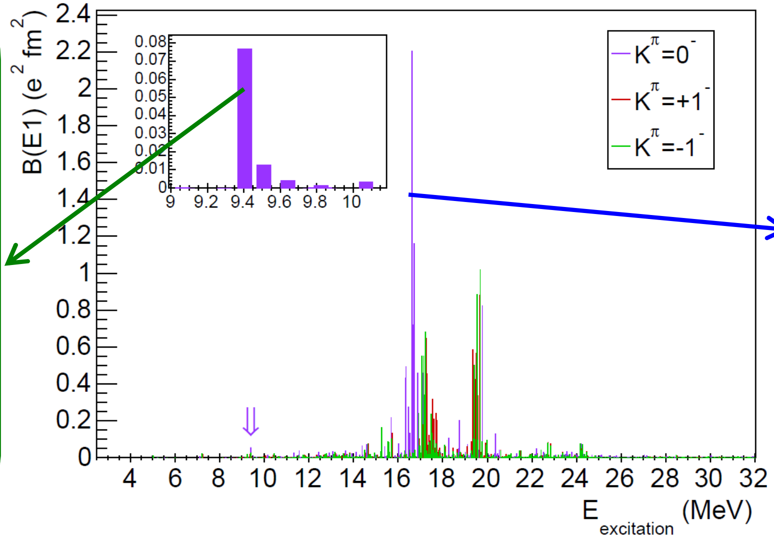


H. Utsunomiya et al, PRL 100, 162502 (2008)

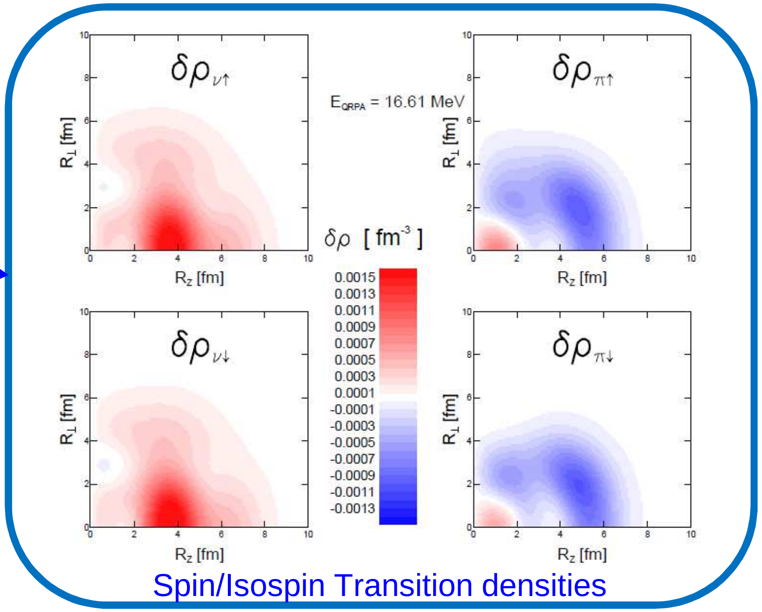
Low Energy Enhancement in the γ Strength of the Odd-Even Nucleus ^{115}In

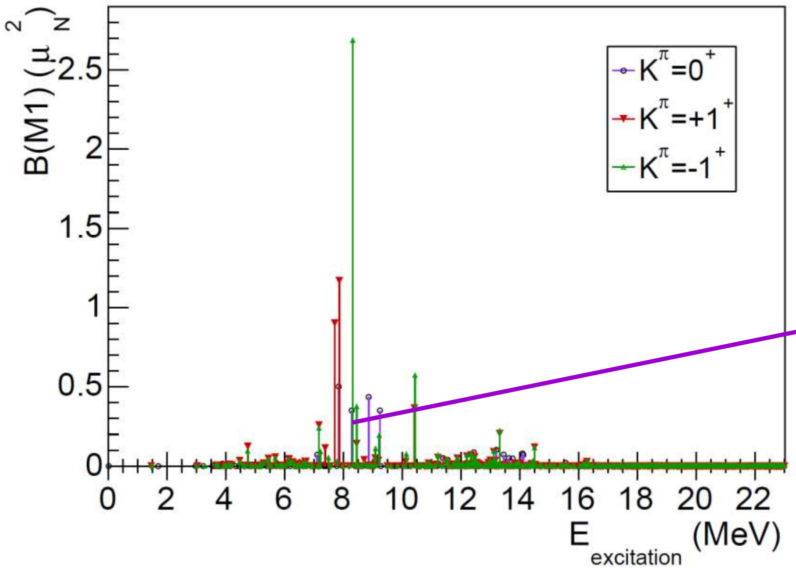
PDR Iso Scalar dipole

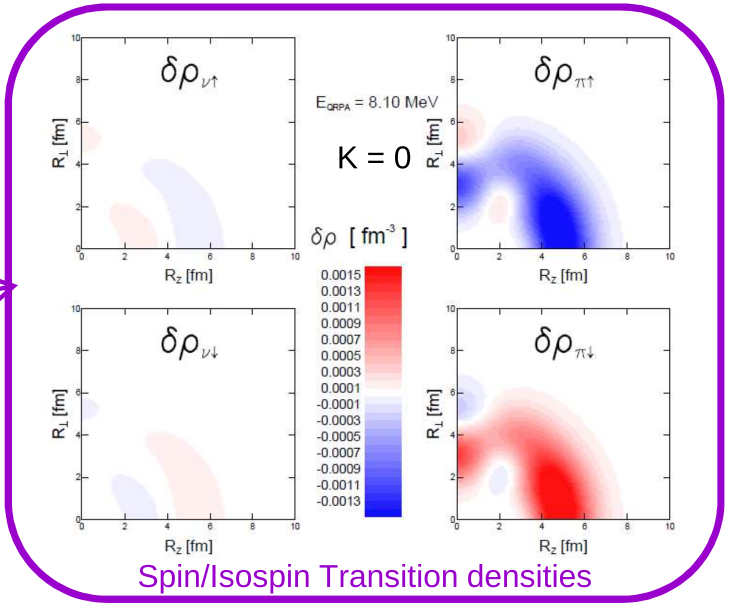




Iso Vector dipole





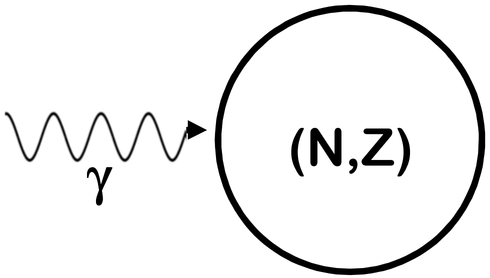


Spin flip

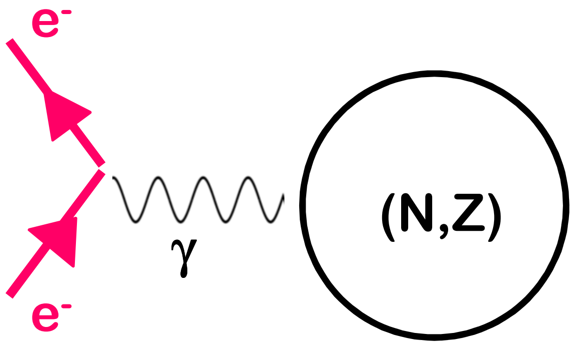
M. Versteegen et al, PRC 94, 044325 (2016)

Nuclear Excitations

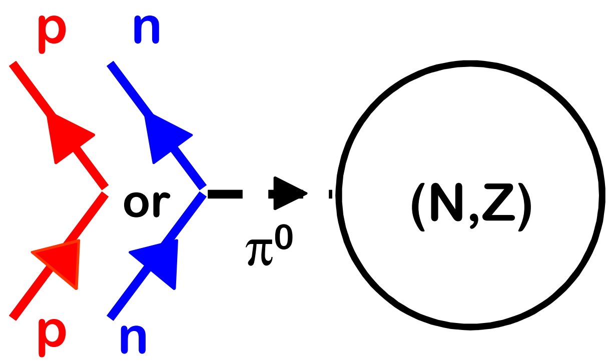
Photo-absorption

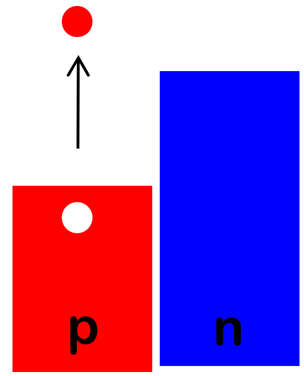


Electron scattering



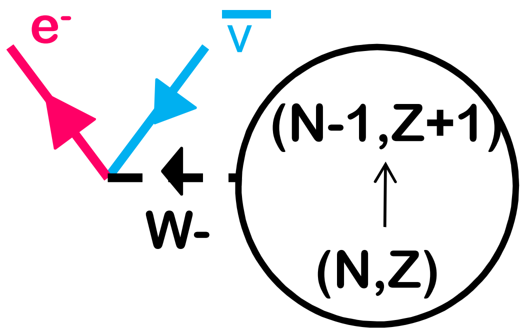
(p,p) or (n,n)



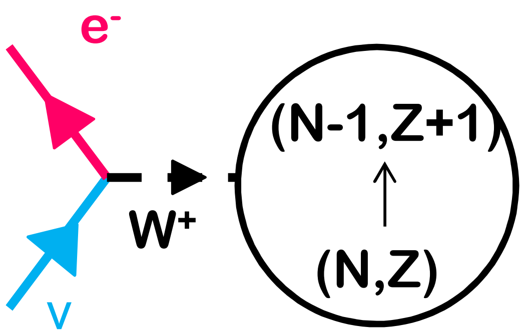


Charge exchange:

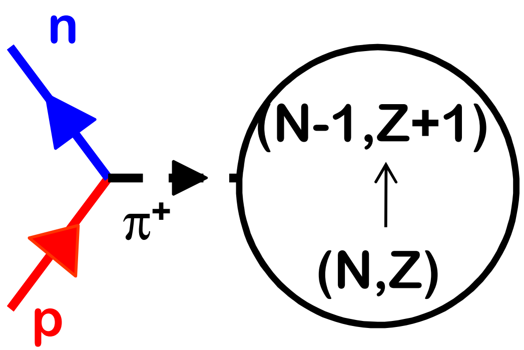
β decay

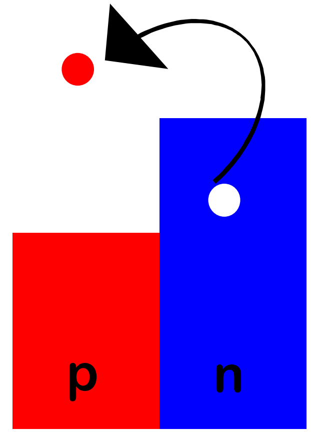


Neutrino scattering



(p,n) or (^{3}He,t)

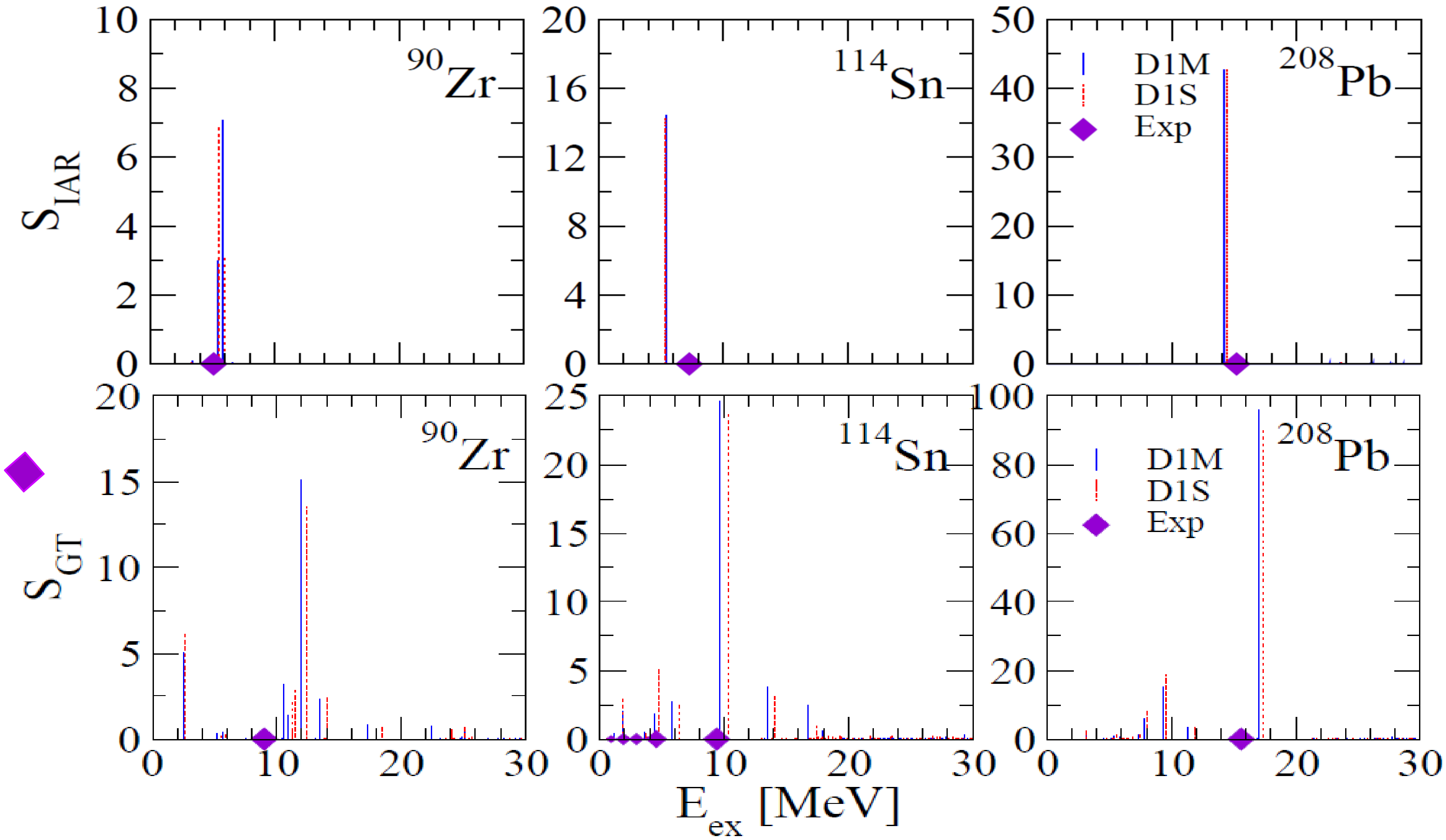




Gogny pnQRPA Strength Distributions

M. Martini, S. Péru and S. Goriely, Phys. Rev. C **89**, 044306 (2014)

Good agreement with experimental data



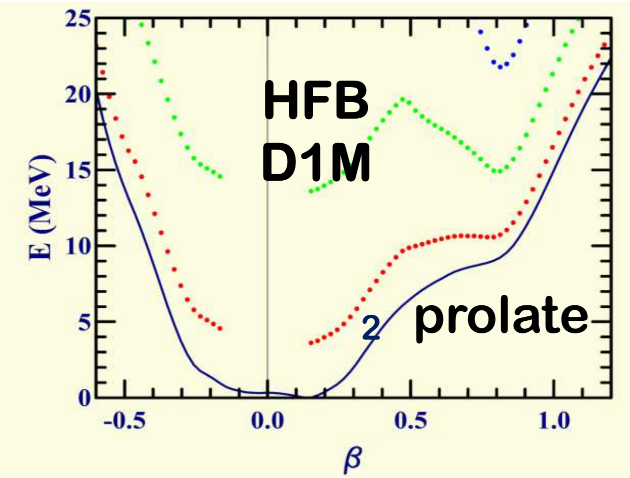
Here, the reference energy corresponds to the lowest 2-qp excitation associated with the ground state of the odd-odd daughter nucleus in which the quantum numbers of the single quasi-proton and neutron states are obtained from the self-consistent HFB calculation of the odd-odd system.

Charge exchange QRPA : GT resonances for β decay

M. Martini, S. Péru and S. Goriely, Phys. Rev. C **89**, 044306 (2014)

An example of deformed nucleus : ^{76}Ge

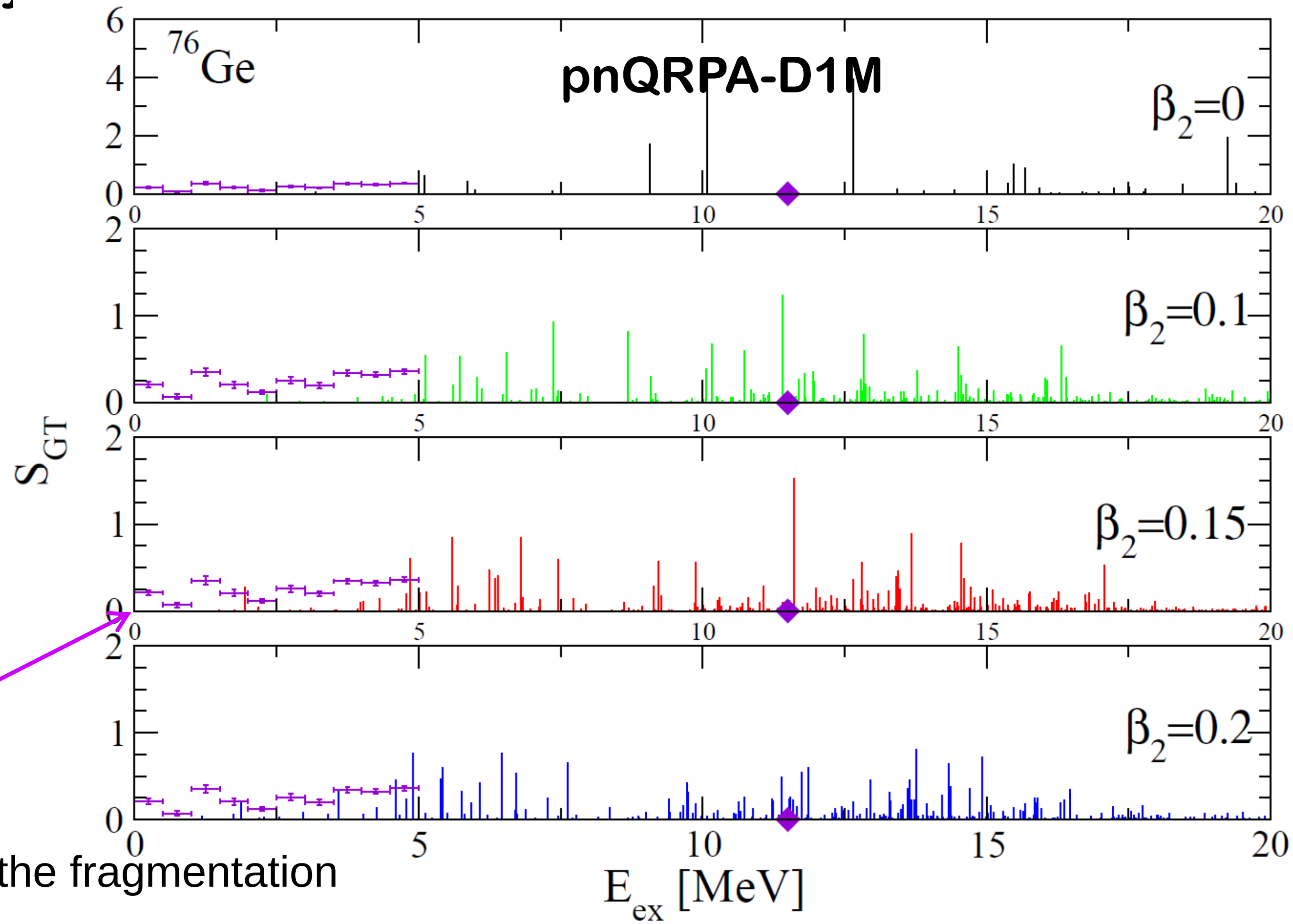
GT $J^{\pi}=1^+$ distributions obtained by adding twice the $K^{\pi}=1^+$ result to the $K^{\pi}=0^+$ one



β_2(min. HFB) = 0.15 **γ(min.HFB) = 0°**

β_2 (0^+_1:5DCH) = 0.26 **γ(0^+_1:5DCH) = 26°**

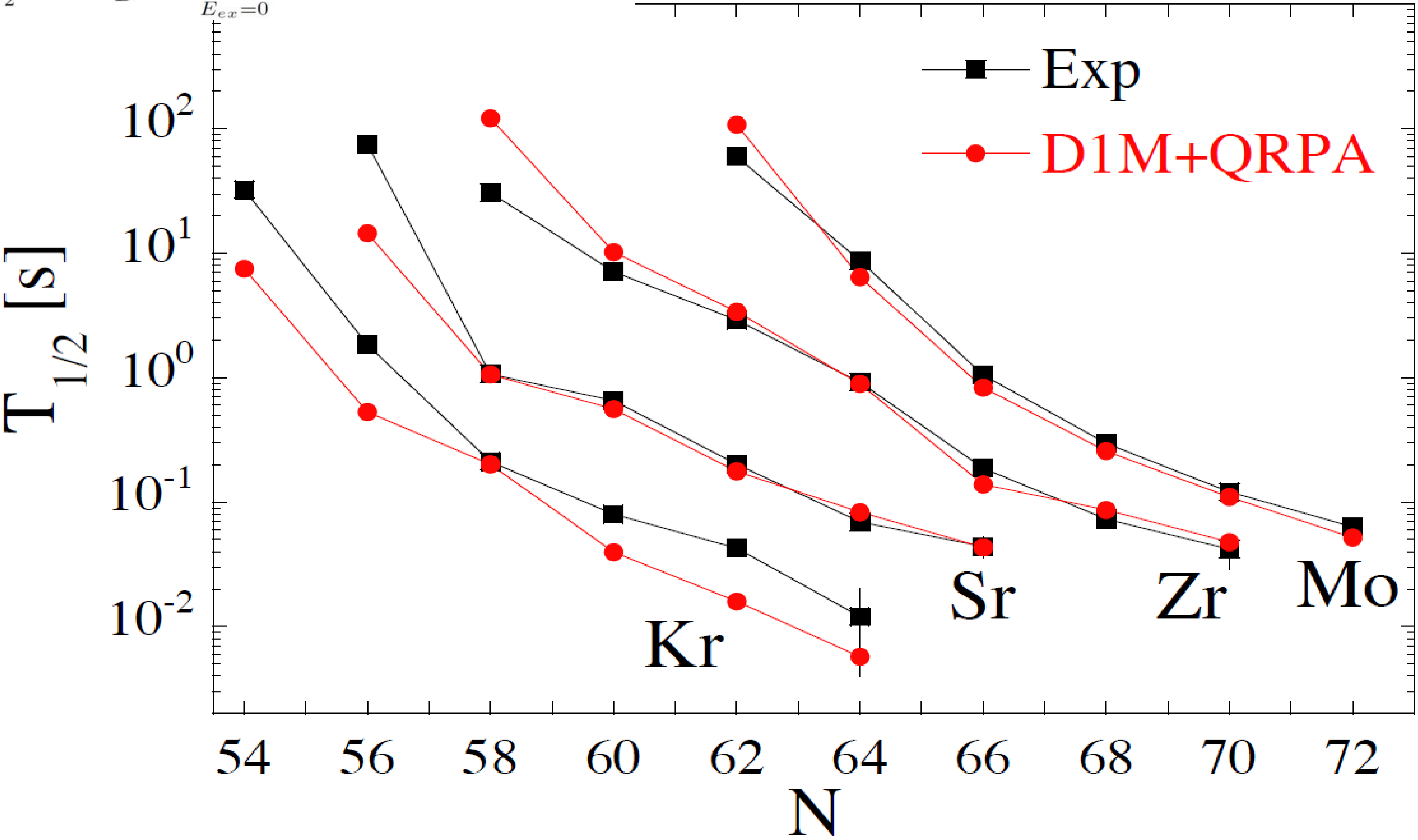
Experiment
Thies et al., Phys. Rev. C 86, 014304 (2012)



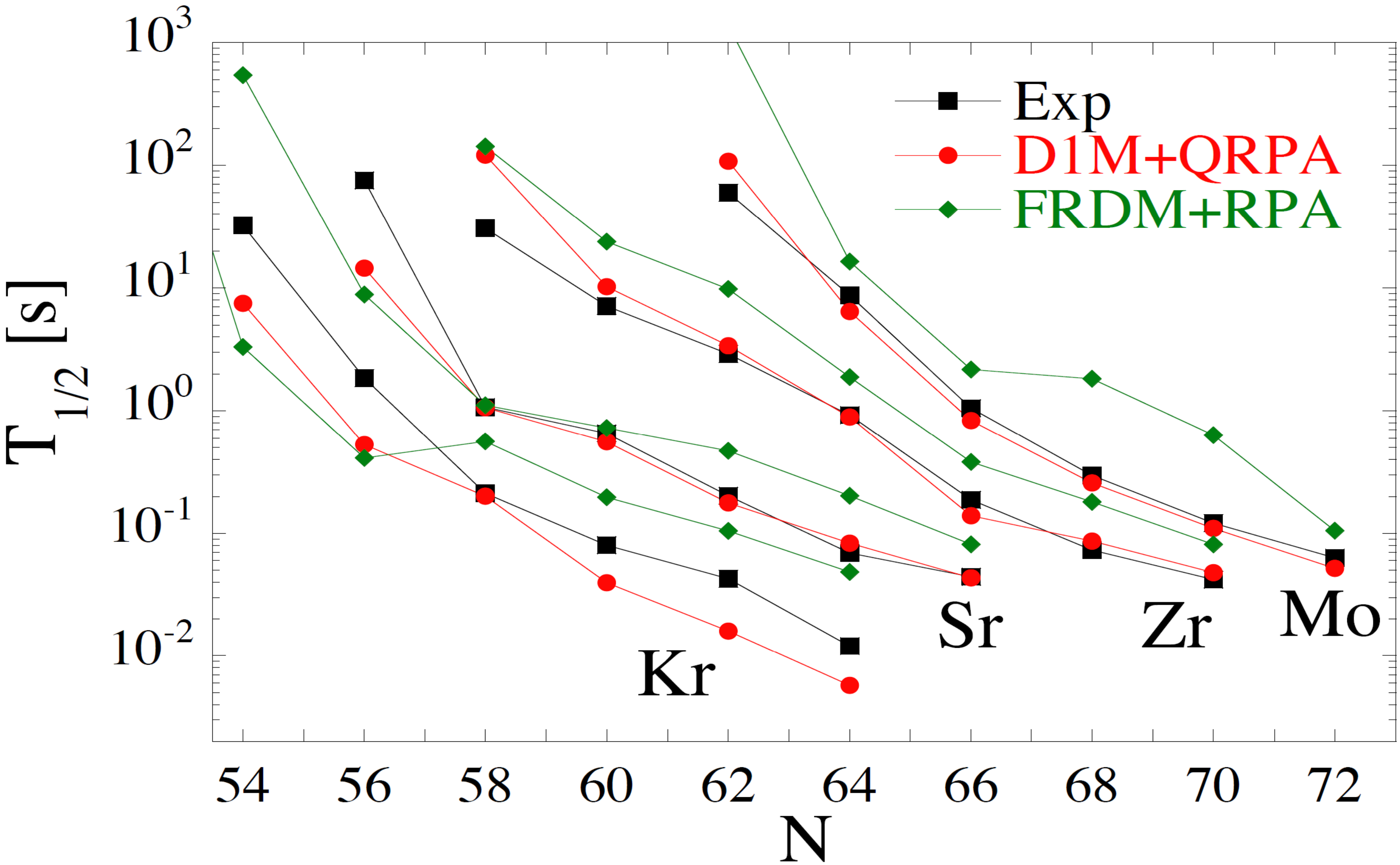
- The deformation tends to increase the fragmentation
- Displacements of the peaks
- Deformation influences the low energy strength hence β decay half-lives are expected to be affected

β^- decay half-lives of deformed isotopic chains

$$\frac{\ln 2}{T_{1/2}} = \frac{(g_A/g_V)^2_{\rm eff}}{D} \sum_{E_{ex}=0}^{Q_\beta} f_0(Z, A, Q_\beta - E_{ex}) S_{GT}(E_{ex})$$

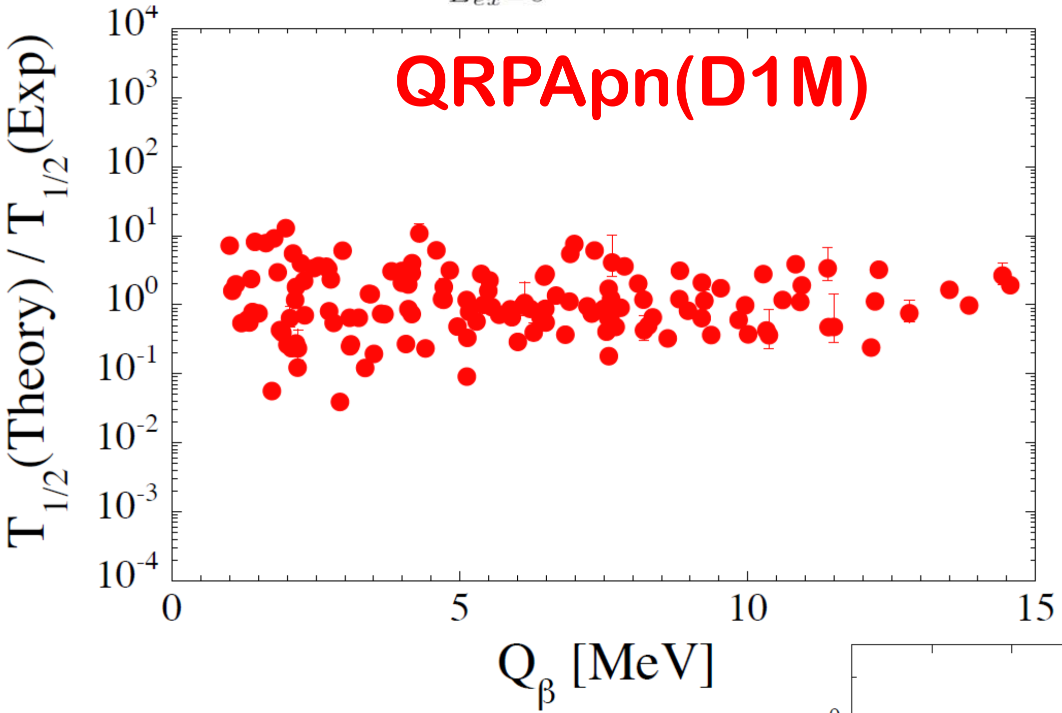


β^- decay half-lives of deformed isotopic chains

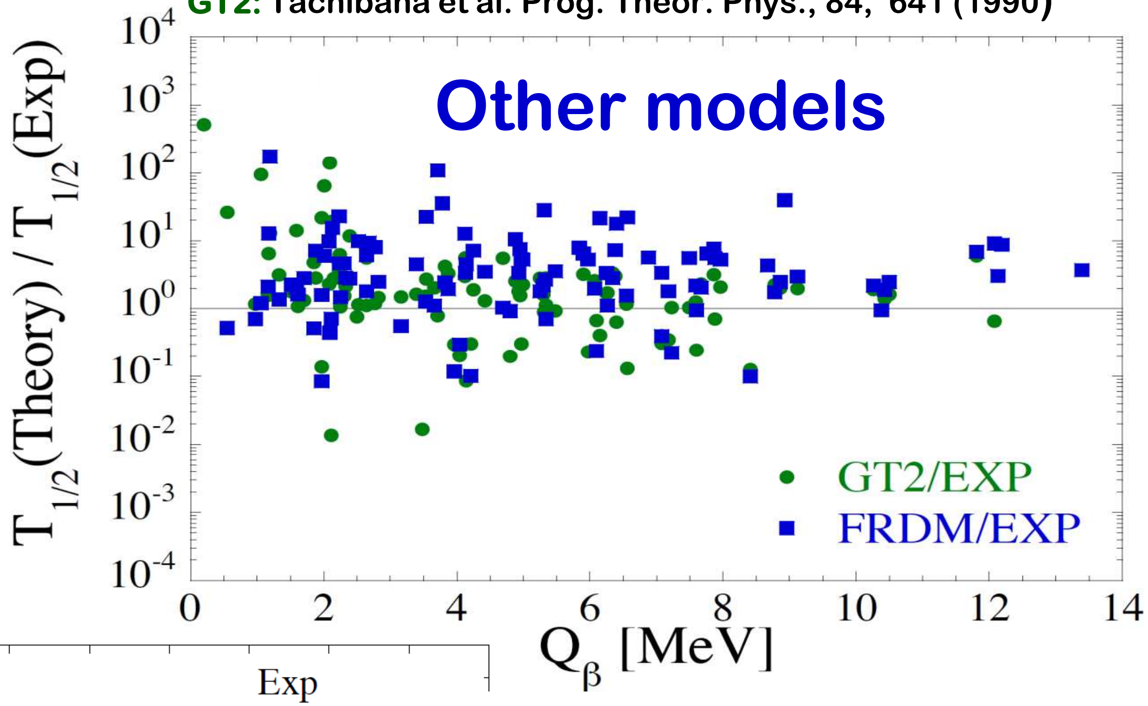


β^- decay half-life $T_{1/2}$: Comparison with other models

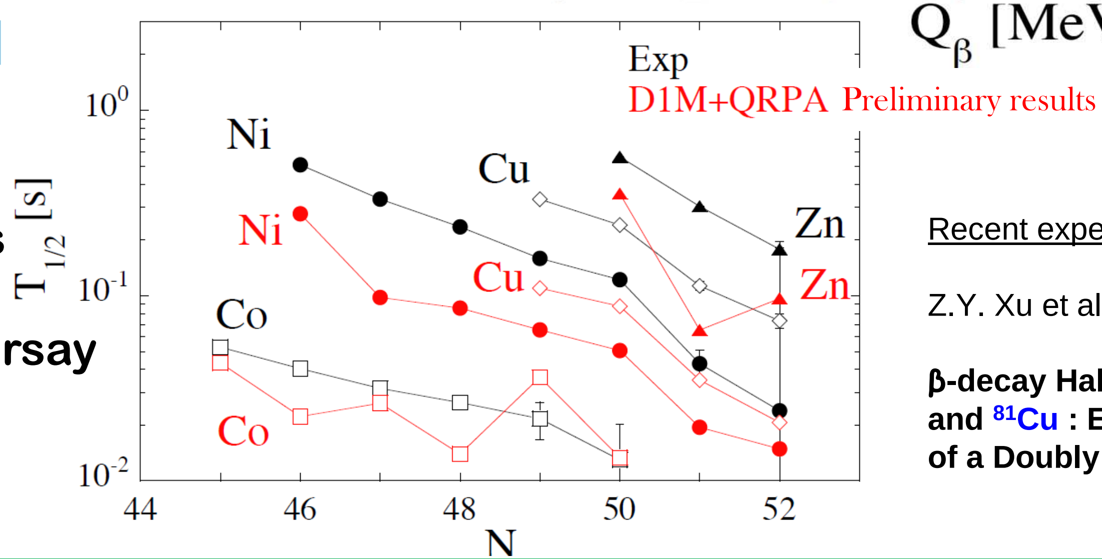
$$\frac{\ln 2}{T_{1/2}} = \frac{(g_A/g_V)^2_{\text{eff}}}{D} \sum_{E_{ex}=0}^{Q_\beta} f_0(Z,A,Q_\beta - E_{ex}) S_{GT}(E_{ex})$$



FRDM: Moller et al., ADNDT, 66,131 (1997)
GT2: Tachibana et al. Prog. Theor. Phys., 84, 641 (1990)



Extension to odd systems in collaboration with Isabelle Deloncle (CSNSM) Orsay



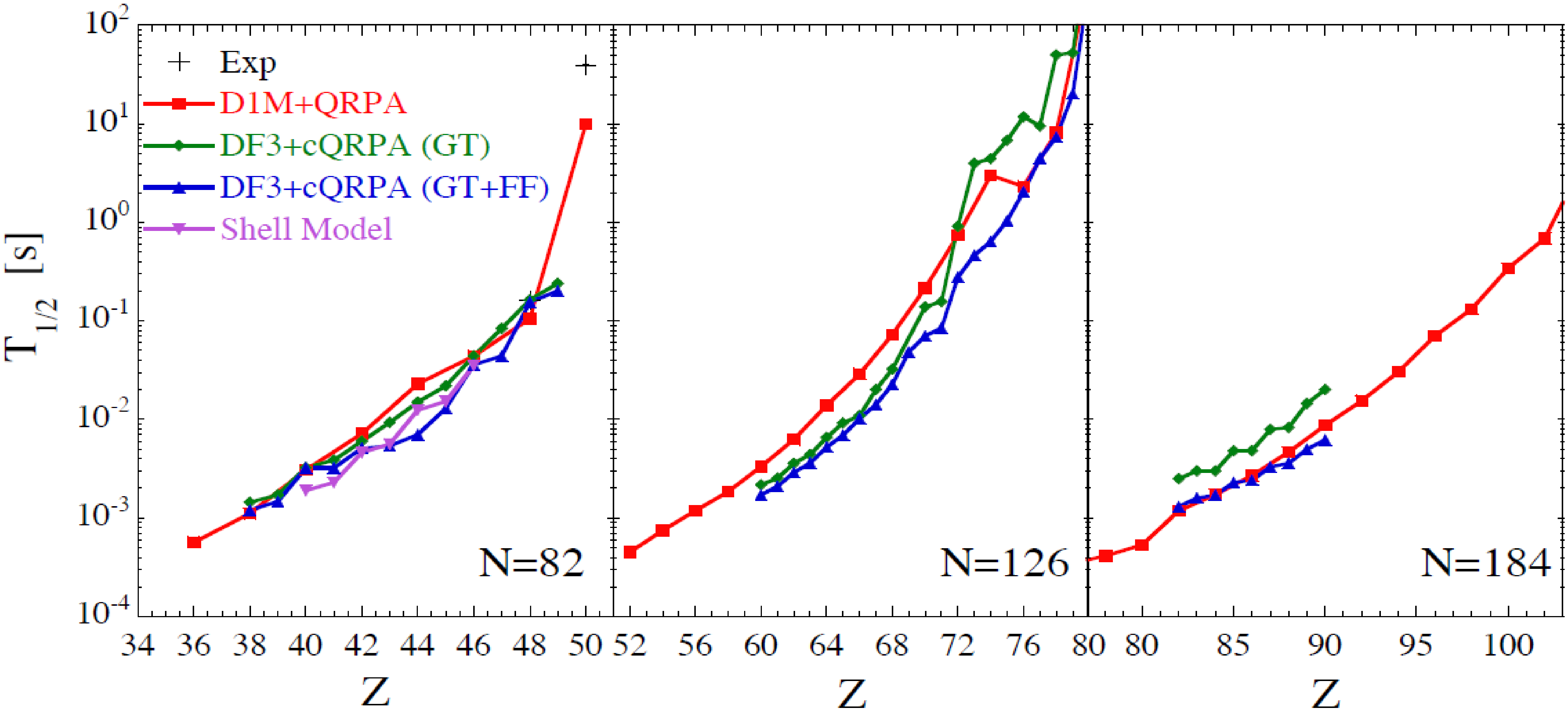
Recent experimental results

Z.Y. Xu et al, PRL 113, 032505 (2014)

β-decay Half lives of ^{76,77}Co, ^{79,80}Ni and ^{81}Cu : Experimental indication of a Doubly Magic ^{78}Ni

β^- decay half-lives of the N=82, 126, 184 isotones

Relevance for the r-process nucleosynthesis



DF3+cQRPA: Borzov et al., PRC 62, 035501 (2000)

Shell Model: Martinez-Pinedo et al., PRL 83, 4502 (1999)

Possible origins of differences: GT Strengths, estimation of Q_β values, …

Thanks for your attention