

Benoît BRAÏDA

How VB theory can help to understand multicenter bonding

**Laboratoire de Chimie Théorique
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Purpose

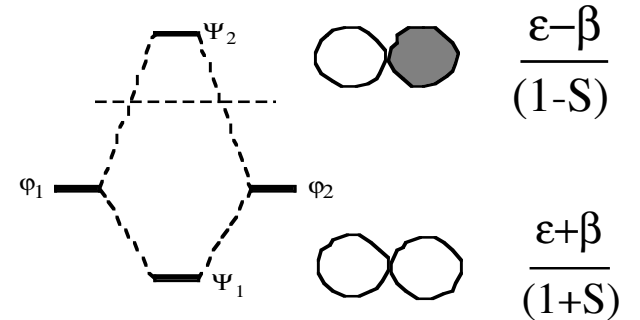
How VB theory can elucidate electronic structure of large & complex systems

- VB description of elementary interactions
- Qualitative VB & MO analysis of «Pancake bonding»
- Quantitative VB results, and conclusion

Qualitative VB

Effective Hamiltonian : $H^{\text{eff}} = (h(1) + h(2) + h(3) + \dots)$

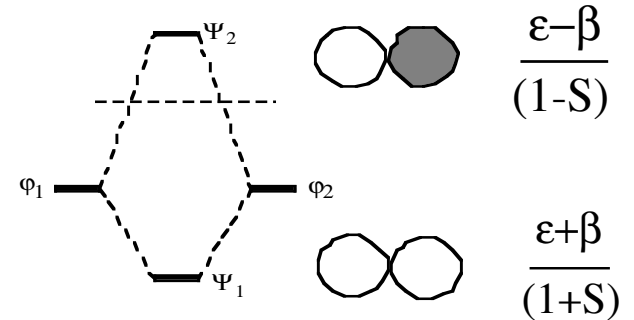
Parameters: β , S , ϵ (same as in the MO framework)



Qualitative VB

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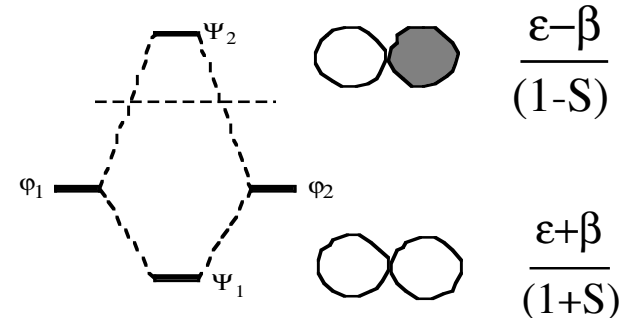
1) Energy of a determinant :

$$\langle D_i | H | D_i \rangle = \frac{-2n\beta S}{1 - S^2} \quad n = \text{N}^{\#} \text{ of neighboring } (\uparrow\uparrow) \text{ pairs}$$

Qualitative VB

Effective Hamiltonian : $H^{\text{eff}} = (\mathbf{h}(1) + \mathbf{h}(2) + \mathbf{h}(3) + \dots)$

Parameters: β , S , ϵ (same as in the MO framework)



1) Energy of a determinant :

$$\langle D_i | H | D_i \rangle = \frac{-2n\beta S}{1 - S^2} \quad n = \text{N}^{\#} \text{ of neighboring } (\uparrow\uparrow) \text{ pairs}$$

2) Off diagonal terms :

- Determinants differ by 2 spinorbitals:

$$\langle (| a \bar{b} |) | H | (| b \bar{a} |) \rangle = 2\beta_{ab} S_{ab}$$

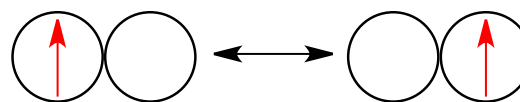
- Determinants differ by + than 2 spinorbitals :

$$\langle D_i | H | D_j \rangle = 0$$

Qualitative VB

- Elementary interactions :

1-e bond ($A\uparrow B$) =



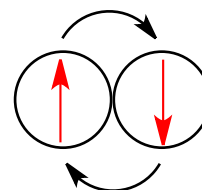
VB

$$\frac{\beta}{1+S}$$

MO

$$\frac{\beta}{1+S}$$

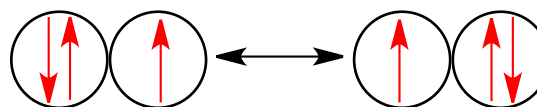
2-e bond ($A-B$) =



$$\frac{2\beta S}{1+S^2}$$

$$\frac{2\beta}{1+S}$$

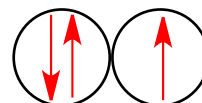
3-e bond ($A{:}\cdot B$) =



$$\frac{\beta(1-3S)}{1-S^2}$$

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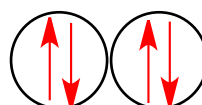
3-e repulsion ($A\uparrow\downarrow \uparrow B$) =



$$\frac{-2\beta S}{1-S^2}$$

$$\frac{-2\beta S}{1-S^2}$$

4-e repulsion ($A\uparrow\downarrow \downarrow\uparrow B$) =

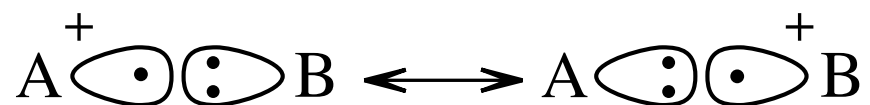


$$\frac{-4\beta S}{1-S^2}$$

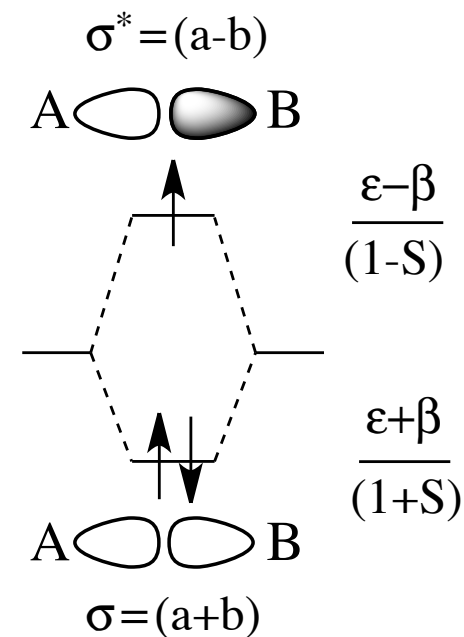
$$\frac{-4\beta S}{1-S^2}$$

The three-electron bond*

- VB vs. MO qualitative descriptions :



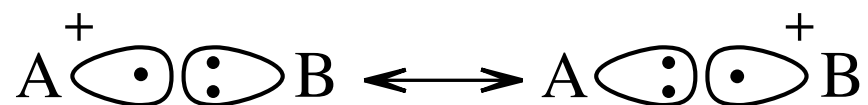
$$\begin{aligned} \Psi_{MO} &= |\sigma \bar{\sigma} \sigma^*| = |(a+b) \overline{(a+b)} (a-b)^*| \\ &= \dots = -|a\bar{a}b| - |b\bar{b}a| = \Psi_{VB} \end{aligned}$$



* Pauling, L. J. Am. Chem. Soc. **1931**, 53, 3225

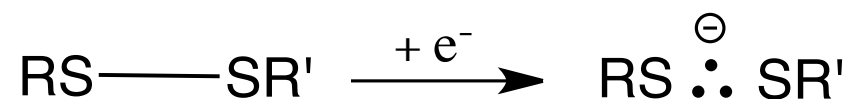
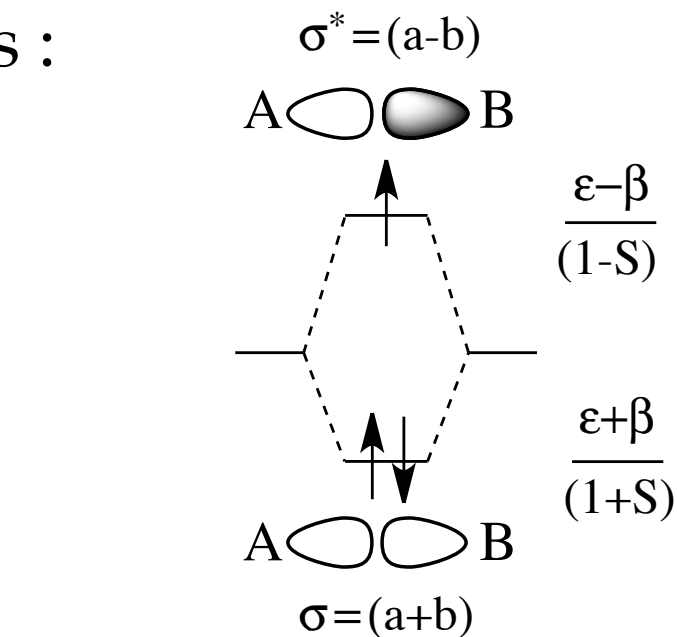
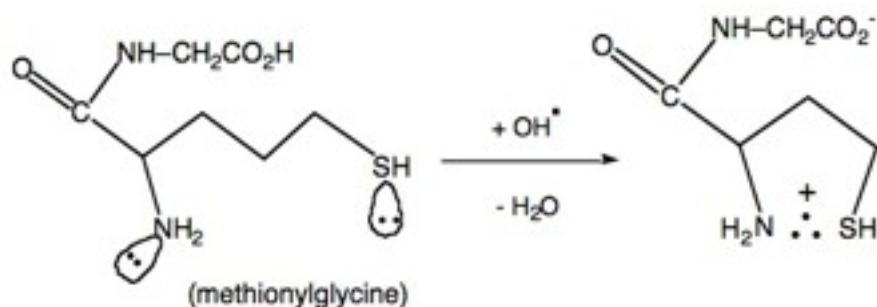
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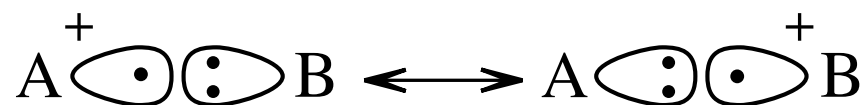
- Examples :



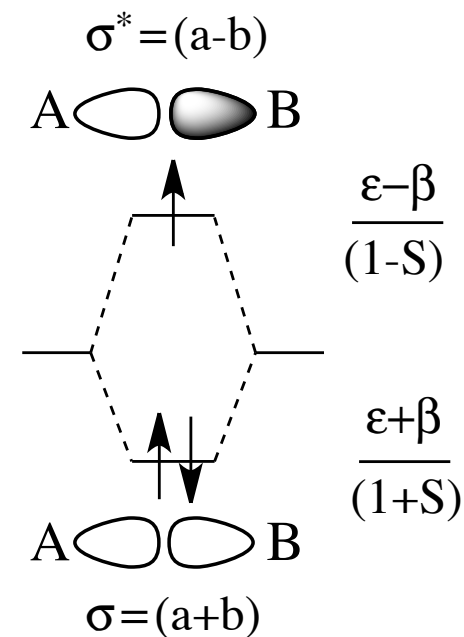
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The three-electron bond*

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$$\begin{aligned} \Psi_{MO} &= |\sigma \bar{\sigma} \sigma^*| = |(a+b)(\overline{a+b})(a-b)^*| \\ &= \dots = -|a\bar{a}b| - |b\bar{b}a| = \Psi_{VB} \end{aligned}$$



- Some features :

- Long bonds : $R_{eq} S-S(2e) \approx 2\text{\AA}$ $R_{eq} S \text{ : } S(2e) \approx 2.8\text{\AA}$
- Small S_{ab} overlaps : $E = \frac{\beta(1-3S)}{(1-S^2)} \Rightarrow S_{opt} \approx 0.17$
- Correlation purely dynamical : $D_e F_2^- : UHF \approx 0. ; MP2 \approx 30. \text{ kcal/mol}$

* Pauling, L. J. Am. Chem. Soc. **1931**, 53, 3225

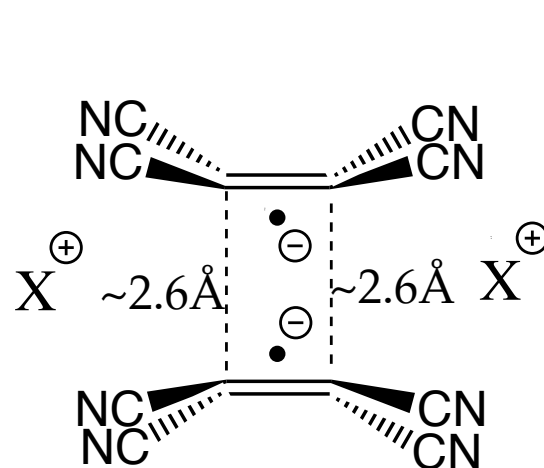
«Pancake bonding»

- Prototype : TCNE_2^{2-}

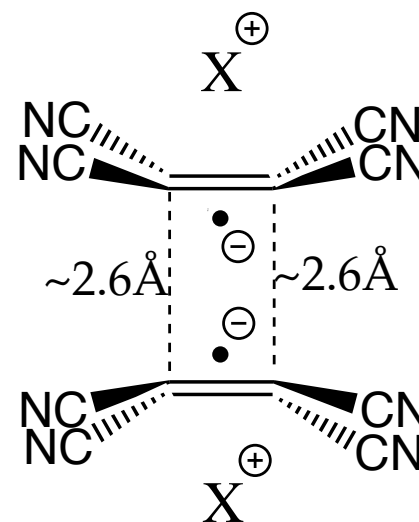
- $R_{\text{C}-\text{C}} < \text{vdW radii (3.4\text{\AA})}$

- Strongly bonded :

not only electrostatic &
dispersion

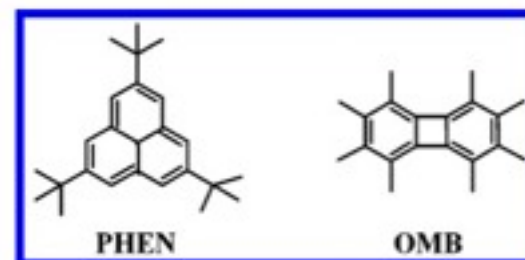
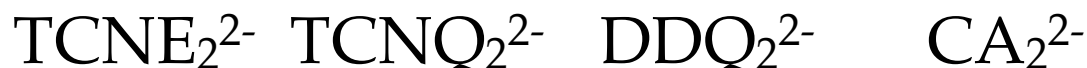


$$D_e \approx 80. \text{ kcal/mol}$$



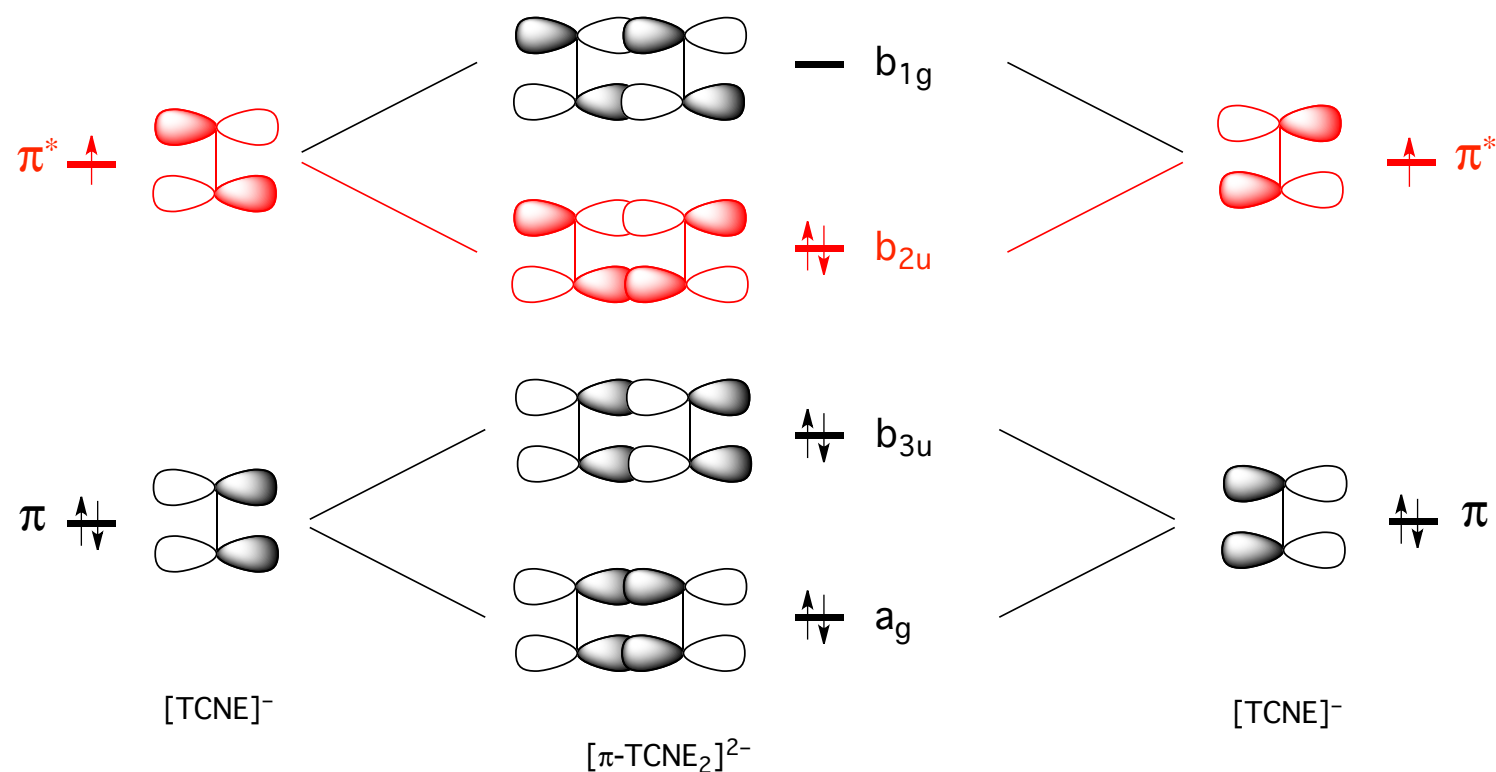
$$D_e \approx 11. \text{ kcal/mol}$$

- Others :



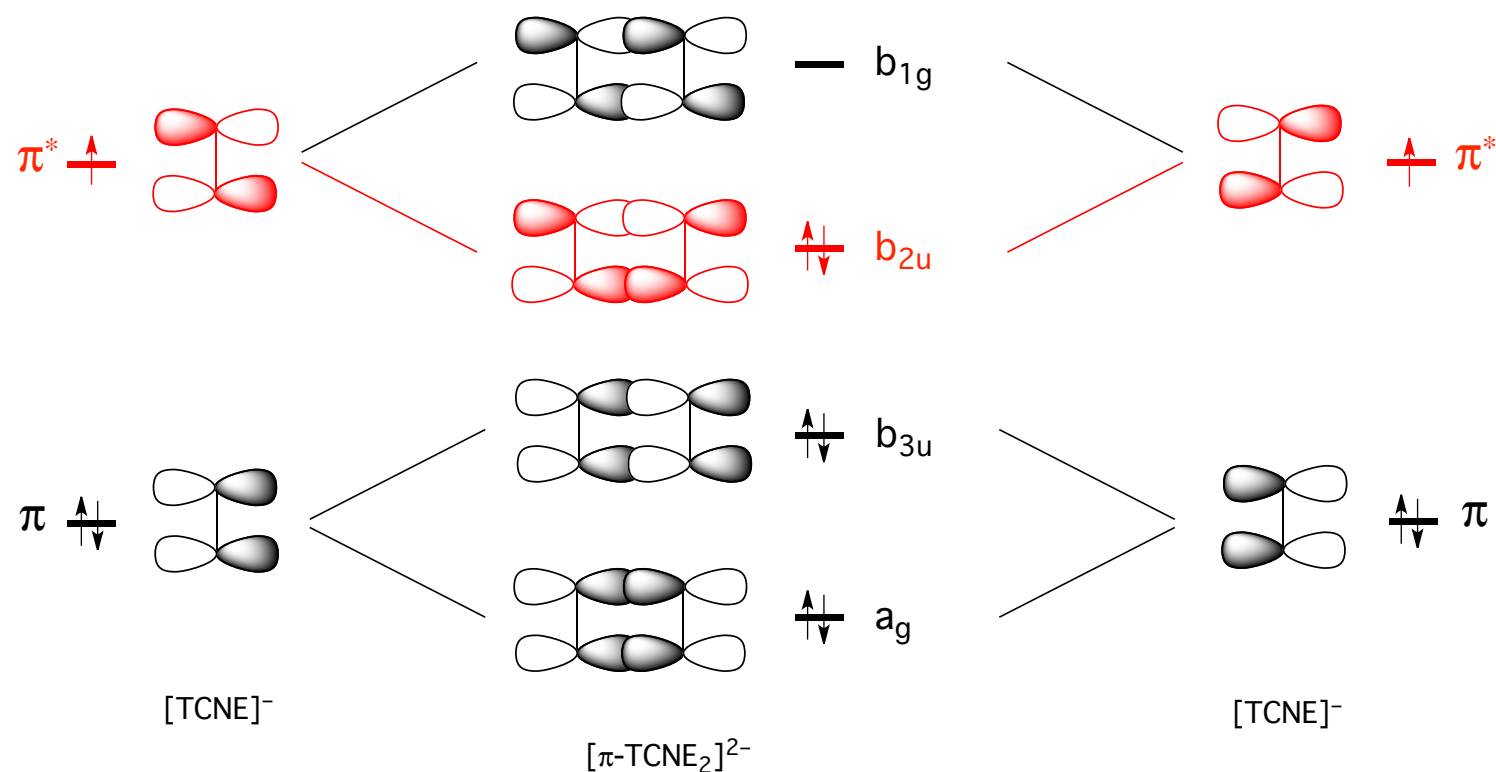
«First sight» MO analysis

- A multicenter 2e⁻ bond ?



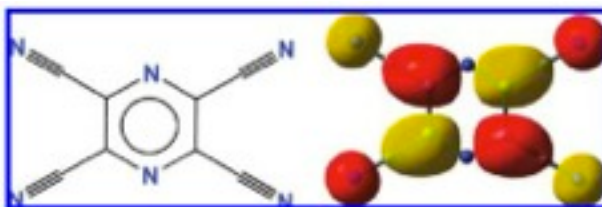
«First sight» MO analysis

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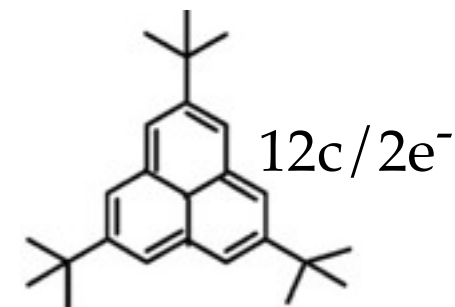


=> a **4c/2e⁻ bond**

extension : **nc/me⁻ bond** :



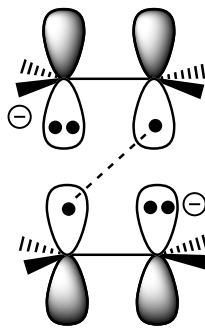
8c/2e⁻



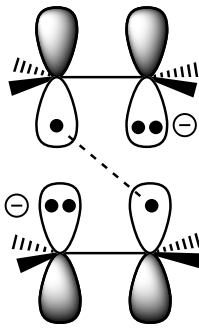
12c/2e⁻

VB analysis

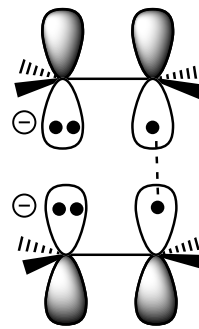
- VB set of structures :



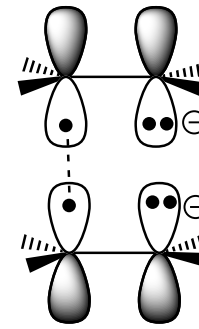
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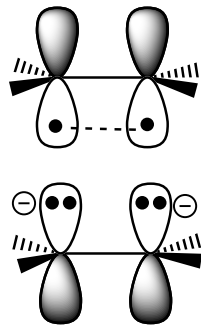
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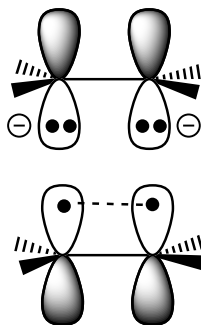
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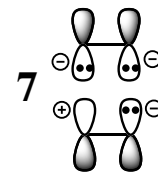
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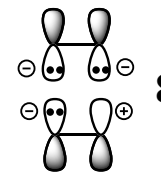
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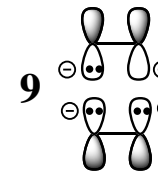
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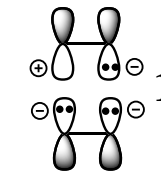
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8



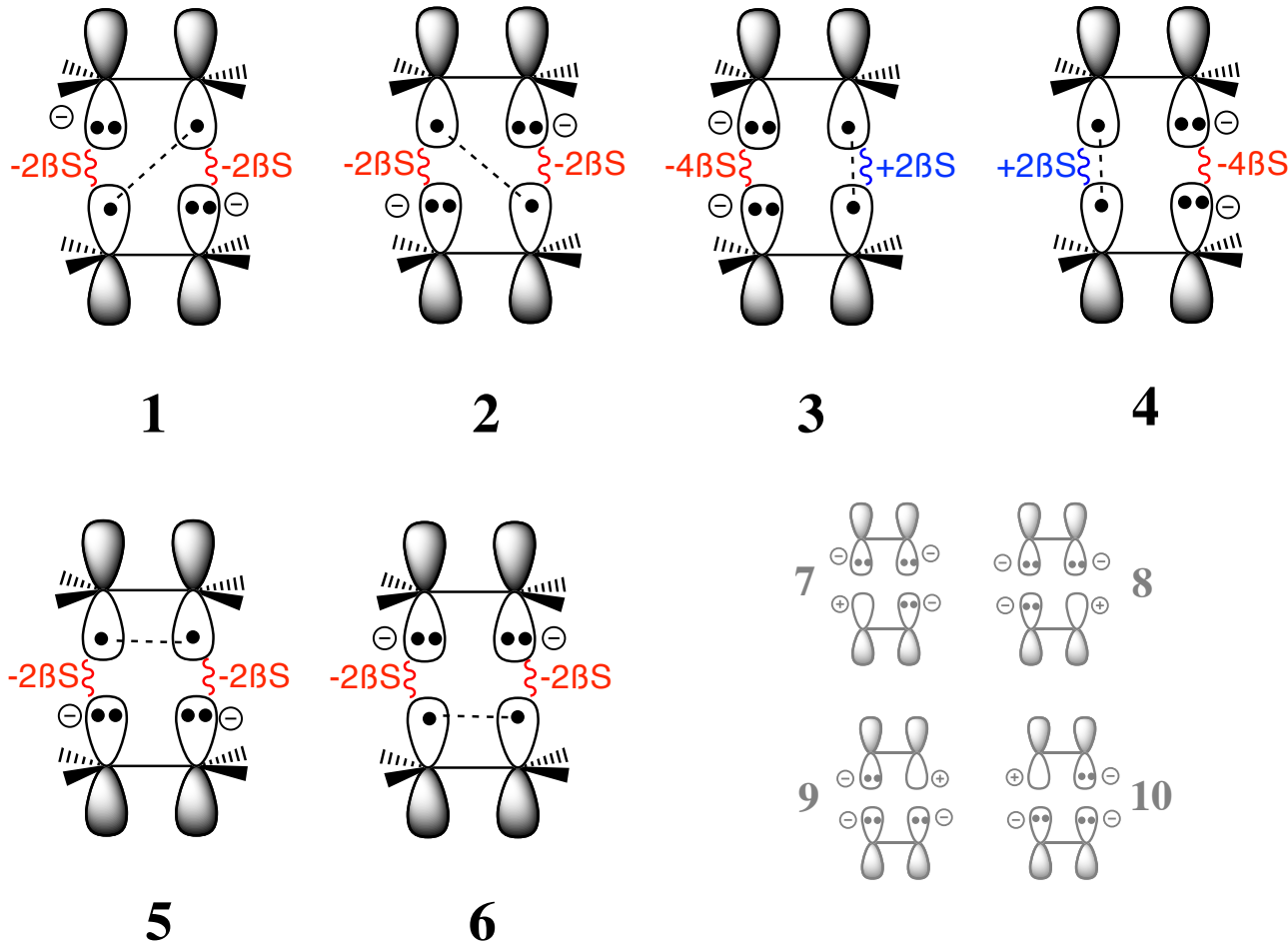
9



10

VB analysis

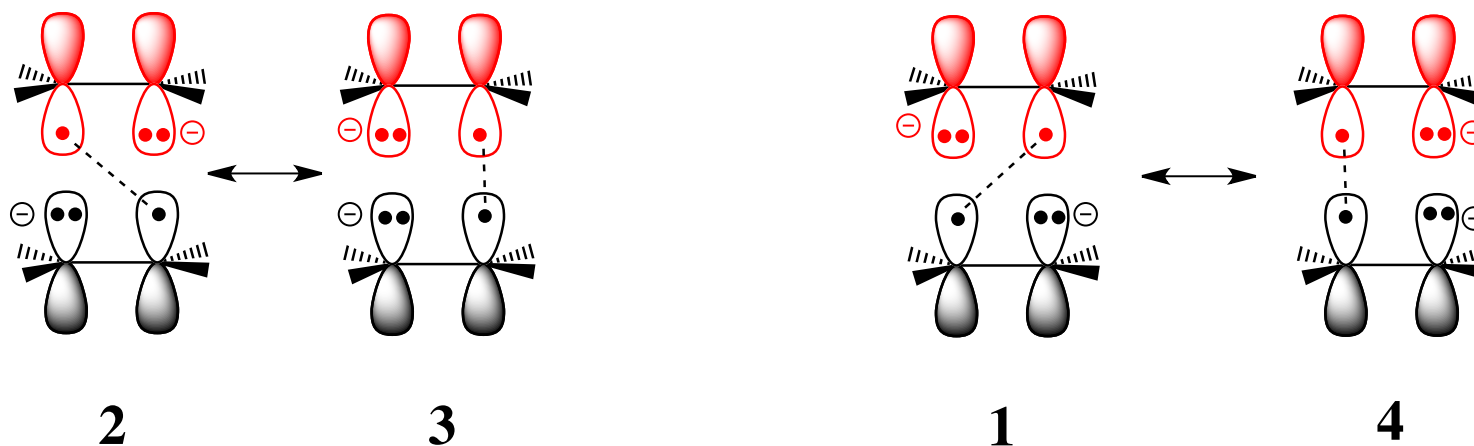
- VB set of structures :



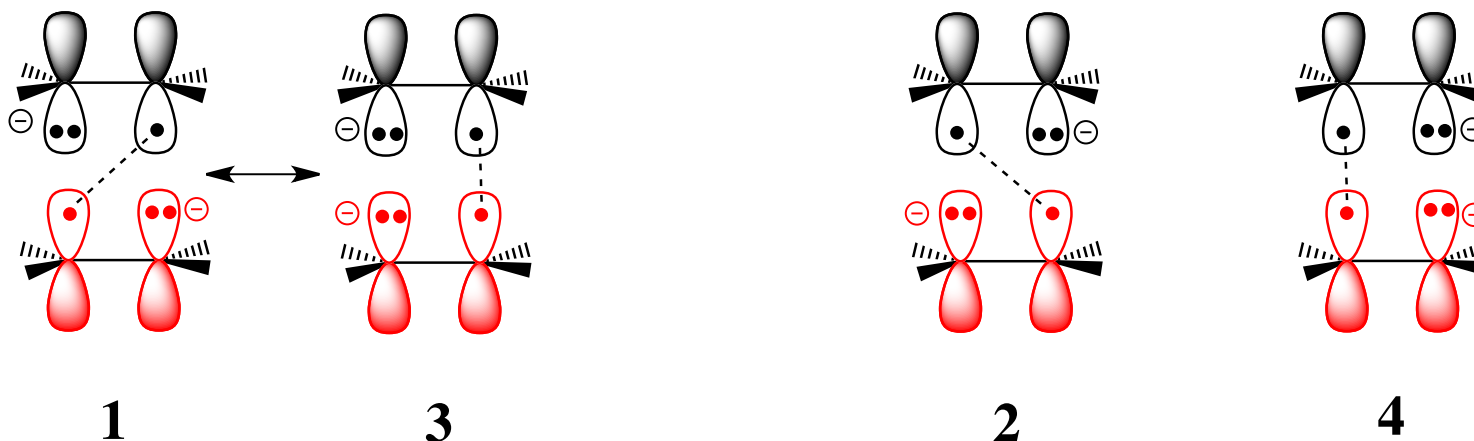
=> No structure is bonding by itself, **all the bonding comes from the resonance**

VB analysis

- $2 \leftrightarrow 3$ and $1 \leftrightarrow 4$: **intra-fragment $3e^- \pi$ bond** (upper part) :

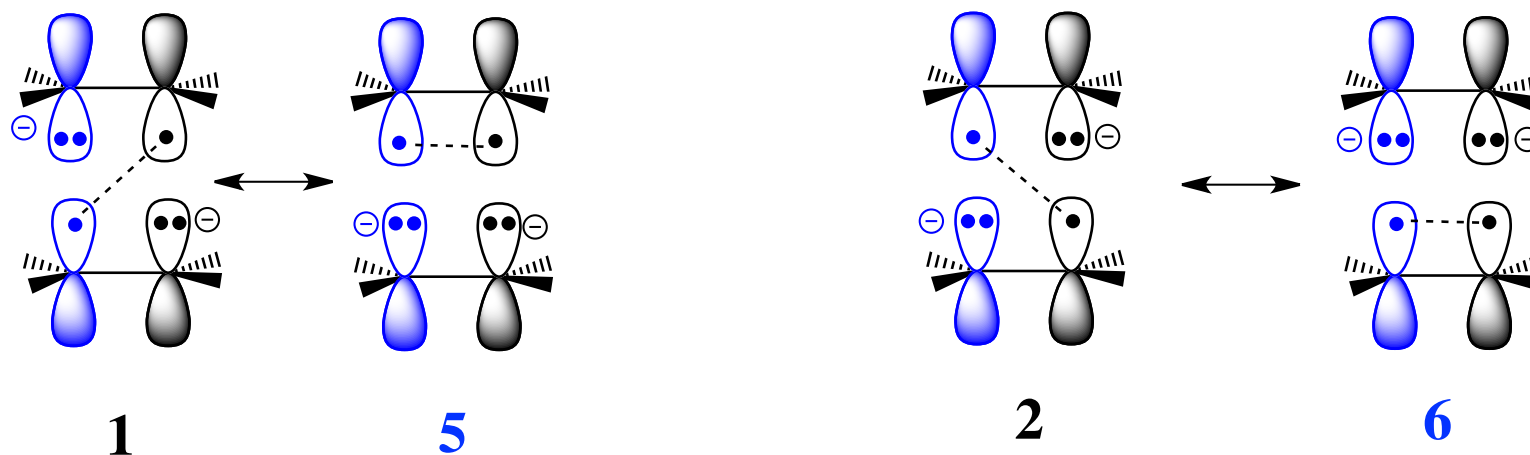


- $1 \leftrightarrow 3$ and $2 \leftrightarrow 4$: **intra-fragment $3e^- \pi$ bond** (lower part) :

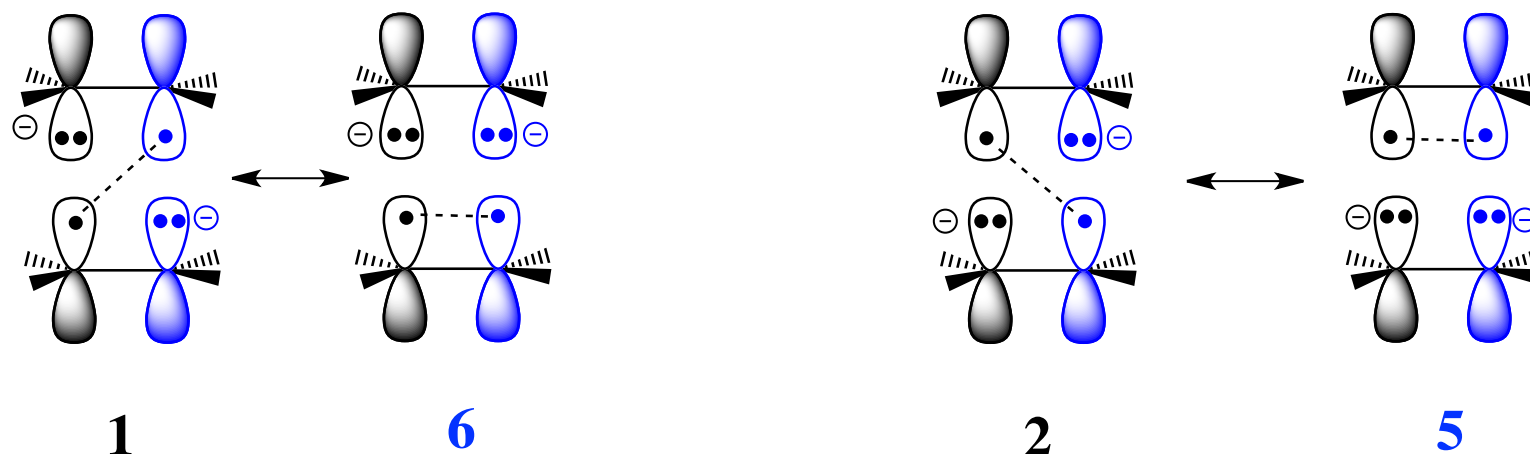


VB analysis

- $1 \leftrightarrow 5$ and $2 \leftrightarrow 6$: **inter-fragment $3e^- \pi$ bond** (left-hand side) :

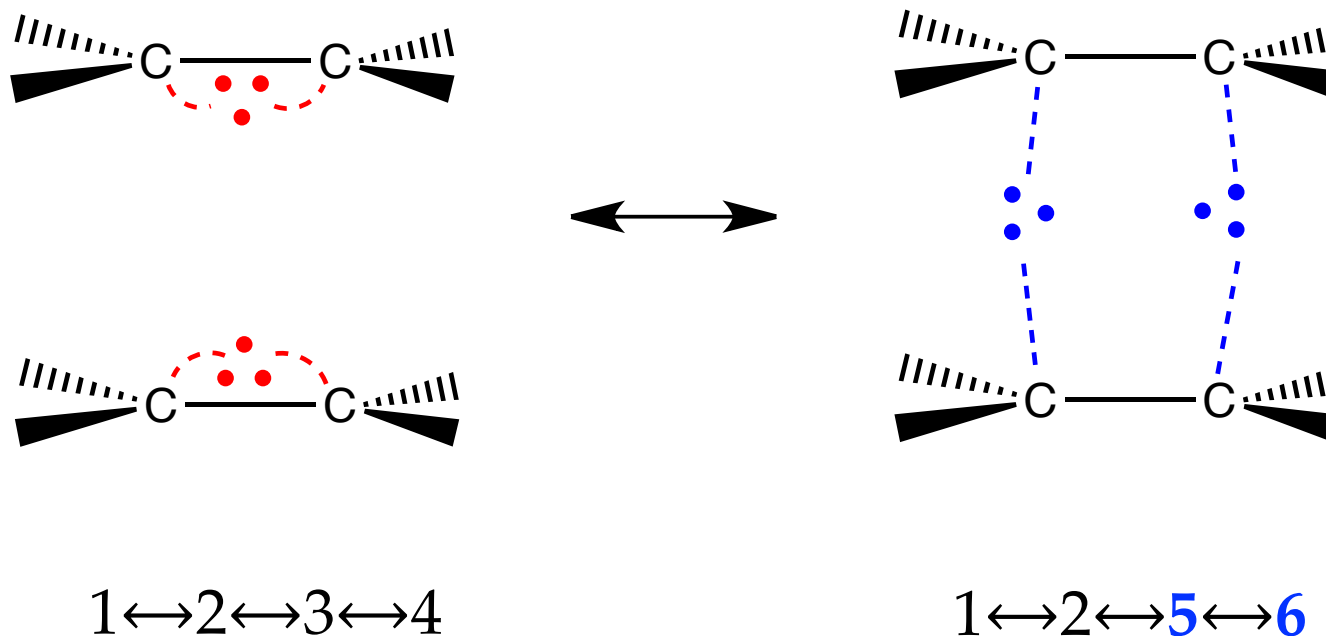


- $2 \leftrightarrow 5$ and $1 \leftrightarrow 6$: **inter-fragment $3e^- \pi$ bond** (right-hand side) :



VB analysis

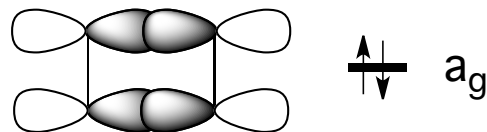
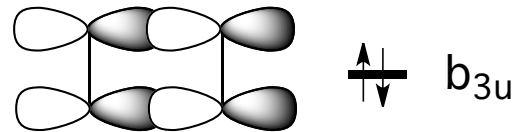
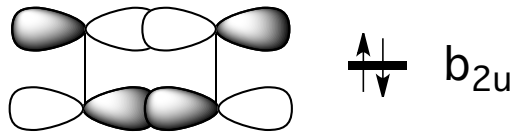
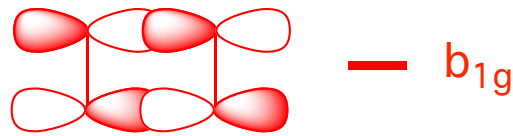
- Conclusion of the qualitative VB analysis :



$\Rightarrow$ 3e⁻ bonding nature of TCNE anion dimer

MO/VB mapping

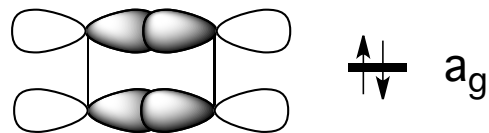
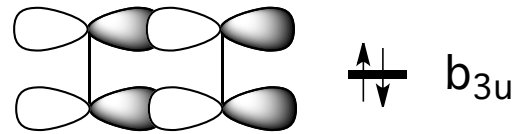
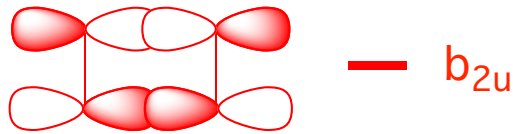
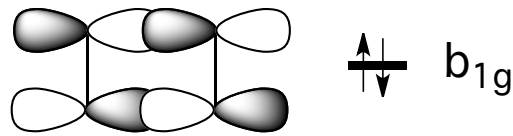
- HF determinant - development in VB basis of structures :



$$\Psi_{HF} = |a_g \bar{a}_g b_{3u} \bar{b}_{3u} b_{2u} \bar{b}_{2u}| = \dots = +\Psi_1^{VB} + \Psi_2^{VB} - \Psi_3^{VB} - \Psi_4^{VB} - \Psi_5^{VB} - \Psi_6^{VB}$$

MO/VB mapping

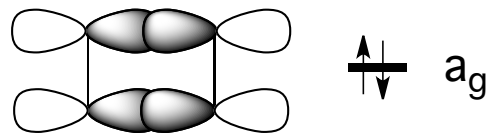
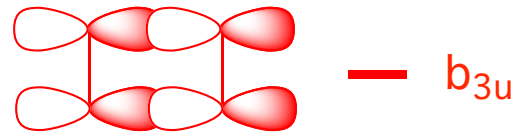
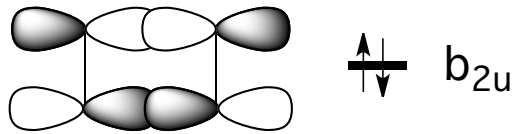
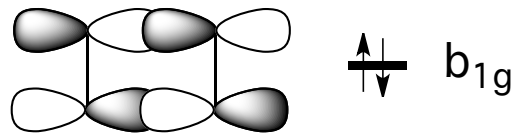
- 1st excited det. - development in VB basis of structures :



$$\Psi_{1-ext} = \left| a_g \bar{a}_g b_{3u} \bar{b}_{3u} b_{1g} \bar{b}_{1g} \right| = \dots = +\Psi_1^{VB} + \Psi_2^{VB} - \Psi_3^{VB} - \Psi_4^{VB} + \Psi_5^{VB} + \Psi_6^{VB}$$

MO/VB mapping

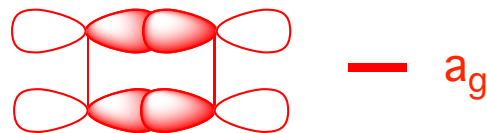
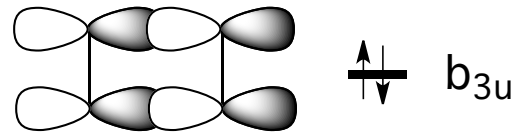
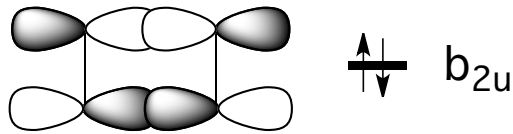
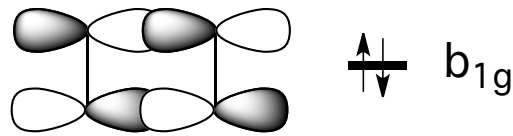
- 2nd excited det. - development in VB basis of structures :



$$\Psi_{2-ext} = \left| a_g \bar{a}_g b_{2u} \bar{b}_{2u} b_{1g} \bar{b}_{1g} \right| = \dots = +\Psi_1^{VB} + \Psi_2^{VB} + \Psi_3^{VB} + \Psi_4^{VB} - \Psi_5^{VB} - \Psi_6^{VB}$$

MO/VB mapping

- 3rd excited det. - development in VB basis of structures :



$$\Psi_{3-ext} = |b_{3u} \bar{b}_{3u} b_{2u} \bar{b}_{2u} b_{1g} \bar{b}_{1g}| = \dots = +\Psi_1^{VB} + \Psi_2^{VB} + \Psi_3^{VB} + \Psi_4^{VB} + \Psi_5^{VB} + \Psi_6^{VB}$$

MO/VB mapping

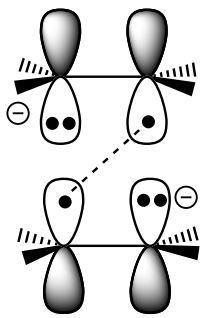
- CAS(4,4) $\Leftrightarrow$ 6 VB structures mixing :

$$\left\{ \begin{array}{l} \Psi_{HF} = |a_g \bar{a}_g b_{3u} \bar{b}_{3u} b_{2u} \bar{b}_{2u}| = \dots = +\Psi_1^{VB} + \Psi_2^{VB} - \Psi_3^{VB} - \Psi_4^{VB} - \Psi_5^{VB} - \Psi_6^{VB} \\ \Psi_{1-ext} = |a_g \bar{a}_g b_{3u} \bar{b}_{3u} b_{1g} \bar{b}_{1g}| = \dots = +\Psi_1^{VB} + \Psi_2^{VB} - \Psi_3^{VB} - \Psi_4^{VB} + \Psi_5^{VB} + \Psi_6^{VB} \\ \Psi_{2-ext} = |a_g \bar{a}_g b_{2u} \bar{b}_{2u} b_{1g} \bar{b}_{1g}| = \dots = +\Psi_1^{VB} + \Psi_2^{VB} + \Psi_3^{VB} + \Psi_4^{VB} - \Psi_5^{VB} - \Psi_6^{VB} \\ \Psi_{3-ext} = |b_{3u} \bar{b}_{3u} b_{2u} \bar{b}_{2u} b_{1g} \bar{b}_{1g}| = \dots = +\Psi_1^{VB} + \Psi_2^{VB} + \Psi_3^{VB} + \Psi_4^{VB} + \Psi_5^{VB} + \Psi_6^{VB} \end{array} \right.$$

- MO description = VB description
- VB analysis reveals the 3e-bond nature

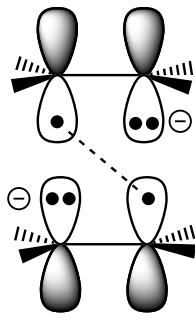
Quantitative proof (1)

- DTCNE computed weights :



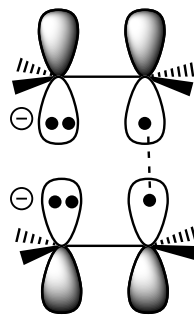
1

20.2%



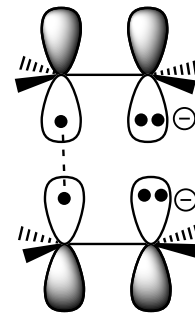
2

20.2%



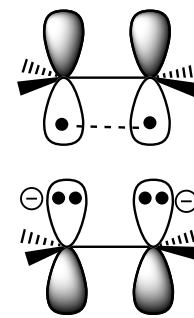
3

16.1%



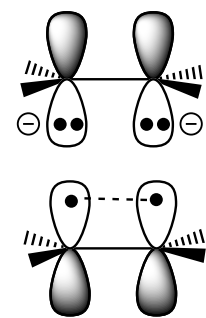
4

16.1%



5

10.1%

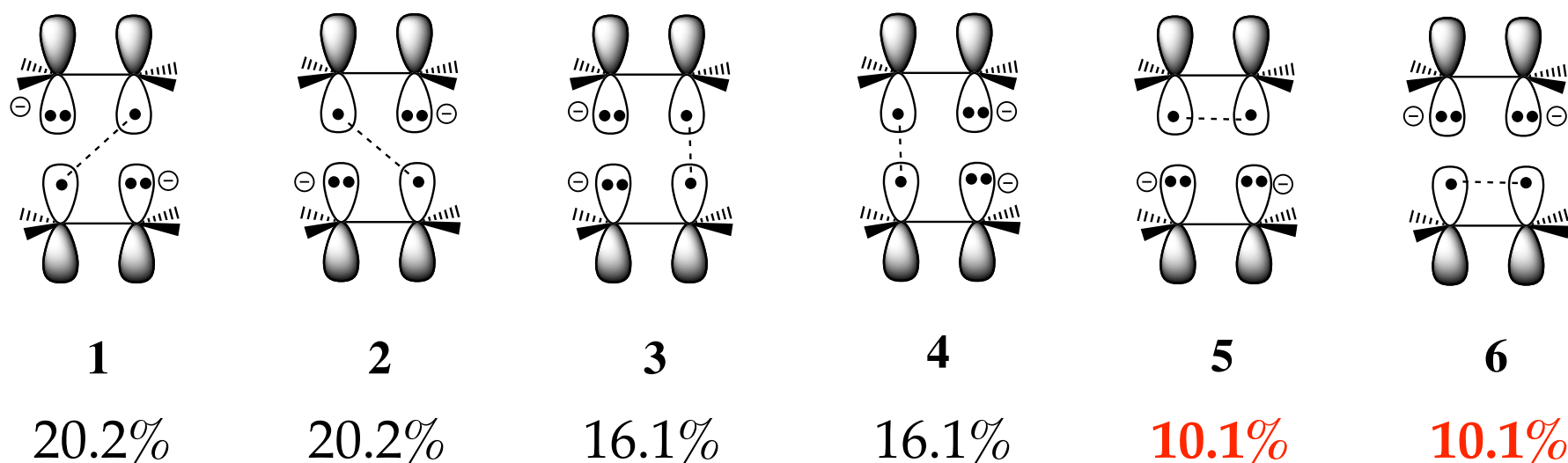


6

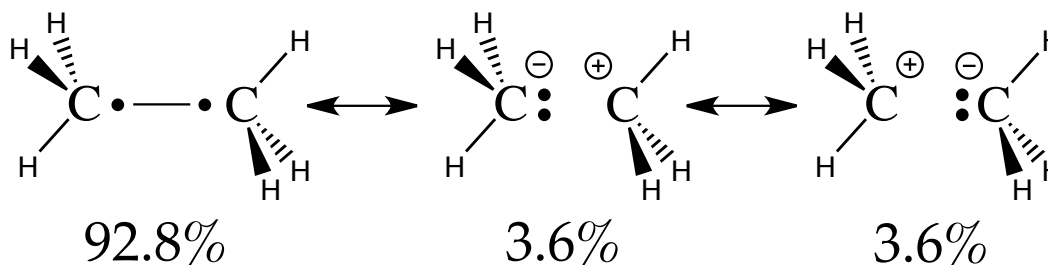
10.1%

Quantitative proof (1)

- DTCNE computed weights :



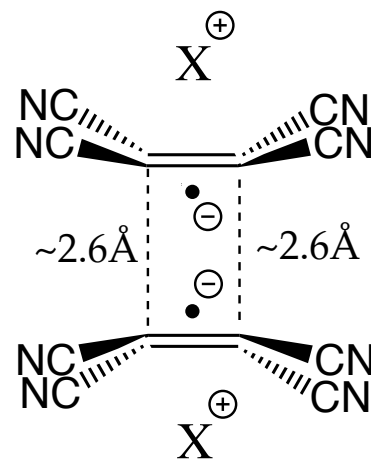
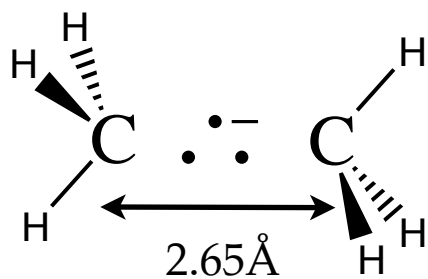
- Stretched ethane :



⇒ Str. 5-6 weights are not compatible with a 2c2e bond

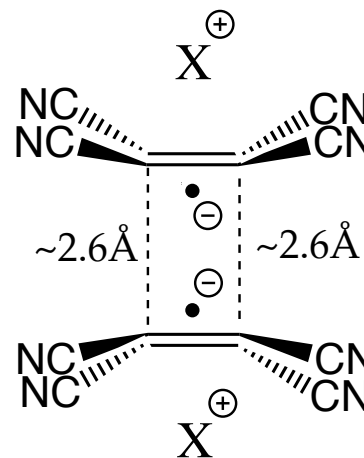
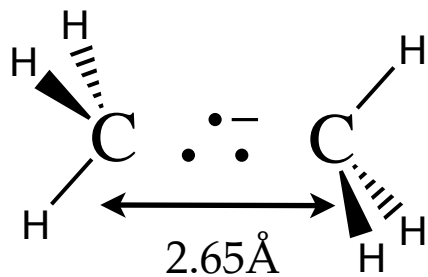
Quantitative proof (2)

1) DTCNE bond length close to $3e^-$ bonded ethane anion :



Quantitative proof (2)

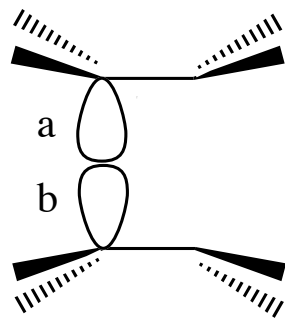
1) DTCNE bond length close to $3e^-$ bonded ethane anion :



2) DTCNE active orb. overlaps close to optimal $3e^-$ bond value :

$$S_{ab} = 0.15$$

(computed)

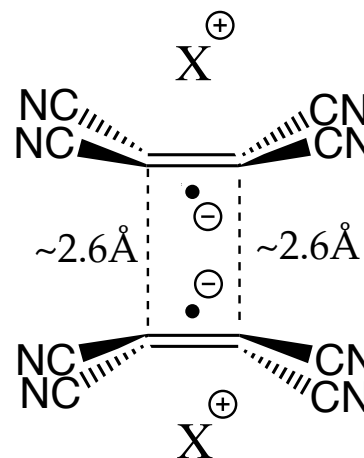
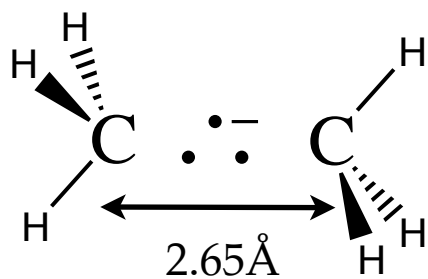


$$S_{opt} \approx 0.17$$

(from qualitative MO and VB)

Quantitative proof (2)

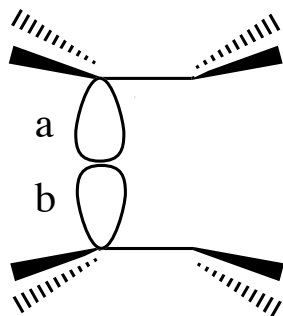
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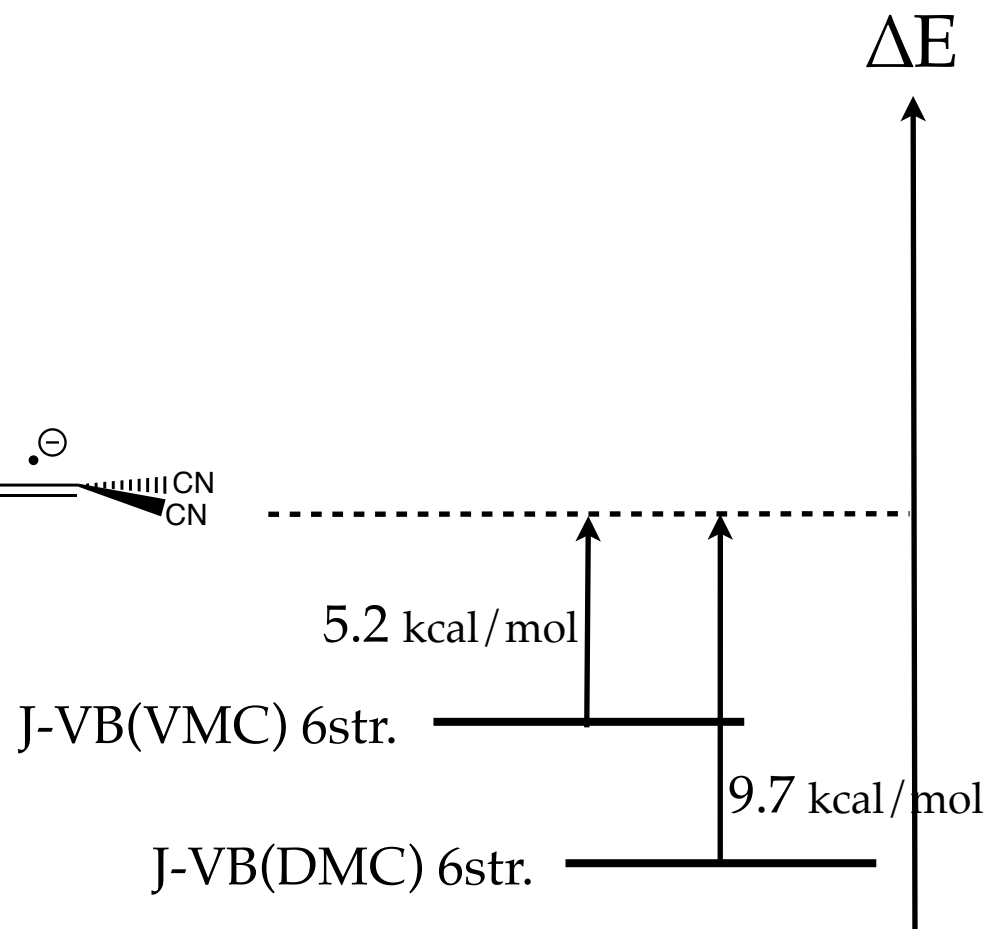
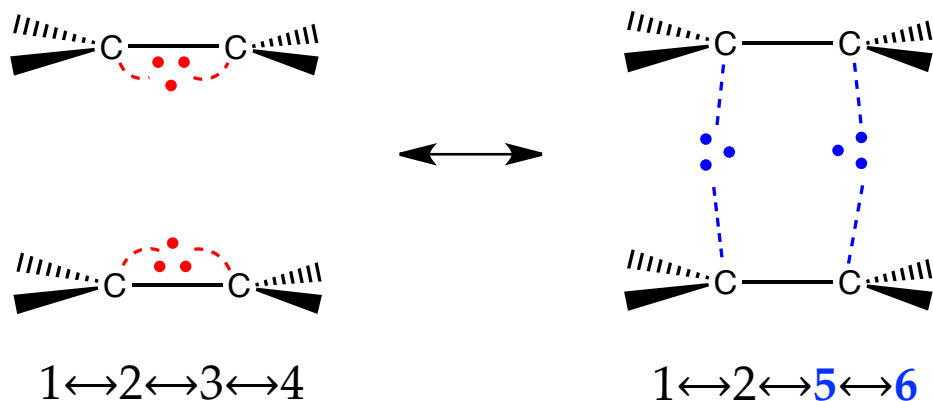


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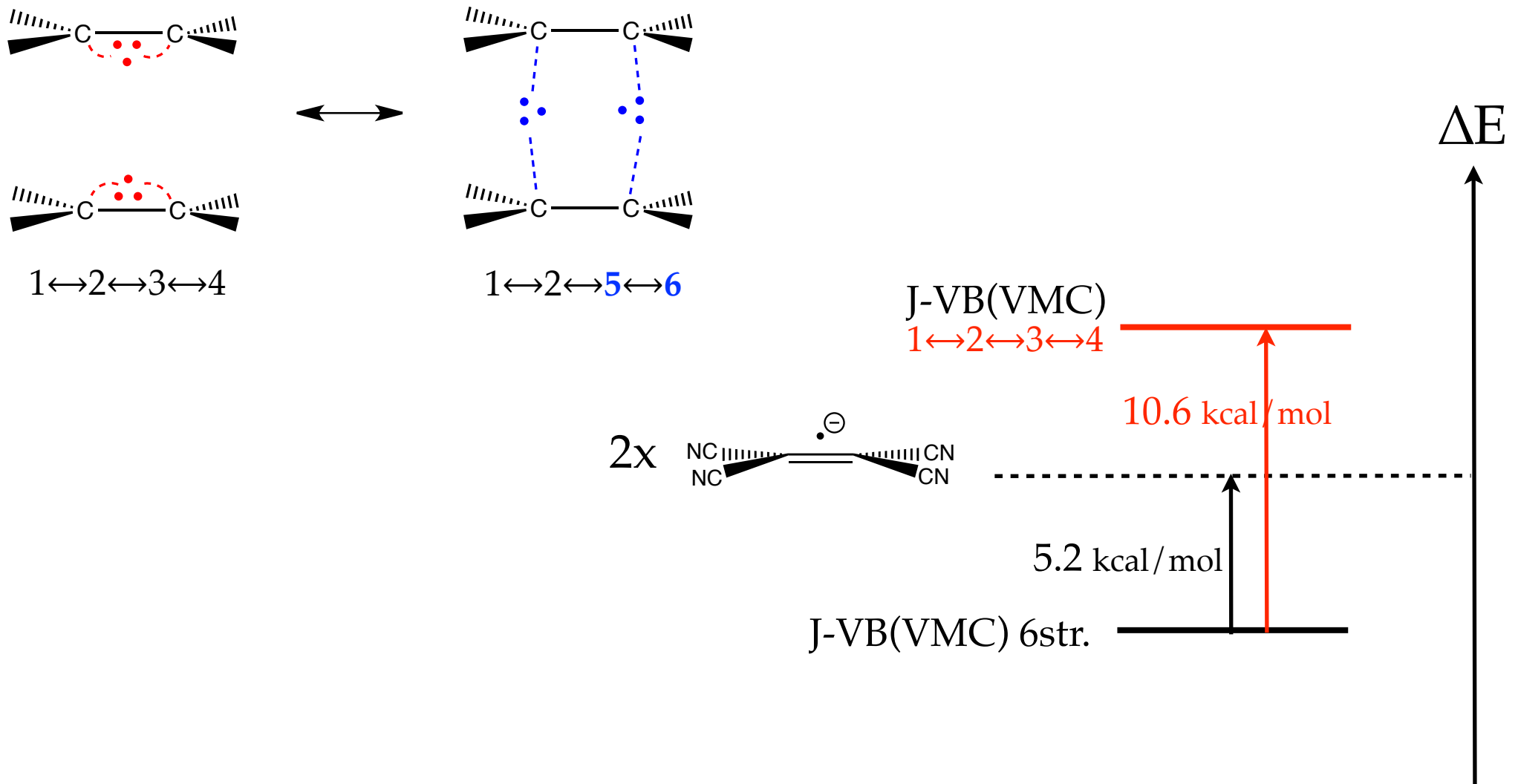
3) Dynamical Correlation Energy important for DTCNE

Quantitative proof (3)



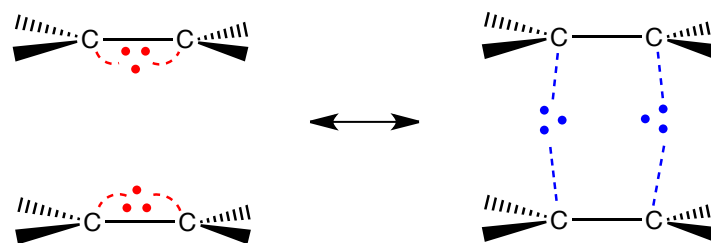
$\Delta E(\text{«exact»}) \approx 11. \text{ kcal/mol}$

Quantitative proof (3)



⇒ Without str. 5-6, DTCNE becomes repulsive !

Conclusion



- Qualitative VB analysis reveals the 3e-bonding nature in multicenter «pancake» bonding systems
- Confirmed by quantitative VB / QMC calculations (resonance energies) - size limit : ~30 atoms / ~100 electrons
- Importance of interpretative models in chemistry :
*«I know that the computer has understood,
but I would like to understand too» (E. Wigner)*

VB workshop in Paris !

- ➔ plenary lectures on Valence Bond theory, methods, and related models,
- ➔ short talks given by participants on a topic relevant to VB theory,
- ➔ ample space dedicated to free discussions,
- ➔ "hands-on" lab : basic initiation to the XMVB and BLW programs (limited to 30 participants)



An Ab Initio Non-orthogonal Valence Bond Program

Paris, July 2012



The name is Bond, Valence Bond !

➔ <http://wiki.lct.jussieu.fr/workshop>

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