[Second] RPA, linear-response theory, and nuclear sound

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Time flies

TBW: Seven years ago

Large-scale Second-RPA calculations for collective excitations

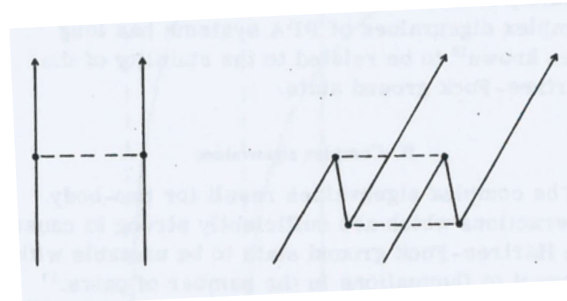
- **Introduction – Motivation**
  - ... and some formalism
- **Large-scale Second RPA**
  - Technical issues
  - Physical aspects via illustrative examples
  - Stability problems and missing correlations
- **Open questions**
  - Range and conditions of validity of SRPA?
  - Any implications for first RPA?
  - ...

❖ SRPA:

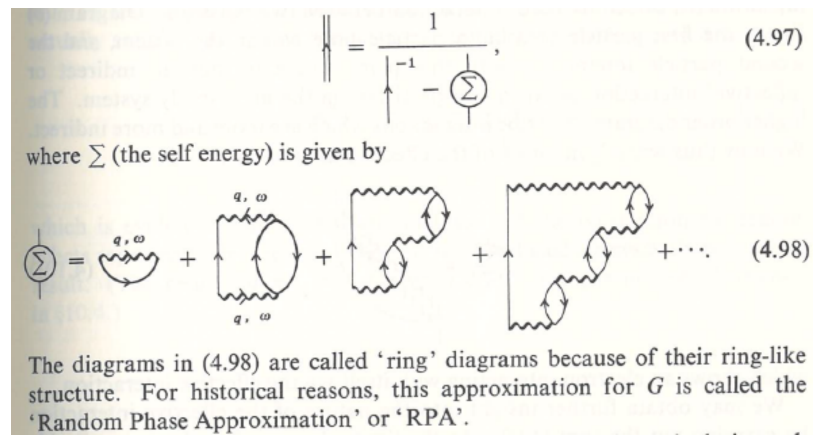
- Range and conditions of validity
- (S)RPA vs linear-response theory vs density-functional theory

Nomenclature: Which RPA?

❖ pp-RPA
- From A to A±2 system
- Pairing interaction
- Ladder diagrams



❖ ph-RPA
- Same number of particles
- Phonon excitations
- Ring diagrams



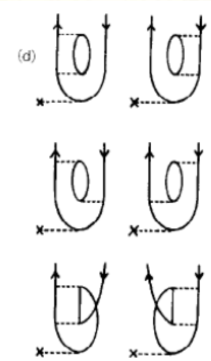
(4.97)

where $\sum$ (the self energy) is given by

(4.98)

The diagrams in (4.98) are called ‘ring’ diagrams because of their ring-like structure. For historical reasons, this approximation for G is called the ‘Random Phase Approximation’ or ‘RPA’.

❖ ph-2p2h-RPA (Second RPA)
- Extension of ph-RPA
- Related to phonon-phonon coupling



Nomenclature: Which RPA?

- pp-RPA
  - From A to A±2 system
  - Pairing interaction
  - Ladder diagrams
- ph-RPA
  - Same number of particles
  - Phonon excitations
  - Ring diagrams
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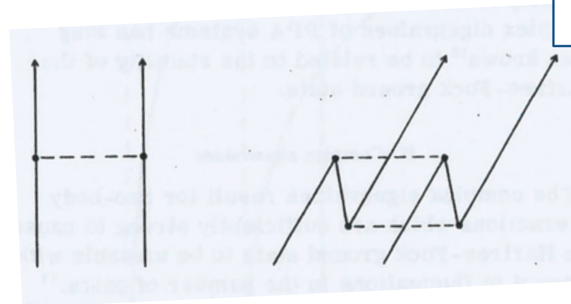
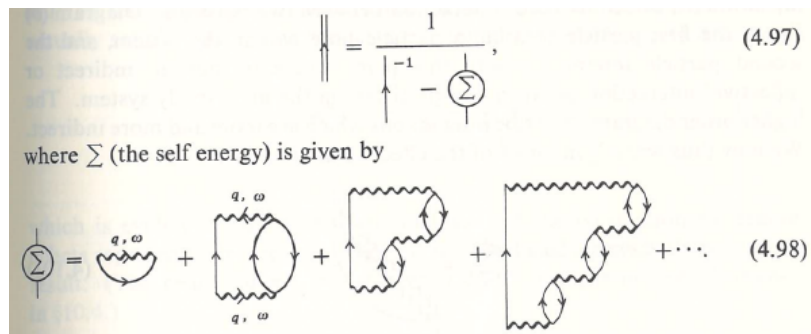
Vary&Ginocchio(1971) etc





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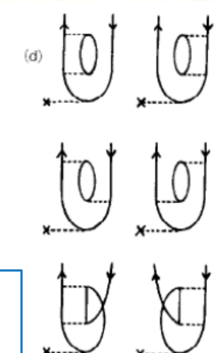

- HF reference state
- Spherical systems (J good quantum number)
- Everything antisymmetrized

And for which purpose?

- Ground-state correlations
  - Correction to the Hartree-Fock energy via the backward amplitudes
  - Correction real in the absence of phase transitions (e.g. superfluidity in pp-RPA)

- Excited states
  - pp-RPA: 2-particle transfer, spectroscopy
  - ph-RPA, ph-2p2h-RPA: Vibrational states, sound waves

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Sum rules and conservation laws

❖ External potential field V probes system with ground-state density $\rho_0(r)$

$$\sum_n (E_n - E_0)|\langle 0|V|n\rangle|^2 = \int d^3r\, \rho_0(\vec{r}) \frac{(\vec{\nabla}V)^2}{2m}$$

- Classical interpretation: relation between the average energy transferred and the impulse given to each nucleon
- Plane-wave field ➔ f-sum rule *[book: Pines&Nozières,1966]*

❖ Sound modes in nuclei: >50% of EWSR ➔ "**Giant resonances**" *[book: Harakeh&van der Woude, 2000]*

Overview

- RPA for sound waves
  - Nuclear sound
  - ph-RPA as linear-response theory

- Second RPA – The past 7 years... and beyond
  - SRPA and Thouless' theorem
  - Implications for first-order RPA
  - Remarks on density-functional theory

- Summary

Unofficial overview

- The purist
  - Thy shall not use SRPA based on the Hartree-Fock reference state. It suffers from inconsistencies and instabilities.
- The practitioner
  - If you are interested in [giant resonances / collective phonons, plasmons, ... / excitations exhausting most of the EWSR] it's fine, it will even give you some fine structure and energy corrections
- The contrarian
  - Then why would I use RPA at all?
  - When should I use what? I'm confused.

Main references

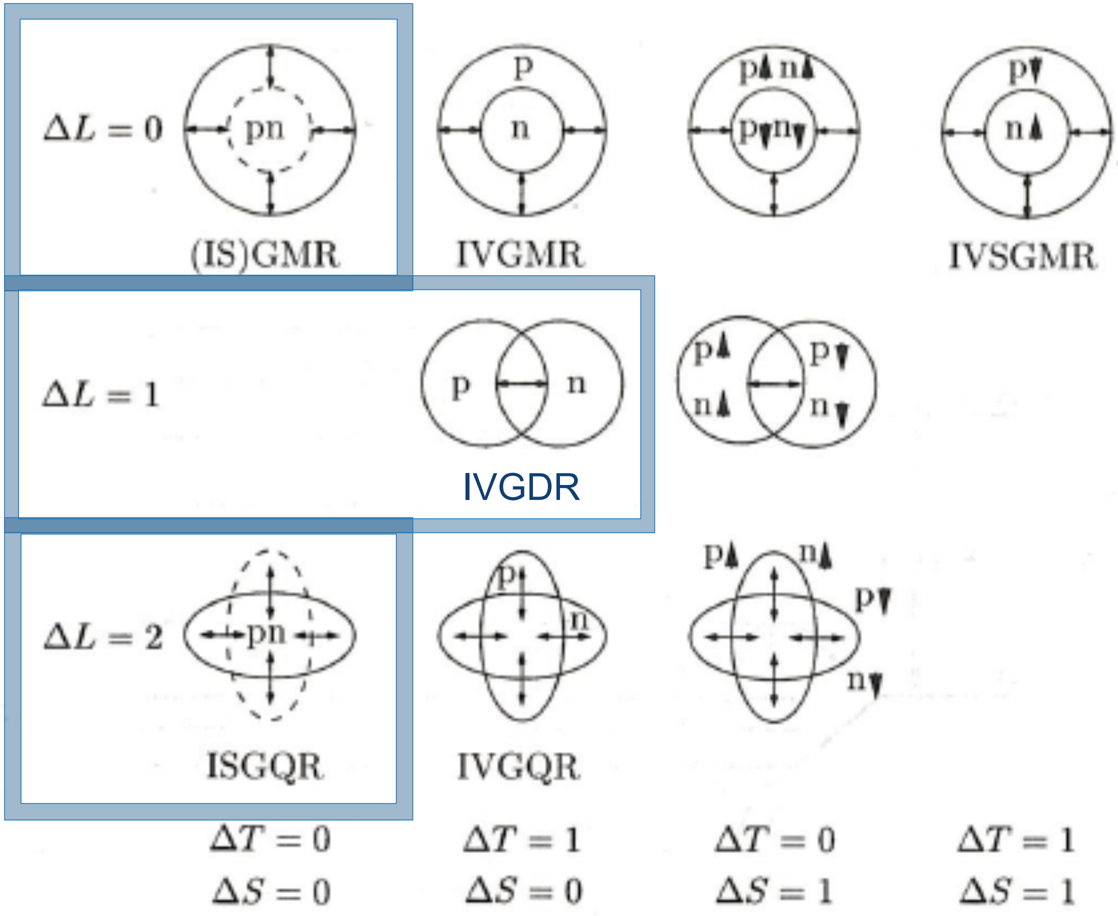
- PP, R.Roth, "*Second random-phase approximation and realistic interactions*", Phys.Lett.B671(2009)356
- PP,R.Roth, "*Large-scale second random-phase approximation calculations with finite-range interactions*", Phys.Rev.C81(2010)024317
- PP, "*Second random-phase approximation, Thouless' theorem, and the stability condition re-examined and clarified*", Phys.Rev.C90(2014)024305

INTRODUCTION

Nuclear sound

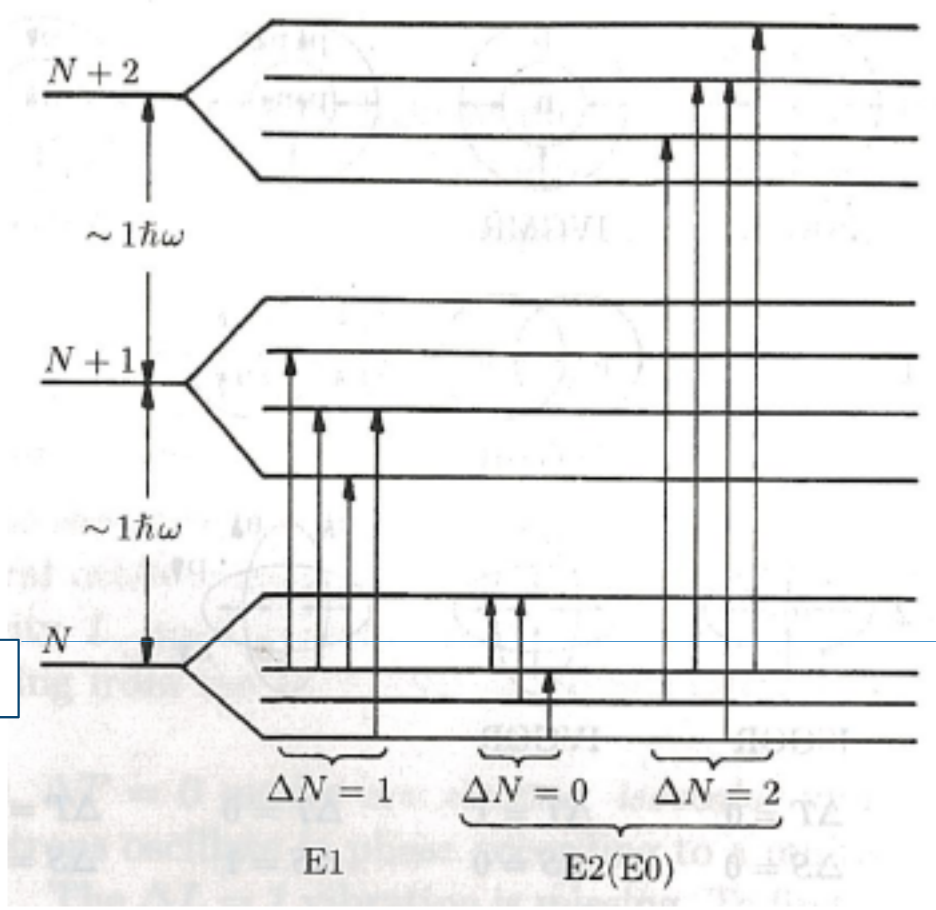
Derivation and properties of (S)RPA

Nuclear Giant Resonances



[“Giant resonances” book by Harakeh and van der Woude]

Nuclear Giant Resonances



occupied (hole) states:

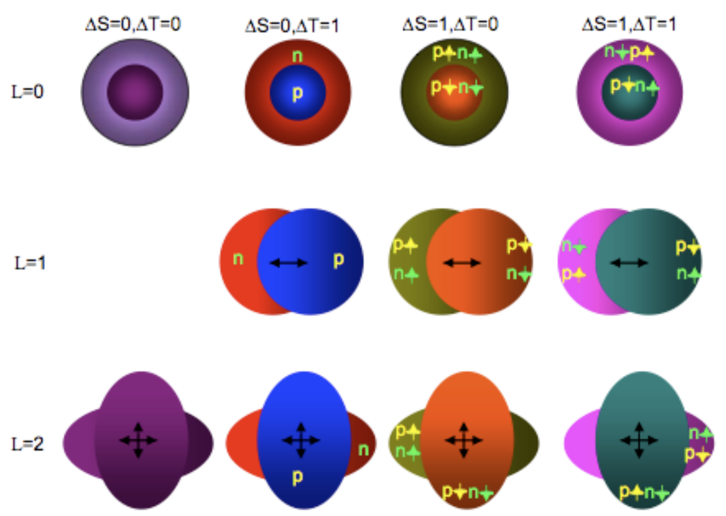
[“Giant resonances” book by Harakeh and van der Woude]

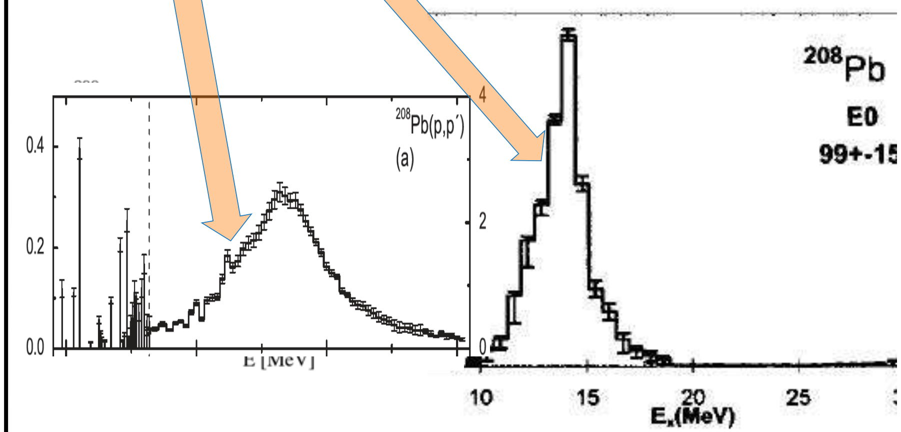
Normal modes of vibration

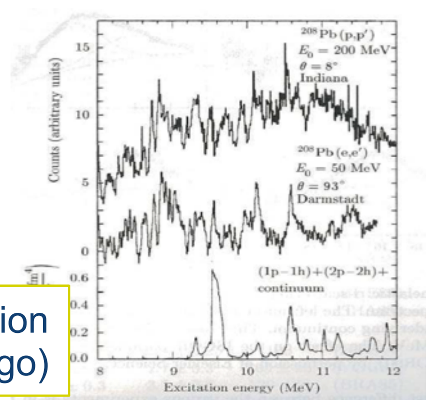
Major part of the EWSR

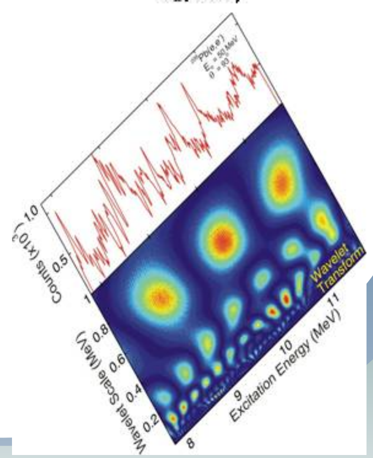
The simplified picture

The reality









SRPA explored to explain attenuation (~30yrs ago)

Equations-of-motion method

- Schrödinger equation:

$$H|\nu\rangle = E_\nu|\nu\rangle$$

- Define creation / annihilation operators

$$|\nu\rangle = Q_\nu^\dagger|0\rangle \quad ; \quad Q_\nu|0\rangle = 0$$

- Rewrite:

$$\langle 0|[R,[H,Q_\nu^\dagger]]|0\rangle = E_\nu\langle 0|[R,Q_\nu^\dagger]|0\rangle \quad ; \quad \forall R$$

- E.g.: $R = a_p^\dagger a_h$, ph operator, $Q_\nu = \sum_{ph} X_{ph}a_p^\dagger a_h - Y_{ph}a_h^\dagger a_p$, and $|0\rangle = |\mathrm{HF}\rangle$: HF-RPA

Standard RPA

- Vibration creation operator:

$$Q_\nu^\dagger = \sum_{ph} X_{ph}^\nu O_{ph}^\dagger - \sum_{ph} Y_{ph}^\nu O_{ph} \quad ; \quad Q_\nu |\mathrm{RPA}\rangle = 0 \quad ; \quad Q_\nu^\dagger |\mathrm{RPA}\rangle = |\nu\rangle$$

- Standard RPA - the RPA vacuum is approximated by the HF ground state:

$$\langle \mathrm{RPA}| \ldots |\mathrm{RPA}\rangle \rightarrow \langle \mathrm{HF}| \ldots |\mathrm{HF}\rangle \quad ; \quad O_{ph}^\dagger \rightarrow a_p^\dagger a_h$$

- RPA equations in $ph-$space:

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \hbar\omega_\nu \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix}$$

$$A_{ph,p'h'} = \delta_{pp'}\delta_{hh'}(e_p - e_h) + H_{hp',ph'} \quad ; \quad B_{ph,p'h'} = H_{hh',pp'} \quad ;$$

For some in-medium Hamiltonian H

☞ Self-consistent HF+RPA: spurious state and sum rules

- Equivalent to small-amplitude Time-Dependent Hartree-Fock
- Linear Response Theory

Second RPA

- Vibration creation operator: Includes $2p2h$ configurations

$$Q^\dagger_\nu = \sum_{ph} X^\nu_{ph} O^\dagger_{ph} - \sum_{ph} Y^\nu_{ph} O_{ph} + \sum_{p_1h_1p_2h_2} \mathcal{X}^\nu_{p_1h_1p_2h_2} O^\dagger_{p_1h_1p_2h_2}$$
$$- \sum_{p_1h_1p_2h_2} \mathcal{Y}^\nu_{p_1h_1p_2h_2} O_{p_1h_1p_2h_2}$$

- The SRPA vacuum is approximated by the HF ground state:

$$\langle \mathrm{SRPA}| \ldots |\mathrm{SRPA}\rangle \rightarrow \langle \mathrm{HF}| \ldots |\mathrm{HF}\rangle$$

- SRPA equations in $ph \oplus 2p2h-$space:

$$\left(\begin{array}{cc|cc} A & \mathcal{A}_{12} & B & 0 \\ \mathcal{A}_{21} & \mathcal{A}_{22} & 0 & 0 \\ \hline -B^* & 0 & -A^* & -\mathcal{A}^*_{12} \\ 0 & 0 & -\mathcal{A}^*_{21} & -\mathcal{A}^*_{22} \end{array}\right) \left(\begin{array}{c} X^\nu \\ \mathcal{X}^\nu \\ \hline Y^\nu \\ \mathcal{Y}^\nu \end{array}\right) = \hbar\omega_\nu \left(\begin{array}{c} X^\nu \\ \mathcal{X}^\nu \\ \hline Y^\nu \\ \mathcal{Y}^\nu \end{array}\right)$$

$A_{ph,p'h'} = \delta_{pp'}\delta_{hh'}(e_p - e_h) + H_{hp',ph'}$; $B_{ph,p'h'} = H_{hh',pp'}$

For some in-medium Hamiltonian H

$\mathcal{A}_{12}$: interactions between ph and $2p2h$ states

$\mathcal{A}_{22}$: $\delta_{p_1p'_1}\delta_{h_1h'_1}\delta_{p_1p'_1}\delta_{h_1h'_1}(e_{p_1} + e_{p_2} - e_{h_1} - e_{h_2})$ + interactions among $2p2h$ states

Drożdż, Nishizaki, Speth, Wambach; Yannouleas; and others, early ‘90s

Second RPA

- Vibration creation operator: Includes $2p2h$ configurations

$$Q_\nu^\dagger = \sum_{ph} X_{ph}^\nu O_{ph}^\dagger - \sum_{ph} Y_{ph}^\nu O_{ph} + \sum_{p_1h_1p_2h_2} \mathcal{X}^\nu_{p_1h_1p_2h_2} O^\dagger_{p_1h_1p_2h_2}$$
$$- \sum_{p_1h_1p_2h_2} \mathcal{Y}^\nu_{p_1h_1p_2h_2} O_{p_1h_1p_2h_2}$$

- The SRPA vacuum is approximated by the HF ground state:

$$\langle \mathrm{SRPA}| \ldots |\mathrm{SRPA}\rangle \to \langle \mathrm{HF}| \ldots |\mathrm{HF}\rangle$$

- SRPA equations in $ph \oplus 2p2h$–space:

$$\left(\begin{array}{cc|cc} A & \mathcal{A}_{12} & B & 0 \\ \mathcal{A}_{21} & \mathcal{A}_{22} & 0 & 0 \\ \hline -B^* & 0 & -A^* & -\mathcal{A}^*_{12} \\ 0 & 0 & -\mathcal{A}^*_{21} & -\mathcal{A}^*_{22} \end{array}\right) \left(\begin{array}{c} X^\nu \\ \mathcal{X}^\nu \\ \hline Y^\nu \\ \mathcal{Y}^\nu \end{array}\right) = \hbar\omega_\nu \left(\begin{array}{c} X^\nu \\ \mathcal{X}^\nu \\ \hline Y^\nu \\ \mathcal{Y}^\nu \end{array}\right)$$

$A_{ph,p'h'} = \delta_{pp'}\delta_{hh'}(e_p - e_h) + H$

$\mathcal{A}_{12}$: interactions between ph

$\mathcal{A}_{22}$: $\delta_{p_1p_1'}\delta_{h_1h_1'}\delta_{p_1p_1'}\delta_{h_1h_1'}(e_{p_1} + $

linear response with collision term

Drożdż, Nishizaki, Speth, Wambach; Yannouleas; and others, early '90s

EoM for the one- and two-body density matrix

- One-body density matrix

$$\rho_{kl}(t) = \langle \psi(t) | a_l^+ a_k | \psi(t) \rangle$$

- Hamiltonian

$$H = \sum_{kl} t_{kl} a_k^+ a_l + \frac{1}{4} \sum_{klpq} \bar{v}_{klpq} a_k^+ a_l^+ a_q a_p$$

- Equation of motion

$$i\hbar \frac{\partial}{\partial t} \rho_{kl}(t) = \langle \psi(t) | [a_l^+ a_k, H] | \psi(t) \rangle$$

- Two-body density matrix

$$\rho^{(2)}_{klpq}(t) = \langle \psi(t) | a_p^+ a_q^+ a_l a_k | \psi(t) \rangle$$

$$i\hbar \frac{\partial}{\partial t} \rho^{(2)}_{klpq}(t) = \langle \psi(t) | [a_p^+ a_q^+ a_l a_k, H] | \psi(t) \rangle$$

- Non-trivial part g

$$\rho^{(2)}_{klpq} = \rho_{kp}\rho_{lq} - \rho_{kq}\rho_{lp} + g_{klpq}$$

- Then EoM takes the form

$$i\hbar \frac{\partial}{\partial t} \rho_{kl}(t) - [h, \rho]_{kl} = I_{kl}$$

- With $h_{kl} = t_{kl} + U_{kl}$, $U_{kl} = \sum_{pq} \bar{v}_{kqlp} \rho_{pq}$
- And a collision term

$$I_{kl} = \frac{1}{2} \sum_{prs} \bar{v}_{kprs} g_{rslp} - \bar{v}_{rslp} g_{kprs}$$

J.Wambach, Rep.Prog.Phys.51(1988)989

Small-amplitude limit: SRPA

- Expand around equilibrium values:

$$\rho = \rho^0 + \delta\rho$$

$$g = g^0 + \delta g.$$

- Assume harmonic dependence:

$$\delta\rho = \delta\rho_{\mathrm{ph}}\, e^{-i\omega t} + \delta\rho_{\mathrm{hp}}\, e^{i\omega t}$$

$$\delta g = \delta g_{\mathrm{pp'hh'}}\, e^{-i\omega t} + \delta g_{\mathrm{hh'pp'}}\, e^{i\omega t}.$$

- Linearization of the EoM yields:

$$\left[\begin{pmatrix} \mathscr{A} & \mathscr{B} \\ -\mathscr{B}^* & -\mathscr{A}^* \end{pmatrix} - \hbar\omega \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right]\begin{pmatrix} \mathscr{X}^\nu \\ \mathscr{Y}^\nu \end{pmatrix} = 0$$

$$\mathscr{A} = \begin{pmatrix} A_{11'} & A_{12'} \\ A_{21'} & A_{22'} \end{pmatrix} \quad \mathscr{B} = \begin{pmatrix} B_{11'} & B_{12'} \\ B_{21'} & B_{22'} \end{pmatrix} \quad \mathscr{X}^\nu = \begin{cases} \delta\rho_{\mathrm{ph}} = X^\nu_{\mathrm{ph}} \\ \delta g_{\mathrm{pp'hh'}} = X^\nu_{\mathrm{pp'hh'}} \end{cases} \quad \mathscr{Y}^\nu = \begin{cases} \delta\rho_{\mathrm{hp}} = Y^\nu_{\mathrm{ph}} \\ \delta g_{\mathrm{hh'pp'}} = Y^\nu_{\mathrm{pp'hh'}} \end{cases}$$

- Linear response with collision term:

$$i\hbar\frac{\partial}{\partial t}\delta\rho_{kl}(t) = [h^0, \delta\rho]_{kl} + \left[\frac{\delta h}{\delta\rho}\delta\rho, \rho_0\right]_{kl} + \left[\frac{\delta I}{\delta\rho}\delta\rho, \rho_0\right]_{kl}$$

J.Wambach, Rep.Prog.Phys.51(1988)989

Formal properties of RPA...

[D.J.Rowe, Rev.Mod.Phys.40(1968)153]
[D.J.Thouless, Nucl.Phys.22(1961)78]

- Solutions appear in adjoint pairs
- If the stability condition is satisfied,

$$\langle 0|[F^\dagger,[H,F]]|0\rangle = \sum_{ab} F_a^* M_{ab} F_b \geq 0$$

solutions are real

- The EWSR is preserved ($\sum_n (E_n - E_0)|\langle 0|V|n\rangle|^2 = \int d^3r \rho_0(\vec{r}) \frac{(\vec{\nabla}V)^2}{2m}$)

$$\sum_{n:N_n=1} |\langle 0|O|n\rangle|^2 E_{n0} = \frac{1}{2}\langle 0|[O,H,O]|0\rangle$$

- ... and therefore spurious transitions have zero energy (restoration of symmetries)

|HF) = vacuum of ph states ➔ stability of RPA for physical states

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- If the stability condition is satisfied,

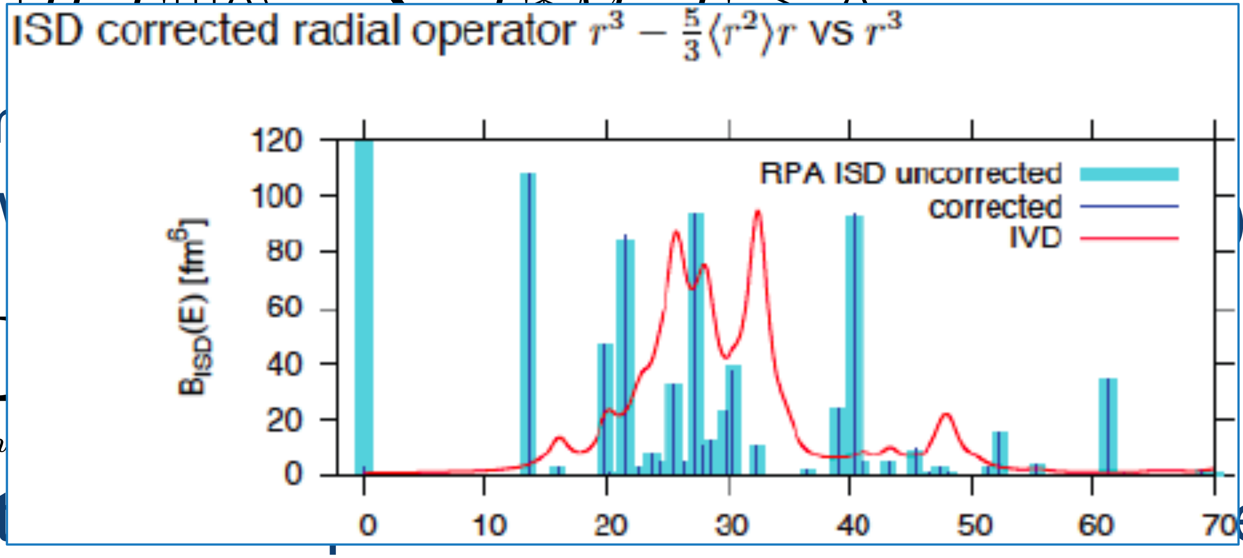
$\langle 0|[F^\dagger,$

solution

- The EW

$\sum$

$n:N_n$



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|HF) = vacuum of ph states ➔ stability of RPA for physical states

... same for SRPA

As is the case with the simple RPA,[2] the search for formal properties of the second RPA is greatly simplified by the fact that the second RPA excitation operator $O_\nu^\dagger$—which creates a one-phonon state—satisfies the following double-commutator equation:

$$\langle \mathrm{HF} | [R,[H,O_\nu^\dagger]] | \mathrm{HF} \rangle = \hbar\omega_\nu \langle \mathrm{HF} | [R,O_\nu^\dagger] | \mathrm{HF} \rangle , \quad \text{for all } R , \tag{1}$$

where $\hbar\omega_\nu$ is the excitation energy, $|\mathrm{HF}\rangle$ is the Hartree-Fock ground state of the nucleus, and R is any operator in the same space as $O_\nu^\dagger$. H is the exact many-body Hamiltonian.

$$O_\nu^\dagger = \sum_{mi} [Y_{mi}(\omega_\nu) a_m^\dagger a_i - Z_{mi}(\omega_\nu) a_i^\dagger a_m] + \sum_{m<n,i<j} [Y_{mnij}(\omega_\nu) a_m^\dagger a_n^\dagger a_j a_i - Z_{mnij}(\omega_\nu) a_i^\dagger a_j^\dagger a_n a_m] . \tag{2}$$

C.Yannouleas, PRC35(1987)1159

Equipped with Eqs. (1) and (2), one can repeat the steps in Sec. III of Ref. 2 and show that all the formal properties familiar from the simple RPA hold for the zero-temperature second RPA as well. In particular, these formal properties are the following:

(1) The solutions of the second RPA appear in pairs having symmetric positive and negative eigenvalues.

(2) The second RPA solutions have real energies when Thouless's stability condition is fulfilled.

(3) Spurious solutions reflecting the center-of-mass motion separate out and have exactly zero energy.

(4) The second RPA solutions are orthonormal.

(5) The nonspurious second RPA solutions form a complete set.

(6) The matrix elements of any operator W calculated in the second RPA preserve the energy-weighted sum rule.

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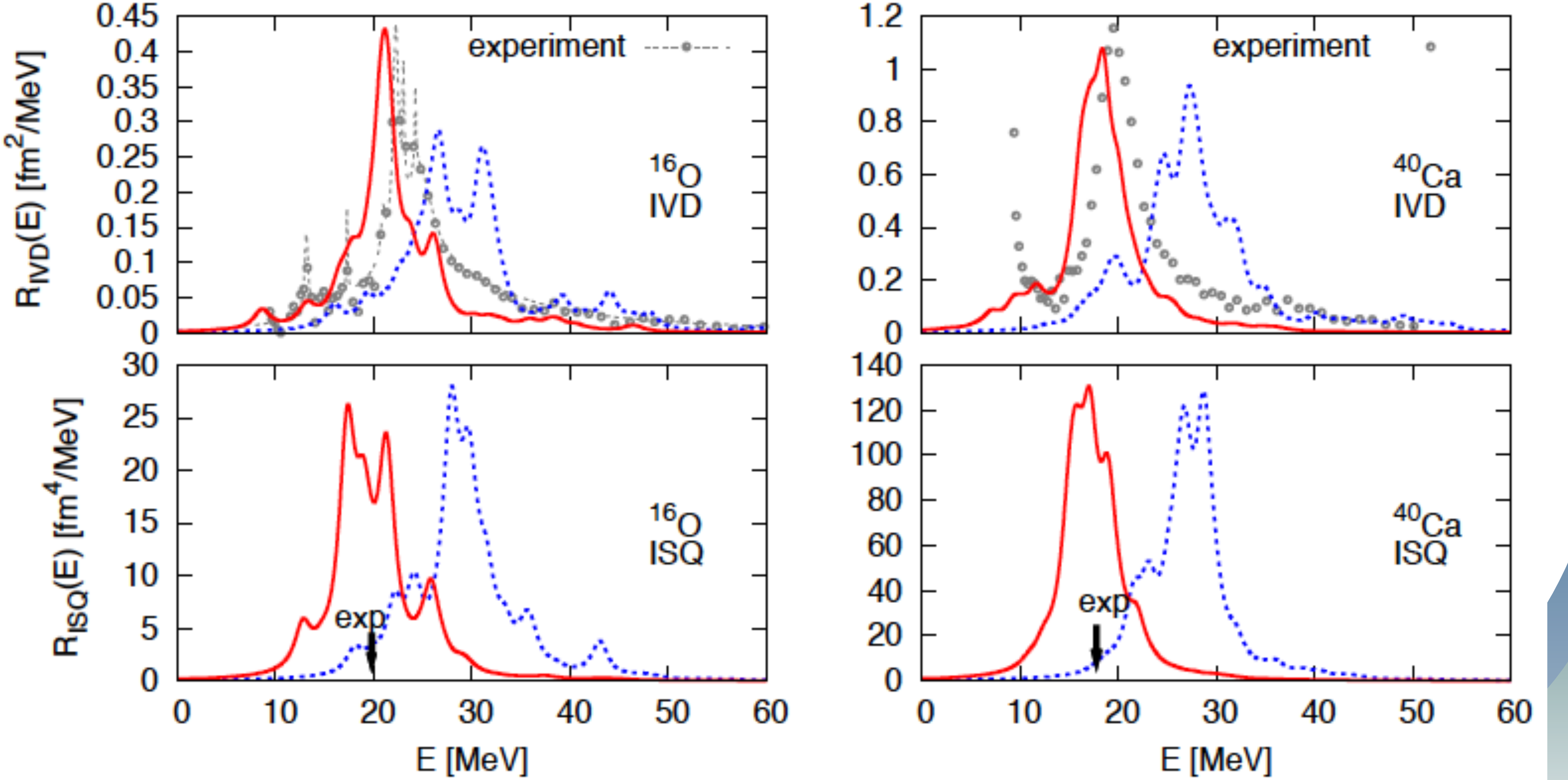
|HF) NOT the vacuum of 2p2h states ➔ ?

my ~2010 results

- Softened realistic Hamiltonian (UCOM)
- No arbitrary truncations
- Explored: Energetic shift; EWSR; ph fragmentation; etc.

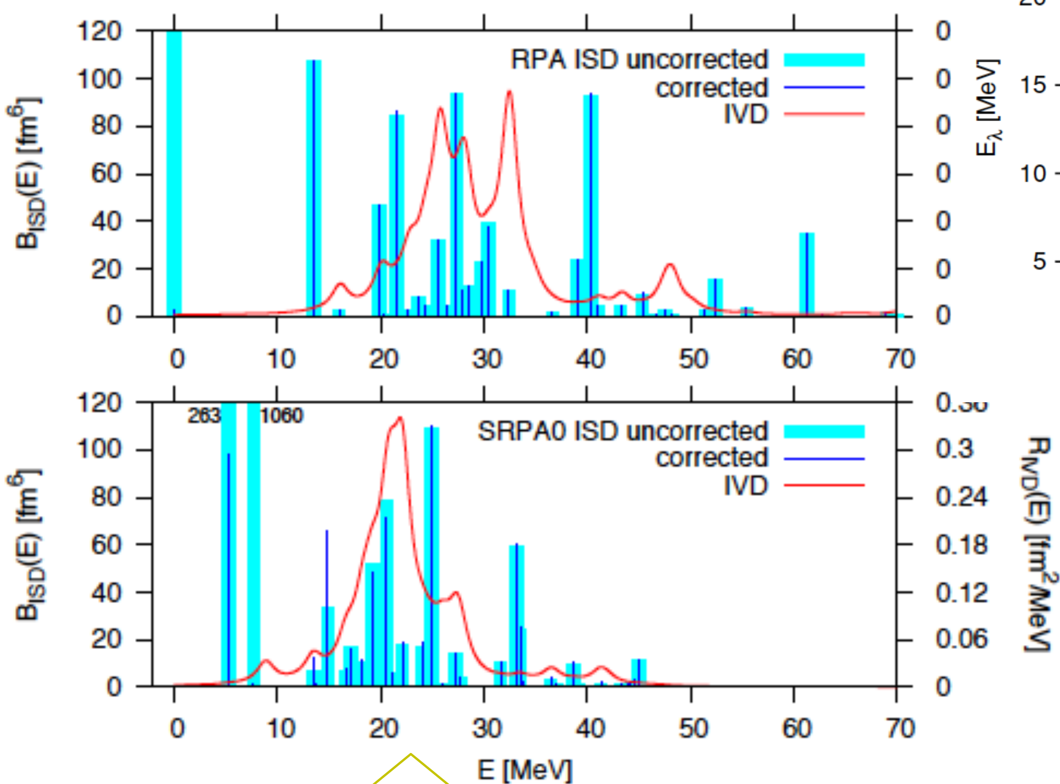


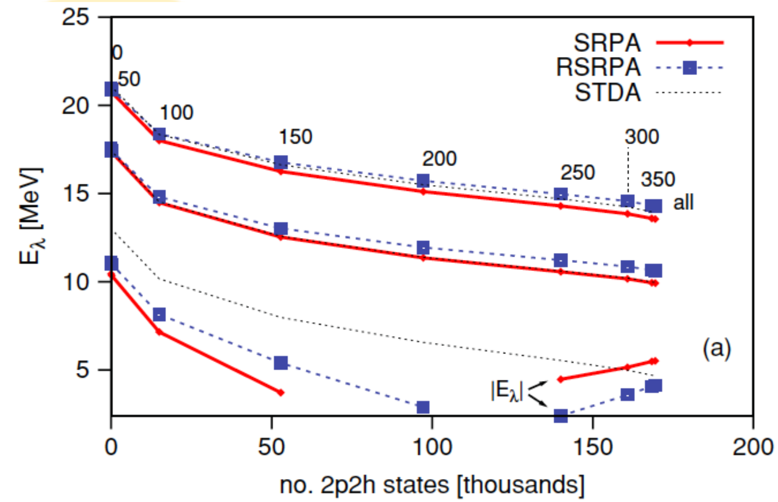
VALIDITY OF SRPA

What is at issue?

Spurious states and instabilities in SRPA

ISD corrected radial operator $r^3 - \frac{5}{3}\langle r^2\rangle r$ vs r^3





Instability of low-energy states

spurious admixtures

Formal properties: like RPA?

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?

We found:

- instances of imaginary 3-, 2+
- spurious state at finite energy

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- spurious state at finite energy

It is not!

P.P., PRC90(2014)024305

Thouless' theorem and the stability condition

PP, PRC90,024305

- **Thouless' theorem still holds:** if all eigenvalues are real, the EWSR satisfies

$$\sum_{\nu:N_\nu=1} |\langle\nu|O|0\rangle|^2 E_\nu = \frac{1}{2}\langle 0|[O,H,O]|0\rangle$$

- For *H* commuting with *O*, this means that the total EWSR must vanish.

- Q: **Then how come there is spurious strength at finite energy?**

- A: Positive-energy solutions with negative norm may exist:
  - (Their negative-energy counterparts will cave postitive norm)
  - Pairs of « antinormal » solutions: $\mathbf{N_\nu E_\nu < 0}$

$$|X_\nu|^2 - |Y_\nu|^2 = N_\nu = \pm 1$$

- **Their contribution to the energy-weighted sum is *negative***

- As a result, **a spurious state can contribute a finite but negative amount to the total spurious EWSR, such that the total spurious EWSR still vanishes ->** Thouless' theorem indeed holds (demonstrated numerically)

Thouless' theorem and the stability condition

PP, PRC90,024305

- **Thouless' theorem still holds:** if all eigenvalues are real, the EWSR satisfies

$$\sum_{\nu:E_\nu>0} |\langle\nu|O|0\rangle|^2 N_\nu E_\nu = \frac{1}{2}\langle 0|[O,H,O]|0\rangle$$

- For *H* commuting with *O*, this means that the total EWSR must vanish.

- Q: **Then how come there is spurious strength at finite energy?**

- A: Positive-energy solutions with negative norm may exist:
  - (Their negative-energy counterparts will cave postitive norm)
  - Pairs of « antinormal » solutions: $\mathbf{N_\nu E_\nu < 0}$

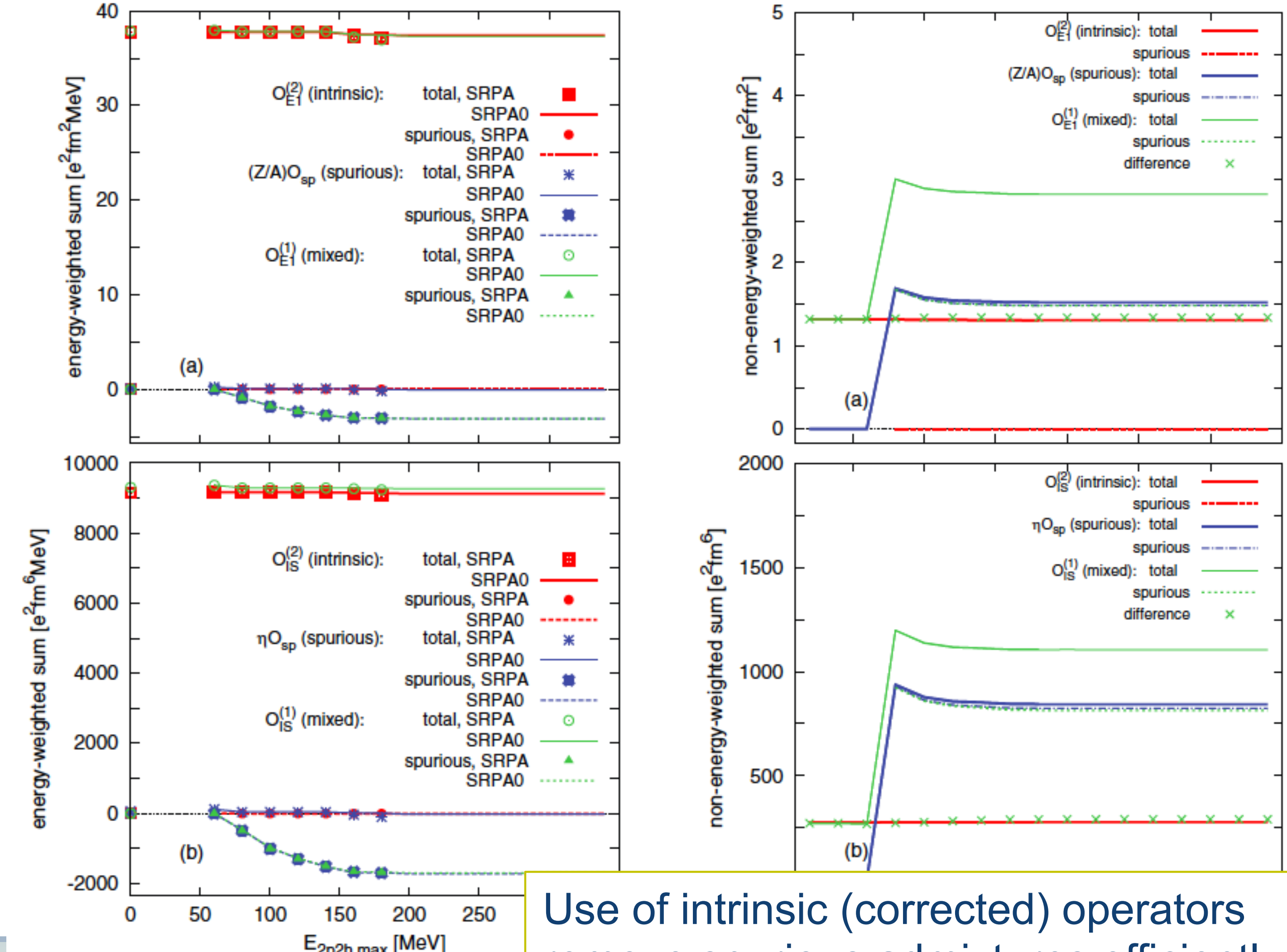
$$|X_\nu|^2 - |Y_\nu|^2 = N_\nu = \pm 1$$

- **Their contribution to the energy-weighted sum is *negative***

- As a result, **a spurious state can contribute a finite but negative amount to the total spurious EWSR, such that the total spurious EWSR still vanishes ->** Thouless' theorem indeed holds (demonstrated numerically)

Spurious strength in SRPA

PP, PRC90,024305

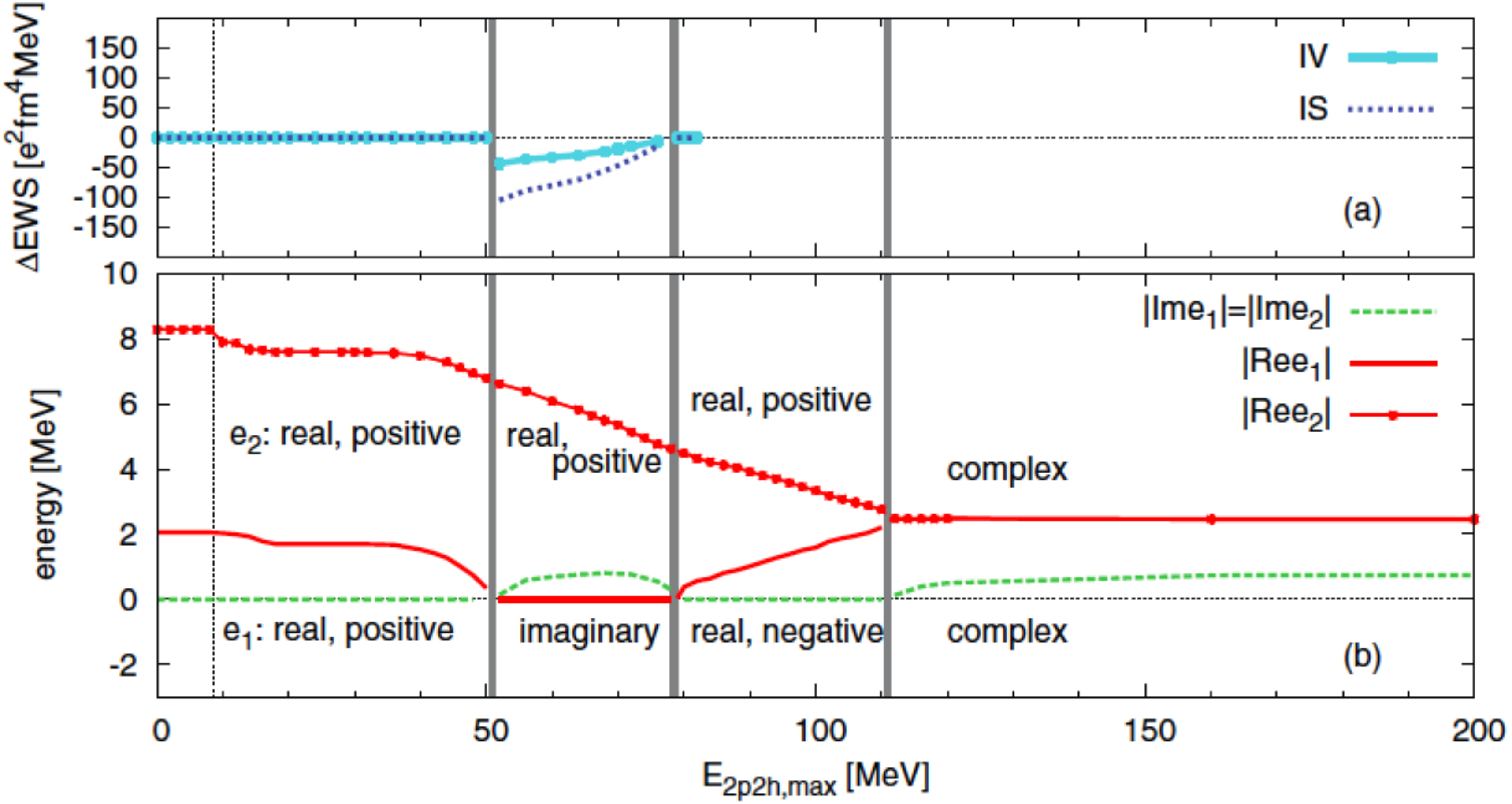


Use of intrinsic (corrected) operators remove spurious admixtures efficiently

Numerical validation of Thouless' theorem

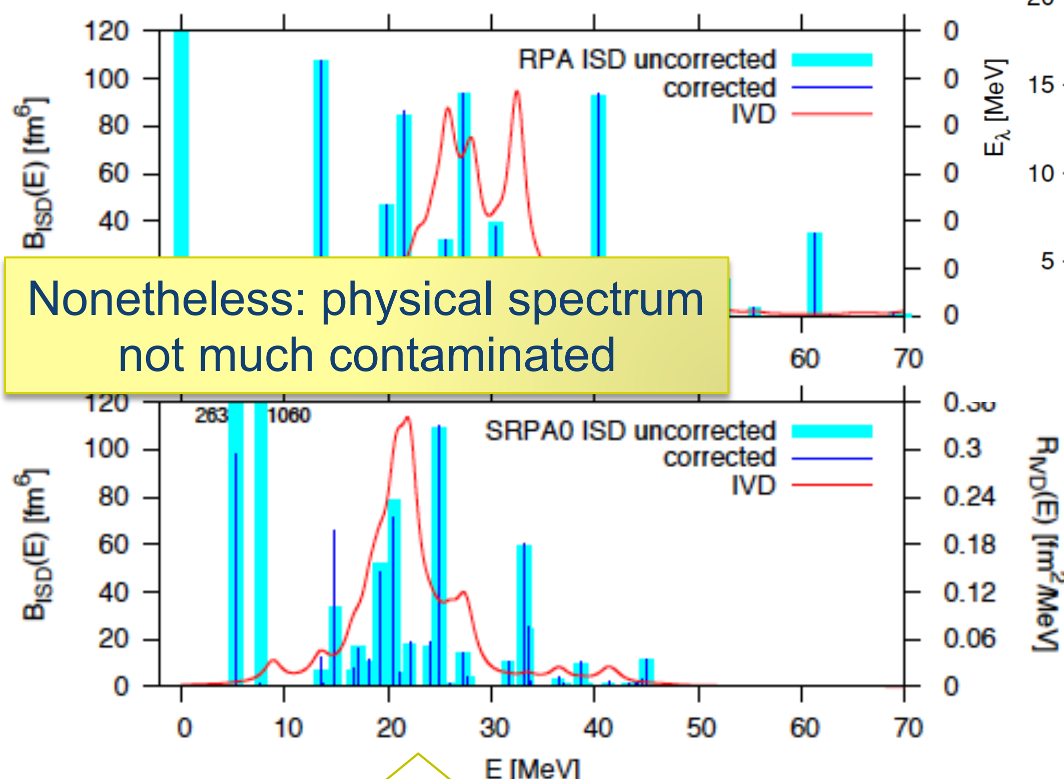
PP, PRC90,024305

2^+ of ^{48}Ca:



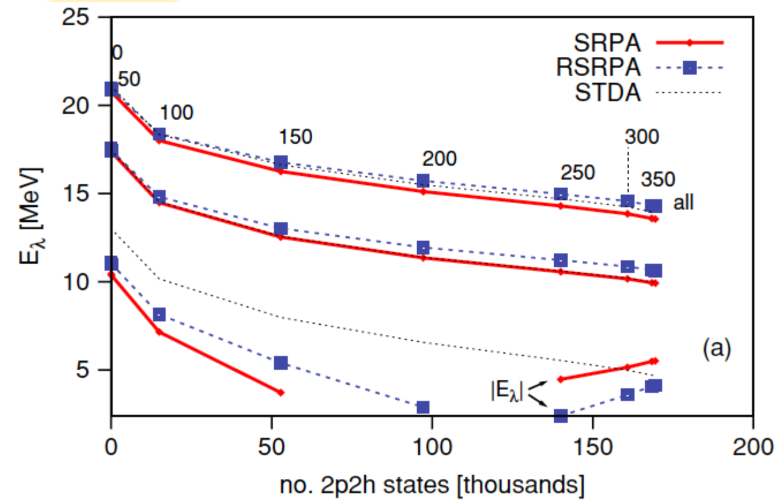
Spurious states and instabilities in SRPA

ISD corrected radial operator $r^3 - \frac{5}{3}\langle r^2 \rangle r$ vs r^3



Nonetheless: physical spectrum not much contaminated

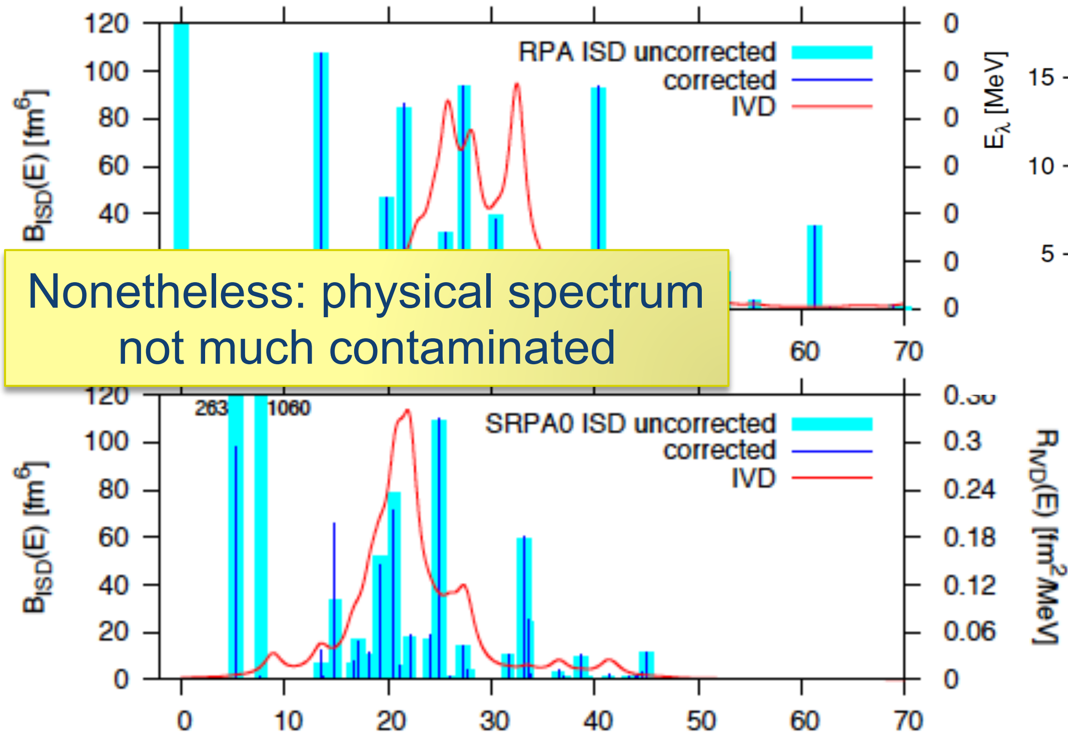
spurious admixtures

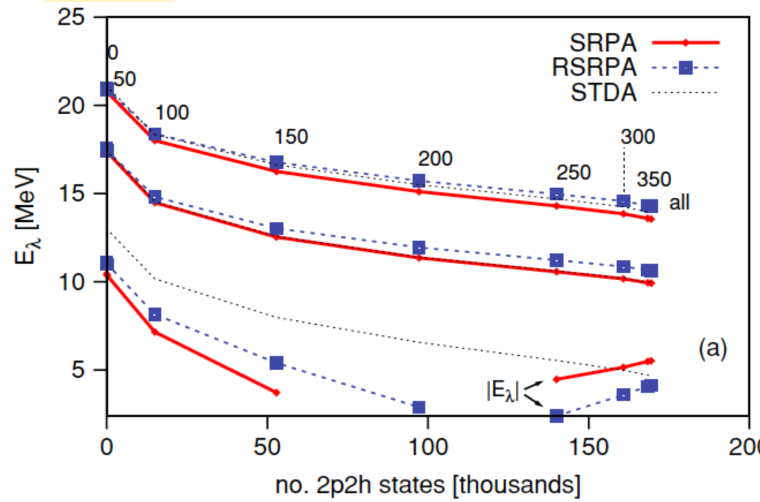


Instability of low-energy states

Spurious states and instabilities in SRPA

ISD corrected radial operator $r^3 - \frac{5}{3}\langle r^2\rangle r$ vs r^3





Nonetheless: physical spectrum not much contaminated

yet to do:
self-consistent second-order formalism

➢ *Correlated ground state [P.Tohyama, P.Schuck; N.Pillet et al.]*

➢ *Subtraction method [Tselayev,...]*

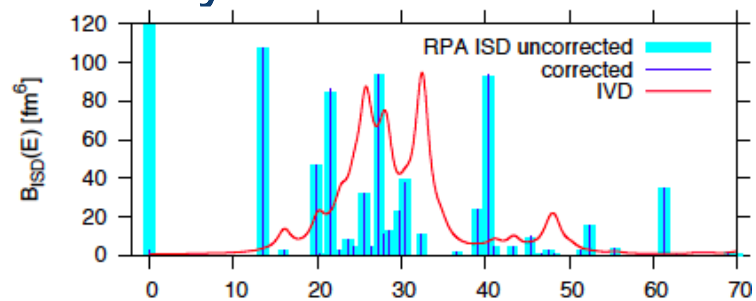
SOME THINGS I DON'T UNDERSTAND YET

The importance of the reference state

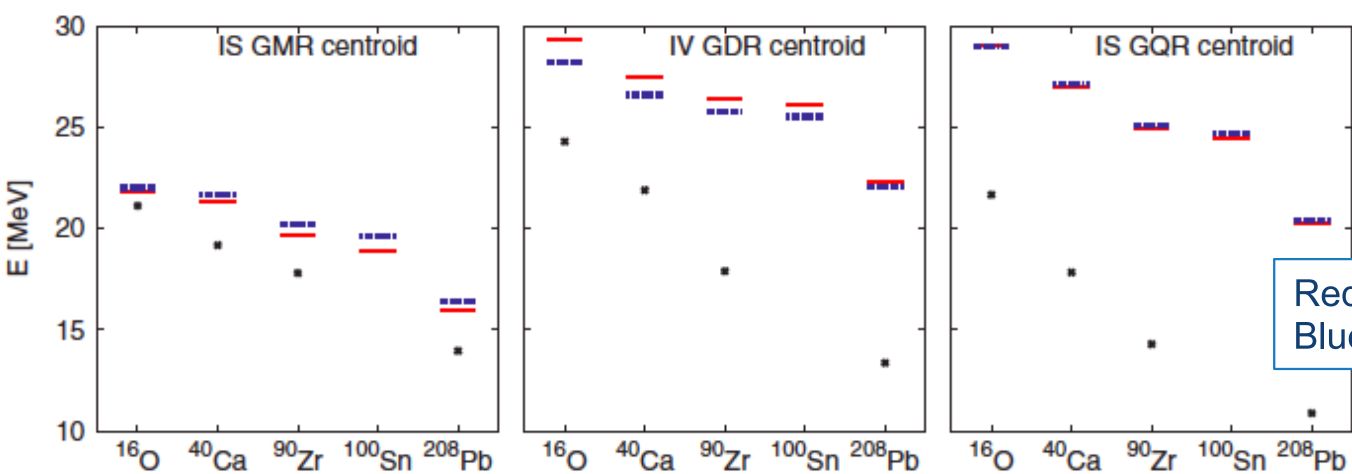
Validity of HF-RPA

❖ HF reference state ensures consistency and good properties

- |HF) = vacuum of ph states ➔ stability of RPA for physical states
- RPA = small-amplitude TDHF



❖ Quality of quasi-boson approximation



Red:HF-RPA
Blue: "renormalized"

PP, R.Roth, N.Paar, PRC75, 014310

HF-RPA, CC-RPA, IM-RPA

R.Trippel, PhD Thesis, TUD, 2016

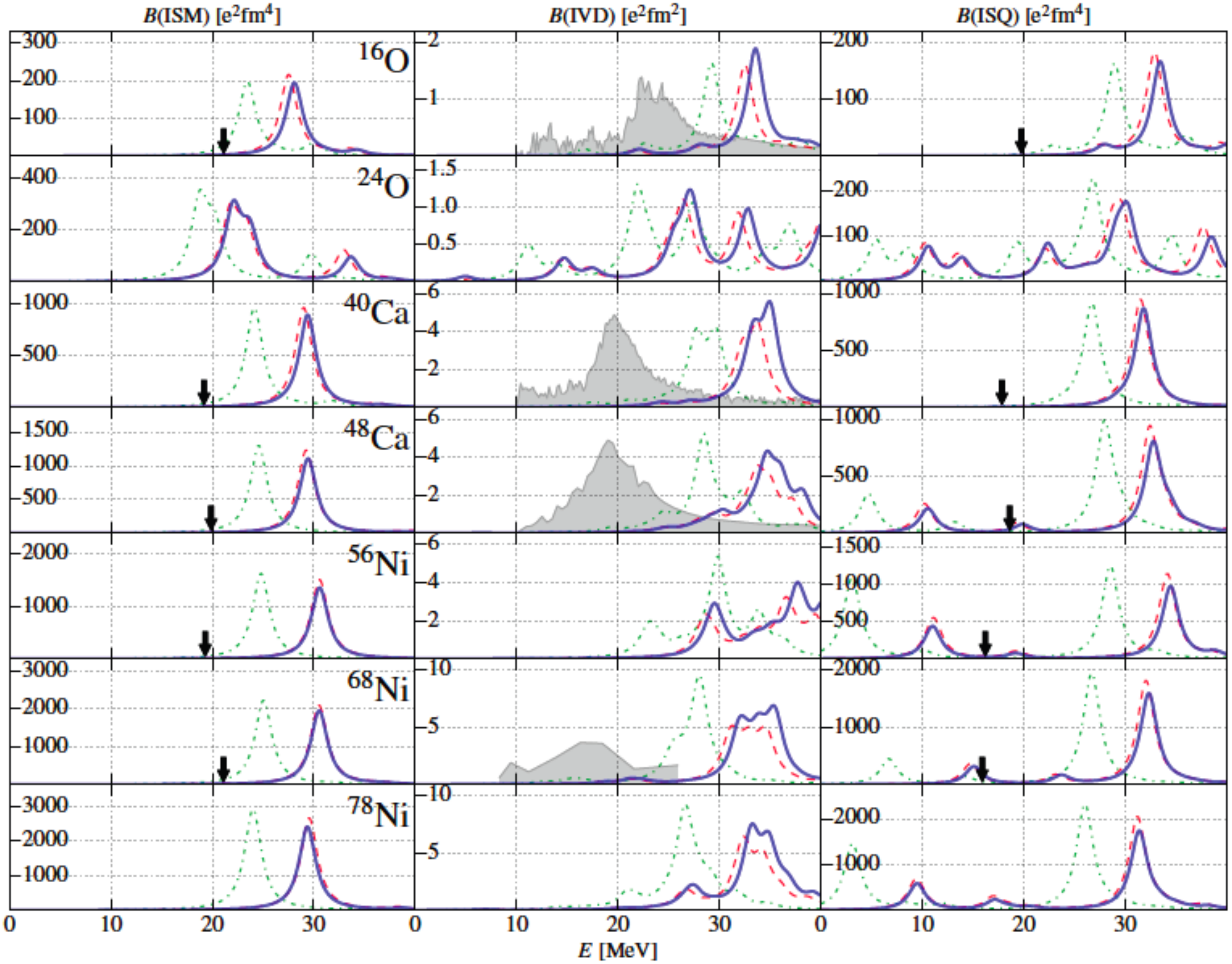


Figure 8.28: Results from HF-RPA (-·-·-), CC-RPA (- - - -), IM-RPA (——). Here we use the EM400 interaction ($\alpha = 0.08\,\mathrm{fm}^4$) with eMax = 12, E3Max = 14 and $\hbar\Omega = 24\,\mathrm{MeV}$.

HF-SRPA vs IM-SRPA (no instabilities?)

R.Trippel, PhD Thesis, TUD, 2016

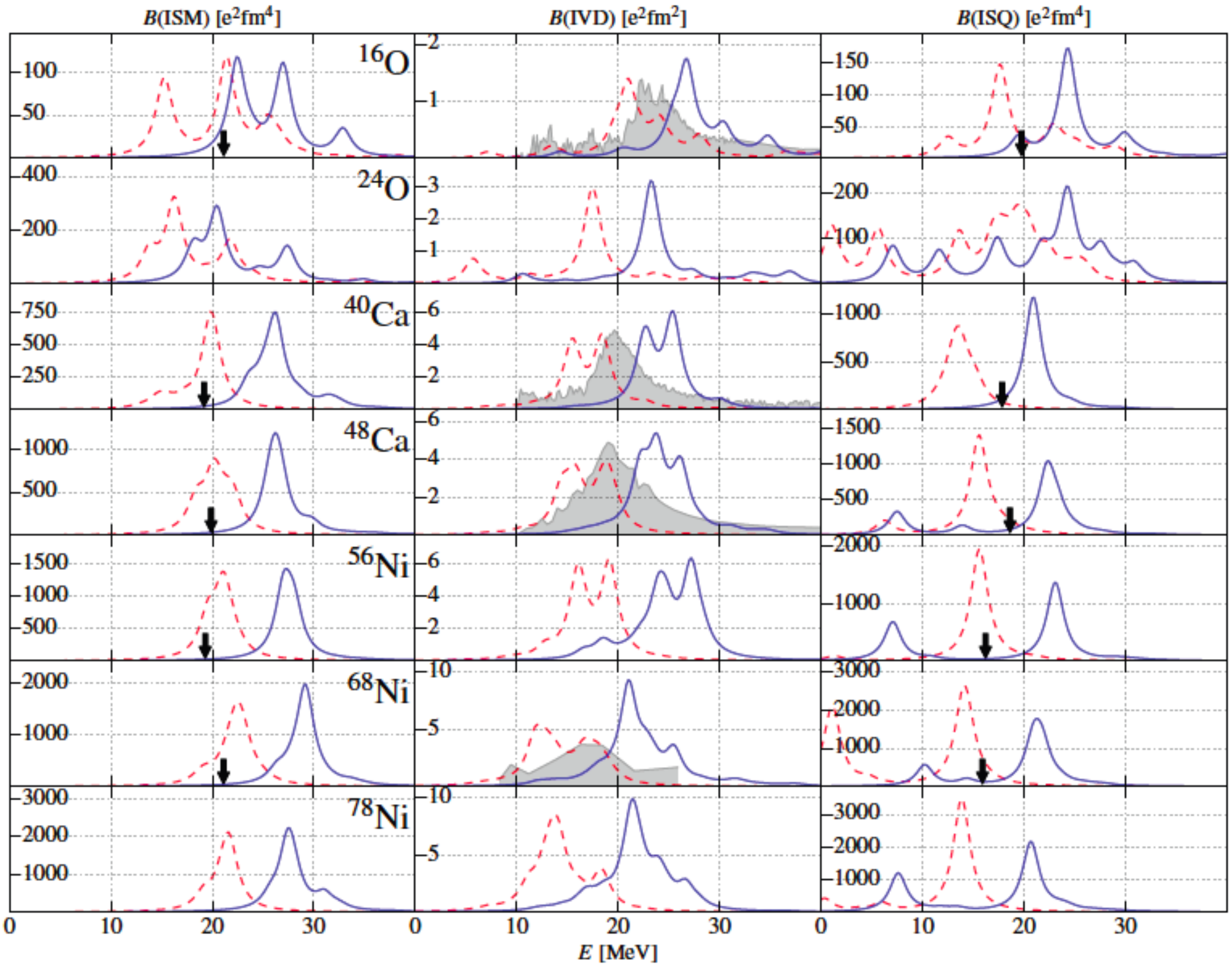


Figure 8.30: Comparison of results from HF-SRPA (- - - -) and IM-SRPA (——). Here we use the SAT interaction ($\alpha = 0.08\,\mathrm{fm}^4$) with eMax = 12, E3Max = 14 and $\hbar\Omega = 22\,\mathrm{MeV}$.

CONCEPTUAL ISSUES

RPA or SRPA?

Unofficial overview

- The purist
  - Thy shall not use SRPA based on the Hartree-Fock reference state. It suffers from inconsistencies and instabilities.
- The practitioner
  - If you are interested in [giant resonances / collective phonons, plasmons, ... / excitations exhausting most of the EWSR] it's fine, it will even give you some fine structure and energy corrections
- The contrarian
  - Then why would I use RPA at all?
  - When should I use what? I'm confused.

EDF basics

- Interested in one-particle observables only
  - Single-particle density encodes **implicitly** all relevant information.
  - Then: E[ρ]; HF, RPA; (SRPA superfluous?)
    - If I determine E[ρ] indirectly (from fits to data), I do not need to solve the A-particle Schrödinger equation!
    - Density fluctuations: $E[\rho_0+\delta\rho] \approx E[\rho_0] + \delta\rho\, E'[\rho_0]$: linear response
- Interested **explicitly** in two-particle observables
  - E.g., 2-phonon states; width of GRs, collisions
  - Two-particle density relevant
  - Then: E[ρ,g]; correlated ground state, SRPA

Roughly two approaches

- Given interaction + many-body method
  - Variational reference state + Equations of Motion
  - To lowest order, HF+RPA
  - Systematic inclusion of correlations / mp-mh until convergence

> - *“Wave-function approach” [JT]*
> - *Known Hamiltonian*

- Energy-density functionals + linear-response theory

> - *Kohn-Sham EFT [JT]*
> - *E[ρ,...] known; Hamiltonian not necessarily known*
> - *“black box” [AG]*

  - The order of truncation depends on the application

Energy density functional for KIDS

- Homogeneous matter -> Ansatz: k_F powers
- Nuclear EDF by **reverse engineering**
- Success in dilute and dense matter and **nuclear ground states**
- ➢ **Poster by Hana Gil**!

❖ KIDS = Korea: IBS - Daegu - Sungkyunkwan

❖ (ή : Kyungpook - IBS - Daegu - Sungkyunkwan)

Unofficial overview

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- The contrarian
  - Then why would I use RPA
  - When should I use what?

RPA:
- *Kohn-Sham EFT*
- *E[ρ] known somehow*
- *Hamiltonian not necessarily known*
- *"black box"*

Unofficial overview

- The purist
  - Thy shall not use SRPA based on the Hartree-Fock reference state. It suffers from inconsistencies and instabilities.
- The practitioner
  - If you are interested in [giant resonances / collective phonons, plasmons, ... / excitations exhausting most of the EWSR] it's fine, it will even give you some fine structure and energy corrections
- The contrarian
  - Then why would I use RP
  - When shoul

RPA:
- *Kohn-Sham EFT*
- *E[ρ] known somehow*

necessarily known

SRPA, etc:
- *"Wave-function approach"*
- *Known, perturbative Hamiltonian*

Unofficial overview

- The purist
  - Thy shall not use SRPA based on the Hartree-Fock reference state. It suffers from inconsistencies and instabilities.
- The practitioner
  - If you are interested in [giant resonances / collective phonons, plasmons, ... / excitations exhausting most of the EWSR] it's fine, it will even give you some fine structure and energy corrections
- The contrarian
  - Then why would I use RP
  - When shoul

RPA:

- *Kohn-Sham EFT*
- *E[ρ] known somehow*

necessarily known

SRPA, etc:

In the middle:

Good E[ρ] and RPA; interested in fragmentation: subtraction method?

Thank you!

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