

**Benoît BRAÏDA**

*QMC in Apuan Alps July 2012*

**A Valence Bond / Quantum Monte Carlo study  
of « pancake » bonding**

**Laboratoire de Chimie Théorique  
Université Pierre et Marie Curie - Paris6**

# The Laboratoire de Chimie Théorique of the Université Pierre et Marie Curie

- Université Pierre et Marie Curie (Paris) :



- $\approx 8000$  permanent (Pr.+researchers) +  $\approx 32000$  students
- Topics : mathematics, physics, chemistry, biology, medecine



# The Laboratoire de Chimie Théorique of the Université Pierre et Marie Curie



# The Laboratoire de Chimie Théorique of the Université Pierre et Marie Curie

- Laboratoire de Chimie théorique :

## **Selectivity & environment**

Complex (bio)organic and  
(bio)inorganic systems

## **Methodology**

DFT, QMC, local  $\Psi$   
method development

## **Concepts & interpretative methods**

ELF, MPD, VB  
develop.+appli.

- 26 permanent (prof.+CNRS)
- ~50 (+students)

## **Chemistry & universe**

Interstellar reactions,  
small molecules...

## **Chemistry & surface**

Reactivity on oxyde  
surfaces...



# The Laboratoire de Chimie Théorique of the Université Pierre et Marie Curie

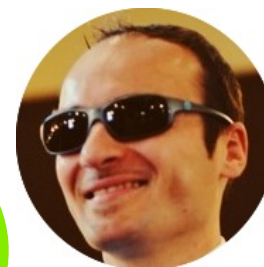
- Laboratoire de Chimie théorique :

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## Concepts & interpretative methods

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- 26 permanent (prof.+CNRS)  
- 52 (permanent+phD+postdocs)  
+ students ( $\approx 10-20$ )

## Chemistry & universe

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# Outline

- **Valence Bond theory :**
  - Motivation and basic concepts
  - Mixed Valence Bond / Quantum Monte Carlo methods
- **Application on «pancake bonding» in  $\text{DTCNE}_2^{2-}$  :**
  - MO analysis and MO-based calculations
  - Qualitative VB analysis
  - Quantitative VB calculations

- Birth and origins:



1916

G.N. Lewis



1928-34

L. Pauling

VB: a quantum dressing of Lewis model



- ~1930-1950s: Rise and glory



L. Pauling



VB dominated the mental map of chemistry



- ~1940-1960: The MO-VB rivalry



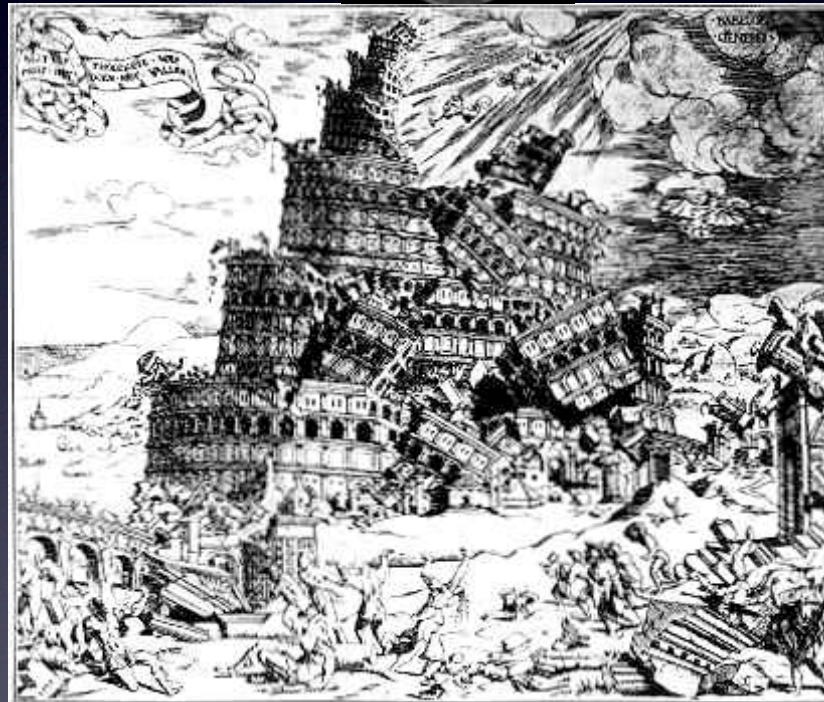
Successes of MO theory vs. VB «failures»



- ~1960-1980: The downfall



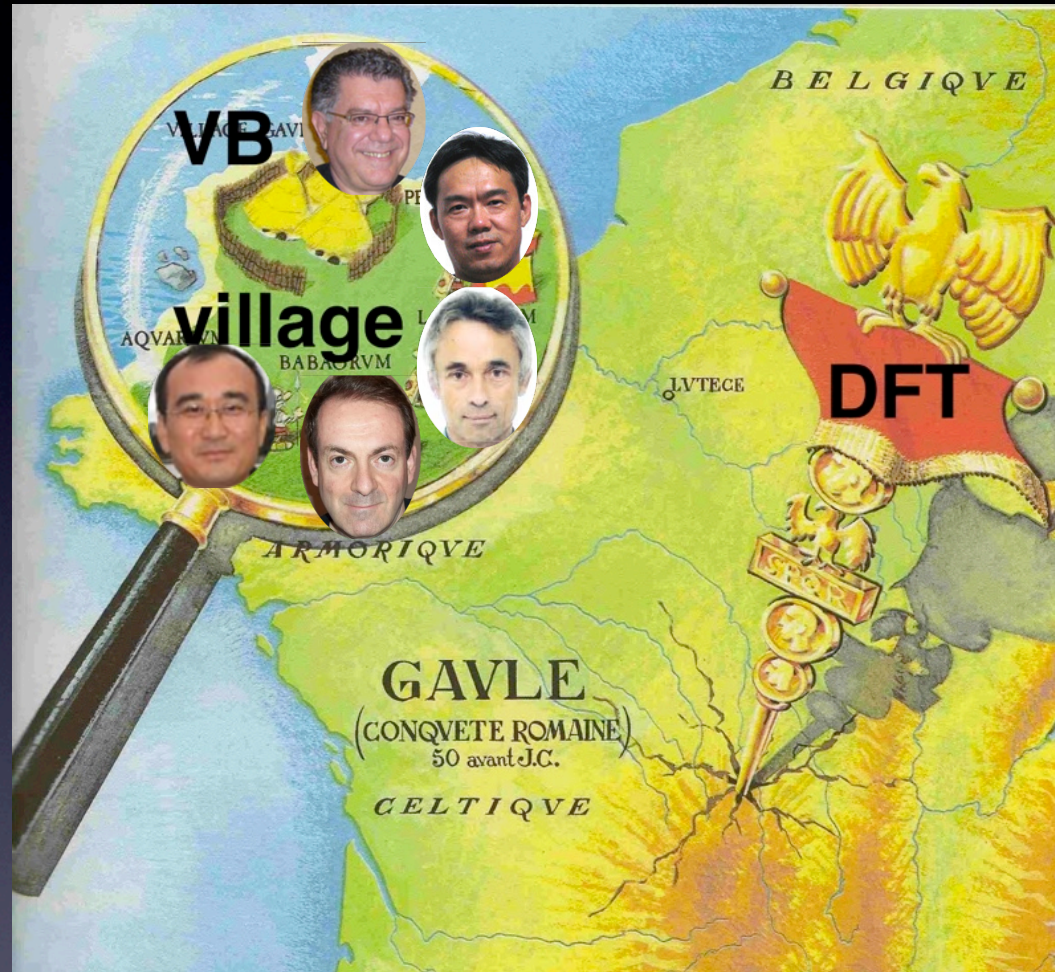
Sir John A. Pople



MO programs are developed, VB had nothing



- ~1980-2010: small but active community



New models, methods, programs, applications



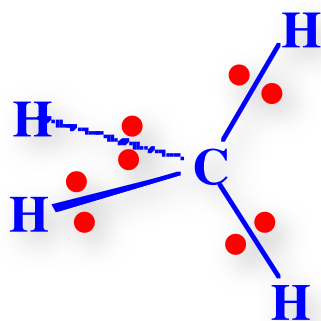
- 2012-...: awakening of the sleeping beauty?



All elements for a Valence Bond revival are ready

# Chemists' «schizophrenia»

- **Concepts and models** based on a localized vision :



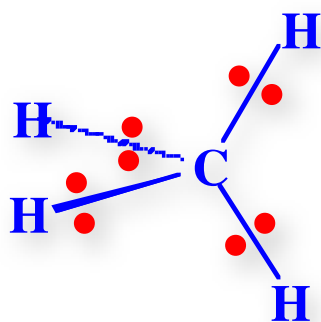
*Lewis model, arrow-pushing language, VSEPR, hybridization,...*

- **Localized** electron pairs
- **Chemical bond** concept



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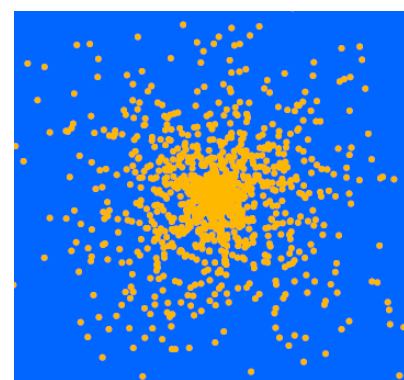


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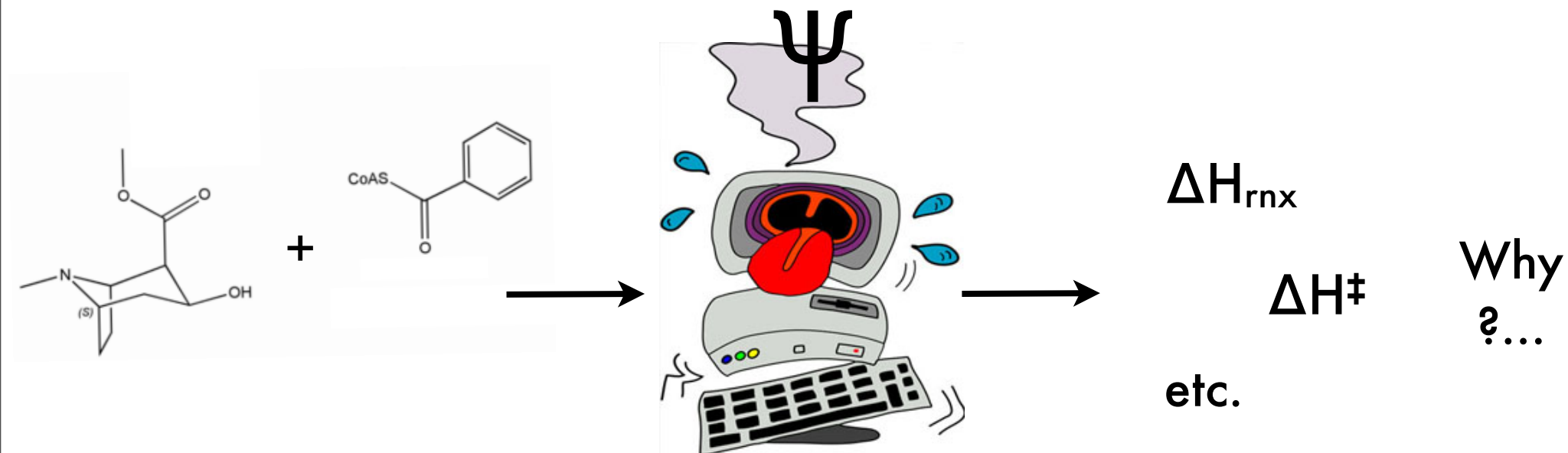
- **Quantitative theory** all-interacting delocalized particles vision :

$$\hat{H}\Psi = E\Psi$$

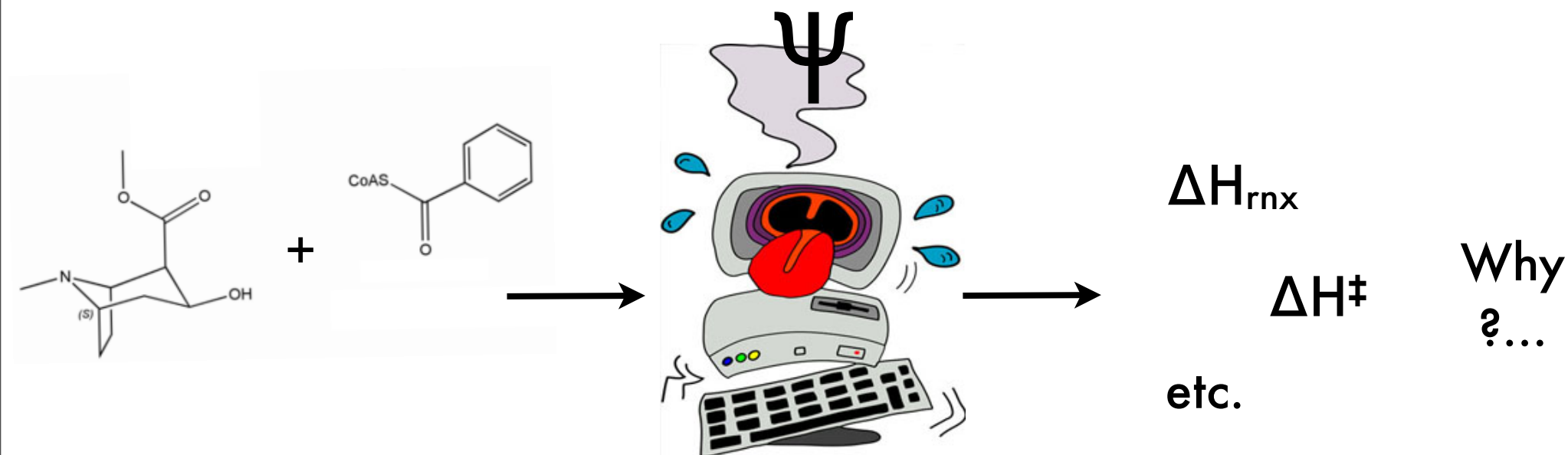


- **delocalized** particles ( $e^-$ ,  $n^+$ )
- **indistinguishable** and all-interacting (no chemical bond)

# Chemists' «schizophrenia»



# Chemists' «schizophrenia»



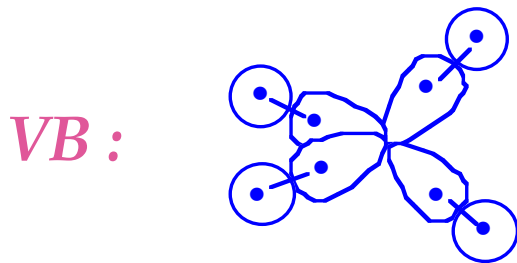
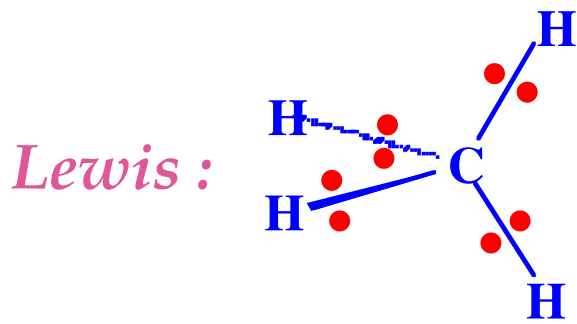
*«I know that the computer has understood, but I would like to understand too» (Eugene Wigner)*

=> how to build a bridge between quantum mechanics and chemists' vision ?



# Valence Bond theory

- Quantum dressing of Lewis' picture :



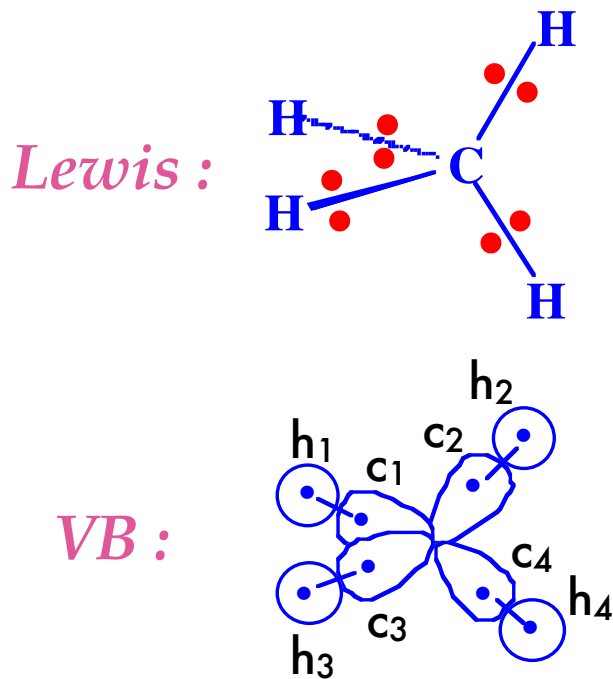
- VB wave functions :

- Electrons occupy **localized** orbitals (atomics, hybrids,...)
- A **bond** = two singlet-coupled electrons in two orbitals

=> bonds are essentially **covalent**  
+ minor ionics

# Valence Bond theory

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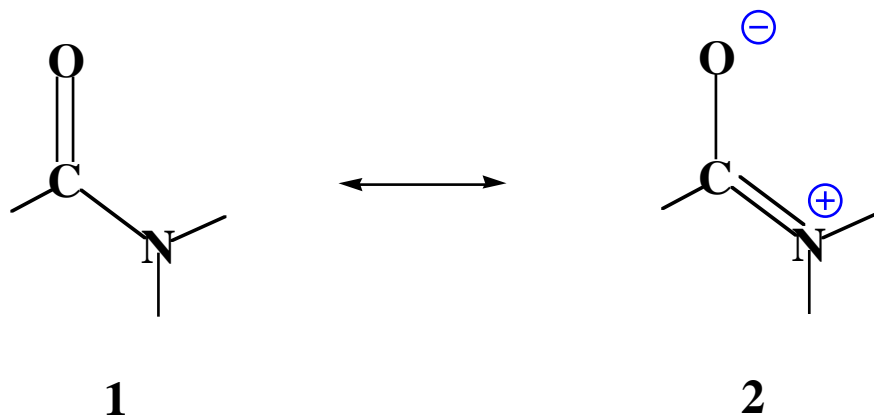
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$$\Psi = \left| (c_1 \bar{h}_1 + h_1 \bar{c}_1)(c_2 \bar{h}_2 + h_2 \bar{c}_2)(c_3 \bar{h}_3 + h_3 \bar{c}_3)(c_4 \bar{h}_4 + h_4 \bar{c}_4) \right|$$

# Valence Bond theory

- When more than one Lewis structure is needed :

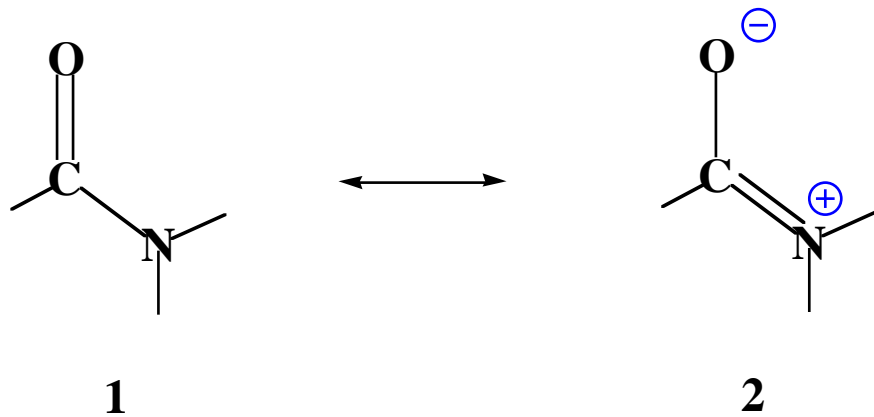


$$\Psi(1 \leftrightarrow 2) = C_1(\Psi_1) + C_2(\Psi_2)$$

- VB wave function :  
two **resonating** components,  
each one corresponding to one  
of the **2 structures**

# Valence Bond theory

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$$\Psi(1 \leftrightarrow 2) = C_1(\Psi_1) + C_2(\Psi_2)$$

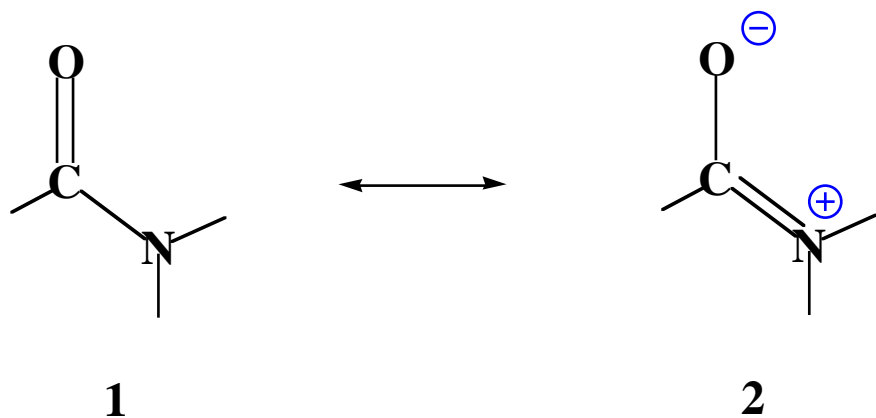
=> structure weights : 73% , 27%

- VB wave function :  
two **resonating** components,  
each one corresponding to one  
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# Valence Bond theory

- When more than one Lewis structure is needed :



$$\Psi(1\leftrightarrow 2) = C_1(\Psi_1) + C_2(\Psi_2)$$

=> structure weights : 73% , 27%

=> resonance energy :  $R.E. = E(\Psi_1) - E(\Psi_{1\leftrightarrow 2}) = 37 \text{ kcal/mol}$

=> rotation barrier due to resonance :

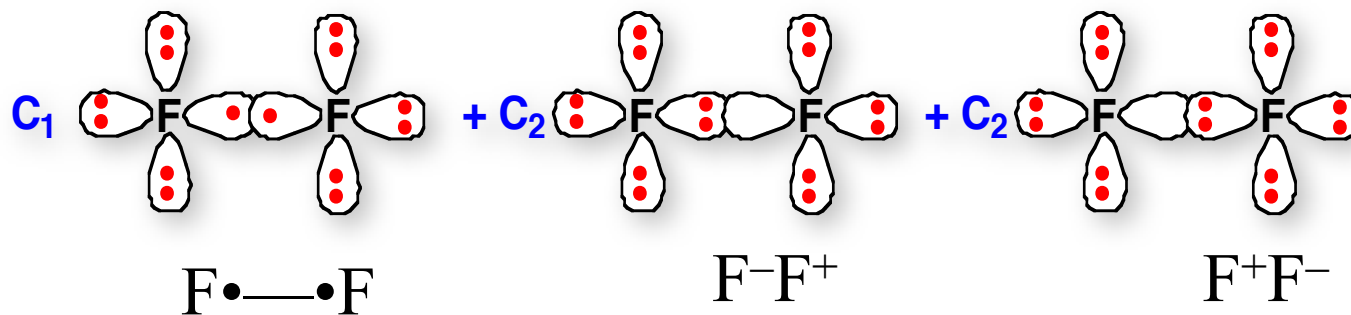
*Lauvergnat+Hiberty,  
JACS 1997, 119, 9478.*

# Ab initio VB methods

- Some *ab initio* Valence Bond methods :

- VBSCF includes **static correlation** : all configurations of electrons into orbitals included, coefs+orbitals optimized, but :

a common set of orbitals is optimized => compromise between  $\neq$  configuration

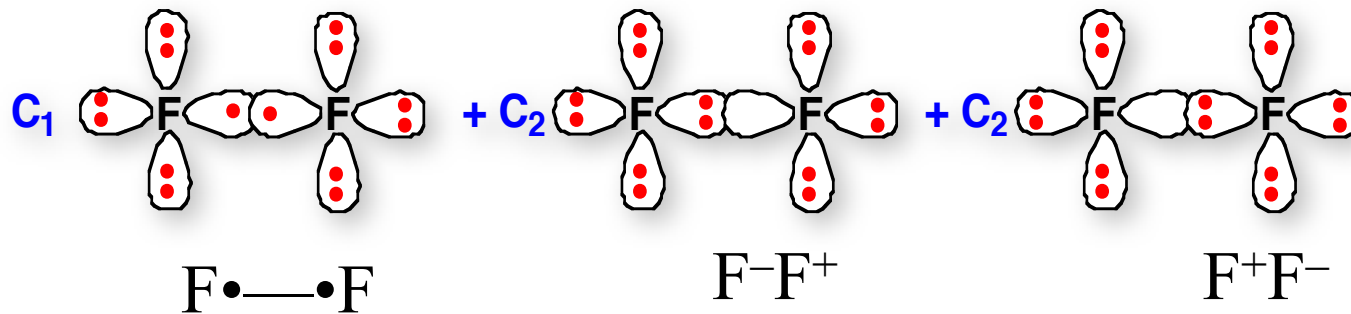


# Ab initio VB methods

- Some *ab initio* Valence Bond methods :

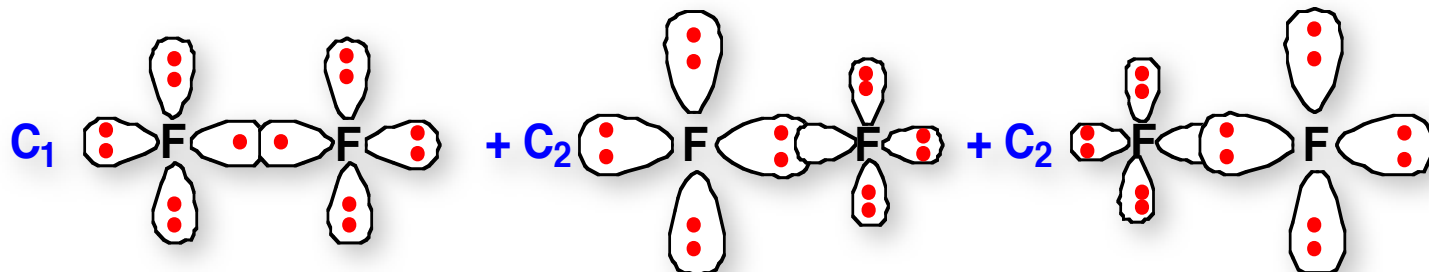
- VBSCF includes **static correlation** : all configurations of electrons into orbitals included, coefs+orbitals optimized, but :

a **common set of orbitals** is optimized => compromise between  $\neq$  configuration



- BOVB (Breathing Orbitals Valence Bond) includes **dynamical correlation** : the coefficients and orbitals are optimized, and furthermore...

**different orbitals for different VB structures** : orbitals adapted to each config.



# Why VB-QMC ?

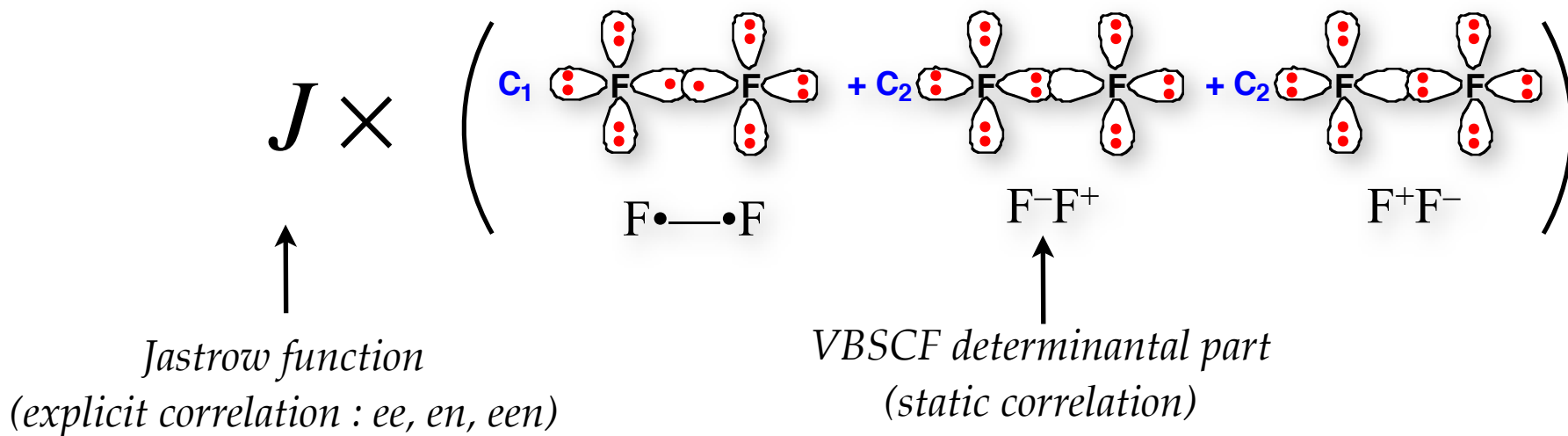
- ★ Interest : interpretative capabilities :
    - deep insight into the electronic structure
    - use the basic language of chemists (Lewis structures,...)
  - ★ Challenges : system size strongly limited :
    - **Non-orthogonality !**
    - Algorithms scaling ( $\sim N^{4-5}$ )
    - Accuracy (inclusion of static & dynamic correlation)
- => interesting way : VB / QMC mariage***



$$\begin{aligned}
& \langle \Psi_{x_1 y_1}^{ij} | \hat{H}_0 | \Psi_{x_2 y_2}^{ij} \rangle = (E_{inact}^{(0)} - \varepsilon_i - \varepsilon_j) \langle \Psi_{x_1 y_1}^{ij} | \Psi_{x_2 y_2}^{ij} \rangle \\
& + \left\{ (4 s_{x_2 x_1} s_{y_2 y_1} - 2 s_{y_2 x_1} s_{x_2 y_1}) E_{act}^{(0)} \right. \\
& + 4 f_{x_2 x_1} s_{y_2 y_1} + 4 f_{y_2 y_1} s_{x_2 x_1} - 2 f_{x_2 y_1} s_{y_2 x_1} - 2 f_{y_2 x_1} s_{x_2 y_1} \\
& + \left[ (f_{tx_1} s_{x_2 y_1} s_{y_2 u} + f_{ty_1} s_{y_2 x_1} s_{x_2 u} - 2 f_{tx_1} s_{y_2 y_1} s_{x_2 u} - 2 f_{ty_1} s_{x_2 x_1} s_{y_2 u}) \right. \\
& + (f_{tx_2} s_{y_2 x_1} s_{y_1 u} + f_{ty_2} s_{x_2 y_1} s_{x_1 u} - 2 f_{tx_2} s_{y_2 y_1} s_{x_1 u} - 2 f_{ty_2} s_{x_2 x_1} s_{y_1 u}) \\
& + (f_{y_2 x_1} s_{y_1 u} s_{x_2 t} + f_{x_2 y_1} s_{x_1 u} s_{y_2 t} - 2 f_{x_2 x_1} s_{y_1 u} s_{y_2 t} - 2 f_{y_2 y_1} s_{x_1 u} s_{x_2 t}) \left. \right] D^{tu} \\
& + \left[ f_{tu} (s_{y_2 x_1} s_{x_2 v} s_{y_1 w} + s_{x_2 y_1} s_{y_2 v} s_{x_1 w} - 2 s_{y_2 y_1} s_{x_2 v} s_{x_1 w} - 2 s_{x_2 x_1} s_{y_2 v} s_{y_1 w}) \right. \\
& + (f_{tx_1} s_{x_2 u} s_{y_1 v} s_{y_2 w} + f_{ty_1} s_{y_2 u} s_{x_1 v} s_{x_2 w} + f_{tx_2} s_{y_2 v} s_{x_1 u} s_{y_1 w} + f_{ty_2} s_{x_2 v} s_{y_1 u} s_{x_1 w}) \left. \right] \Pi^{tv, uw} \\
& + f_{tu} s_{y_2 v_1} s_{x_2 w_1} s_{y_1 v_2} s_{x_1 w_2} \Gamma^{t v_1 w_1, uv_2 w_2} \left. \right\} \\
& + \delta^{ij} \left\{ (4 s_{x_2 y_1} s_{y_2 x_1} - 2 s_{x_2 x_1} s_{y_2 y_1}) E_{act}^{(0)} \right. \\
& + 4 f_{x_2 y_1} s_{y_2 x_1} + 4 f_{y_2 x_1} s_{x_2 y_1} - 2 s_{x_2 x_1} f_{y_2 y_1} - 2 f_{x_2 x_1} s_{y_2 y_1} \\
& + \left[ (f_{tx_2} s_{y_2 y_1} s_{x_1 u} + f_{ty_2} s_{x_2 x_1} s_{y_1 u} - 2 f_{ty_2} s_{x_2 y_1} s_{x_1 u} - 2 f_{tx_2} s_{y_2 x_1} s_{y_1 u}) \right. \\
& + (f_{ty_1} s_{x_2 x_1} s_{y_2 u} + f_{tx_1} s_{y_2 y_1} s_{x_2 u} - 2 f_{ty_1} s_{y_2 x_1} s_{x_2 u} - 2 f_{tx_1} s_{x_2 y_1} s_{y_2 u}) \\
& + (f_{y_2 y_1} s_{x_1 u} s_{x_2 t} + f_{x_2 x_1} s_{y_1 u} s_{y_2 t} - 2 f_{x_2 y_1} s_{x_1 u} s_{y_2 t} - 2 f_{y_2 x_1} s_{y_1 u} s_{x_2 t}) \left. \right] D^{tu} \\
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& + (f_{ty_1} s_{x_2 u} s_{x_1 v} s_{y_2 w} + f_{tx_1} s_{y_2 u} s_{y_1 v} s_{x_2 w} + f_{tx_2} s_{y_1 u} s_{y_2 v} s_{x_1 w} + f_{ty_2} s_{x_1 u} s_{x_2 v} s_{y_1 w}) \left. \right] \Pi^{tv, uw} \\
& + f_{tu} s_{y_2 v_1} s_{x_2 w_1} s_{x_1 v_2} s_{y_1 w_2} \Gamma^{t v_1 w_1, uv_2 w_2} \left. \right\}
\end{aligned}$$

# Why VB-QMC ?

- ★ **No non-orthogonality problem !**  $e_l(w_i) = \left( \frac{\hat{H}\psi_T}{\psi_T} \right)(w_i) ; p = |\psi_T|^2$
- ★ Efficient parallel algorithm ; and scaling :  $N_e^3$  but with a HUGE prefactor !
- ★ New form of correlated VB wave-function : **Jastrow-VBSCF**



- ★ Moderate size basis sets are sufficient (converged at TZP level)
- ★ **We keep all the insight of a real Valence Bond wave function (VMC)** : weights, separate structures calculations...

# The Jastrow function

## ★ Padé expansion :

The Jastrow factor  $J$  is written as

$$J = J_{en} J_{ee} J_{een} = \exp(f_{en} + f_{ee} + f_{een})$$

where

$$f_{en}(R_{i\alpha}) = \sum_{i=1}^{N_{elec}} \sum_{\alpha=1}^{N_{nuc}} \left[ \left( \frac{a_1 R_{i\alpha}}{1 + a_2 R_{i\alpha}} + \sum_{p=2}^{N_{ord}} a_{p+1} R_{i\alpha}^p \right) - \left( \frac{a_1 R_c}{1 + a_2 R_c} + \sum_{p=2}^{N_{ord}} a_{p+1} R_c^p \right) \right]$$

$$f_{ee}(R_{ij}) = \sum_{i=2}^{N_{elec}} \sum_{j=1}^{i-1} \left[ \left( \frac{b_1 R_{ij}}{1 + b_2 R_{ij}} + \sum_{p=2}^{N_{ord}} b_{p+1} R_{ij}^p \right) - \left( \frac{b_1 R_c}{1 + b_2 R_c} + \sum_{p=2}^{N_{ord}} b_{p+1} R_c^p \right) \right]$$

$$f_{een}(R_{i\alpha}, R_{j\alpha}, R_{ij}) = \sum_{i=2}^{N_{elec}} \sum_{j=1}^{i-1} \sum_{\alpha=1}^{N_{nuc}} \sum_{p=2}^{N_{ord}} \sum_{k=p-1}^0 \sum_{l=l_{max}}^0 c_n R_{ij}^k (R_{i\alpha}^l + R_{j\alpha}^l) (R_{i\alpha} R_{j\alpha})^m, \quad \text{where } m = \frac{p-k-l}{2}$$

and  $l_{max}$  is  $p - k$  if  $k \neq 0$  and  $p - k - 2$  if  $k = 0$ . Only terms for which  $m = \frac{p-k-l}{2}$  is an integer are included.



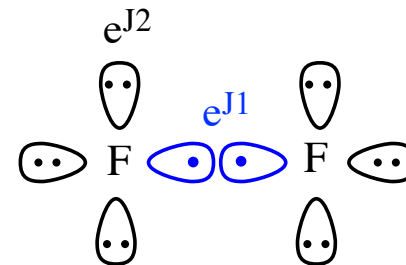
# The Jastrow function

★ Multi-Jastrow approach :

$$\Psi_T = \sum_{k=1}^{N_{\text{det}}} c_k \begin{vmatrix} e^{J_{k_1}^\uparrow(\mathbf{r}_1|\mathbf{r}_i \neq \mathbf{r}_1)} \phi_{k_1}^\uparrow(\mathbf{r}_1) & \dots & e^{J_{k_1}^\uparrow(\mathbf{r}_{N_\uparrow}|\mathbf{r}_i \neq \mathbf{r}_{N_\uparrow})} \phi_{k_1}^\uparrow(\mathbf{r}_{N_\uparrow}) \\ \vdots & \vdots & \vdots \\ e^{J_{k_{N_\uparrow}}^\uparrow(\mathbf{r}_1|\mathbf{r}_i \neq \mathbf{r}_1)} \phi_{k_{N_\uparrow}}^\uparrow(\mathbf{r}_1) & \dots & e^{J_{k_{N_\uparrow}}^\uparrow(\mathbf{r}_{N_\uparrow}|\mathbf{r}_i \neq \mathbf{r}_{N_\uparrow})} \phi_{k_{N_\uparrow}}^\uparrow(\mathbf{r}_{N_\uparrow}) \end{vmatrix} \begin{vmatrix} e^{J_{k_1}^\downarrow(\mathbf{r}_{N_\uparrow+1}|\mathbf{r}_i \neq \mathbf{r}_{N_\uparrow+1})} \phi_{k_1}^\downarrow(\mathbf{r}_{N_\uparrow+1}) & \dots & e^{J_{k_1}^\downarrow(\mathbf{r}_N|\mathbf{r}_i \neq \mathbf{r}_N)} \phi_{k_1}^\downarrow(\mathbf{r}_N) \\ \vdots & \vdots & \vdots \\ e^{J_{k_{N_\downarrow}}^\downarrow(\mathbf{r}_{N_\uparrow+1}|\mathbf{r}_i \neq \mathbf{r}_{N_\uparrow+1})} \phi_{k_{N_\downarrow}}^\downarrow(\mathbf{r}_{N_\uparrow+1}) & \dots & e^{J_{k_{N_\downarrow}}^\downarrow(\mathbf{r}_N|\mathbf{r}_i \neq \mathbf{r}_N)} \phi_{k_{N_\downarrow}}^\downarrow(\mathbf{r}_N) \end{vmatrix}.$$

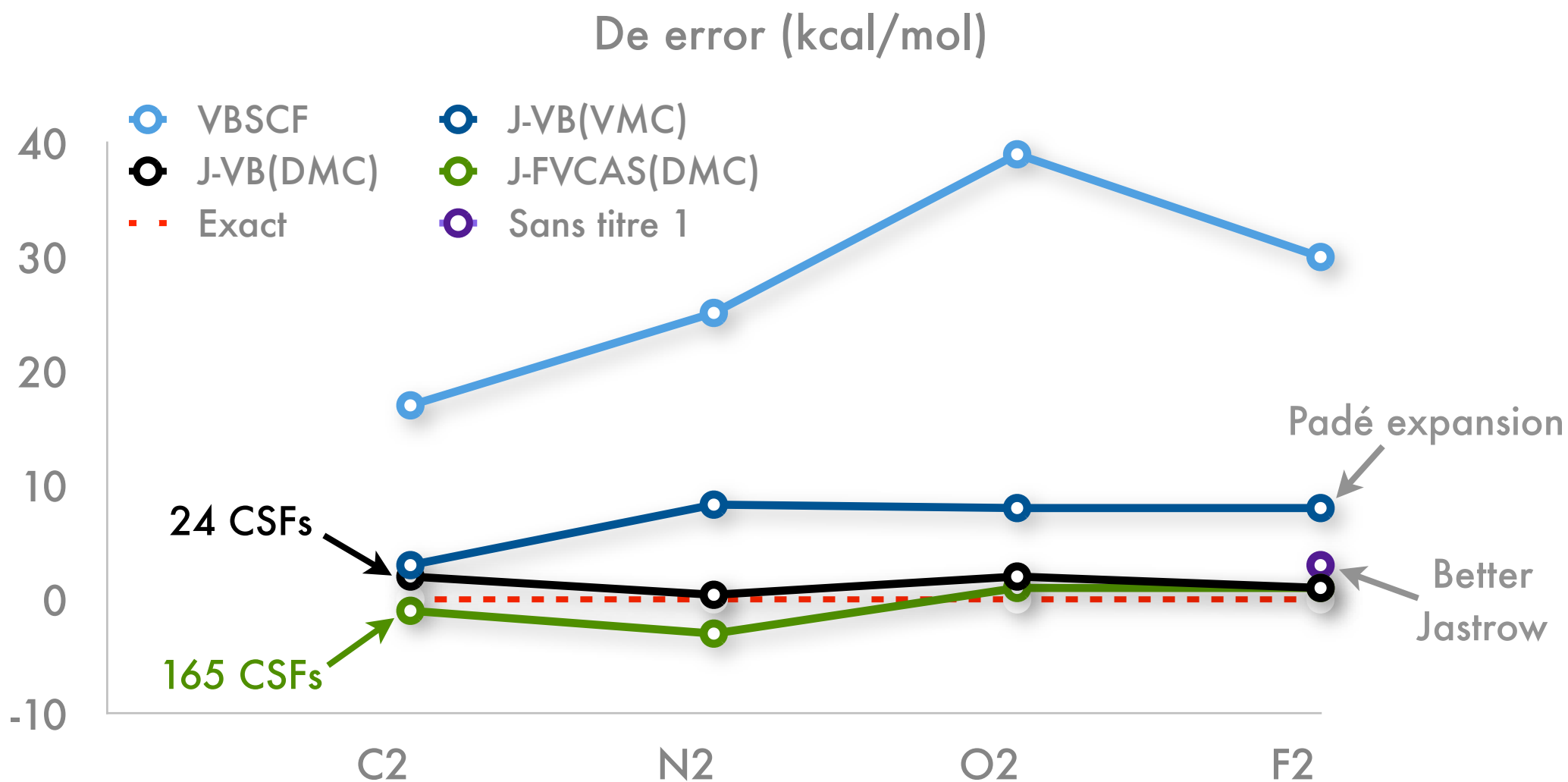
$$J_k(\mathbf{r}_i|\mathbf{r}_j \neq \mathbf{r}_i) = \frac{1}{2} \sum_{j \neq i} \sum_{\alpha} [s_k(\tilde{r}_{ij}) - p_{k\alpha}(\tilde{r}_{i\alpha}) - p_{k\alpha}(\tilde{r}_{j\alpha}) + g_{k\alpha}^{(1)} \tilde{r}_{i\alpha}^2 \tilde{r}_{j\alpha}^2 + g_{k\alpha}^{(2)} (\tilde{r}_{i\alpha}^2 + \tilde{r}_{j\alpha}^2) \tilde{r}_{ij}^2],$$

Different Jastrow for active/inactive...



\* T. Bouabça, B. Braïda, and M. Caffarel, *J. Chem. Phys.* **2010**, 133, 044111

# VB-QMC : benchmarking



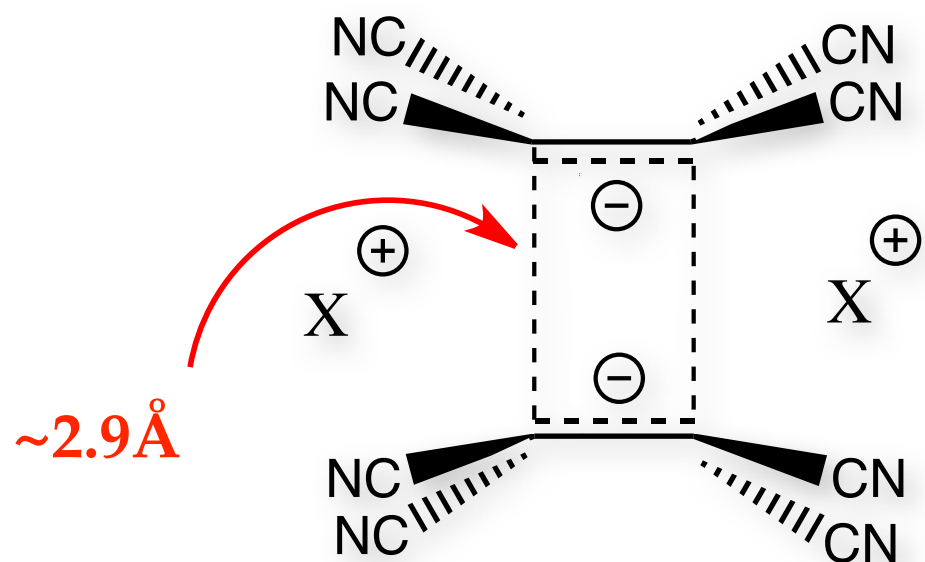
\* B. Braïda, J. Toulouse, M. Caffarel & C. J. Umrigar, J. Phys. Chem., **2011**, 134, 084108

# Outline

- Valence Bond theory :
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  - Mixed Valence Bond / Quantum Monte Carlo methods
- **Application on «pancake bonding» in  $\text{DTCNE}_2^{2-}$  :**
  - MO analysis and MO-based calculations
  - Qualitative VB analysis
  - Quantitative VB calculations

# «Pancake bonding»

- $\text{DTCNE}_2^{2-}$  :



$\text{X}^+ : \text{Na}^+, \text{Cs}^+, [\text{iPr}_4\text{N}]^+,$   
 $[\text{Cr}(\text{C}_6\text{Me}_3\text{H}_3)_2]^+, \dots$

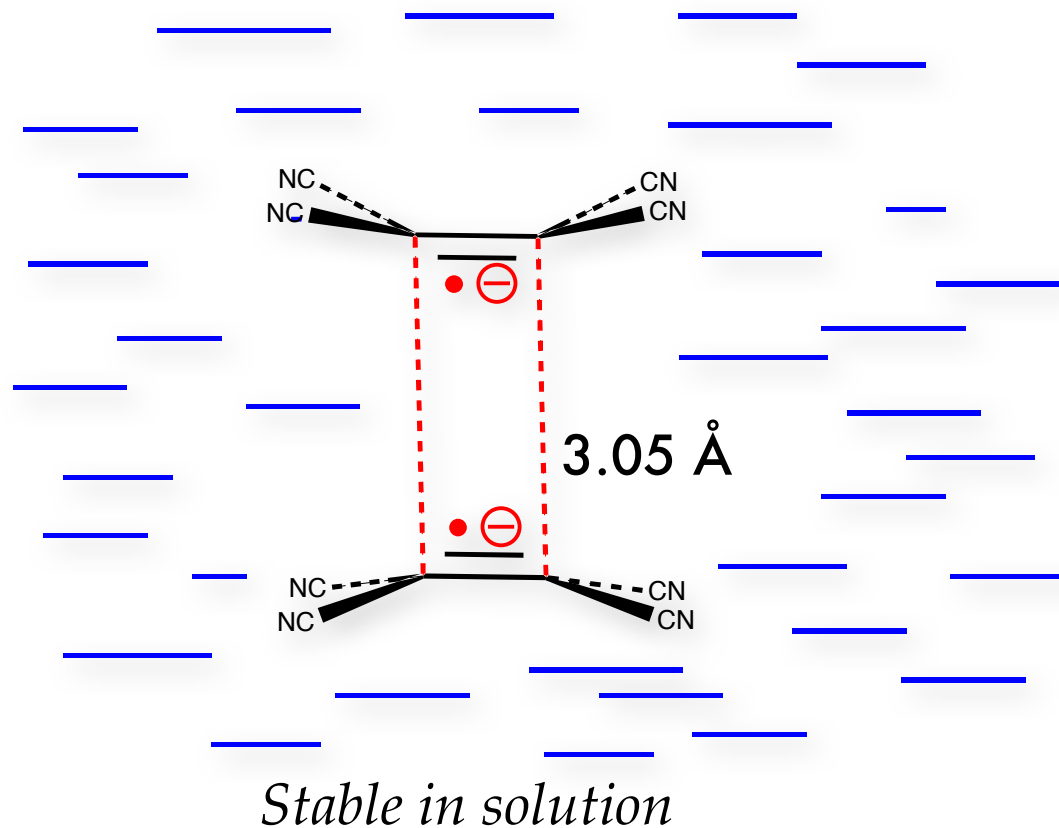
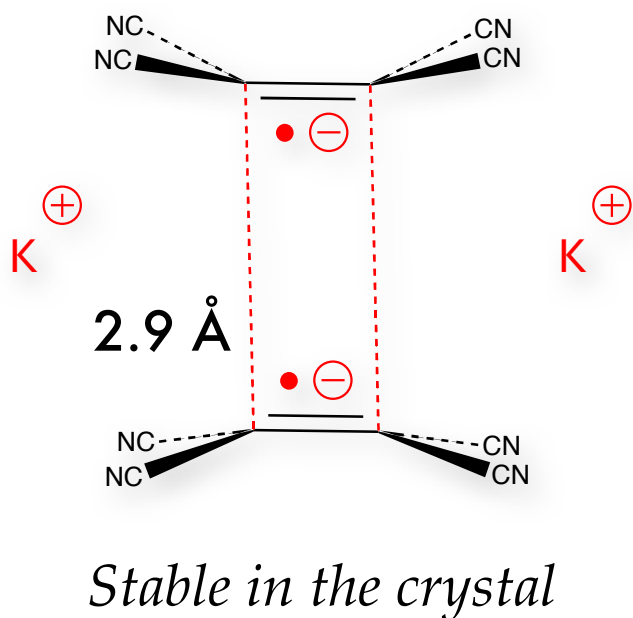
**Electrostatic complex ?...**

- Strong bonds
- Same bonding distance whatever the size of the cations !



# «Pancake bonding»

- $\text{DTCNE}_2^{2-}$  :

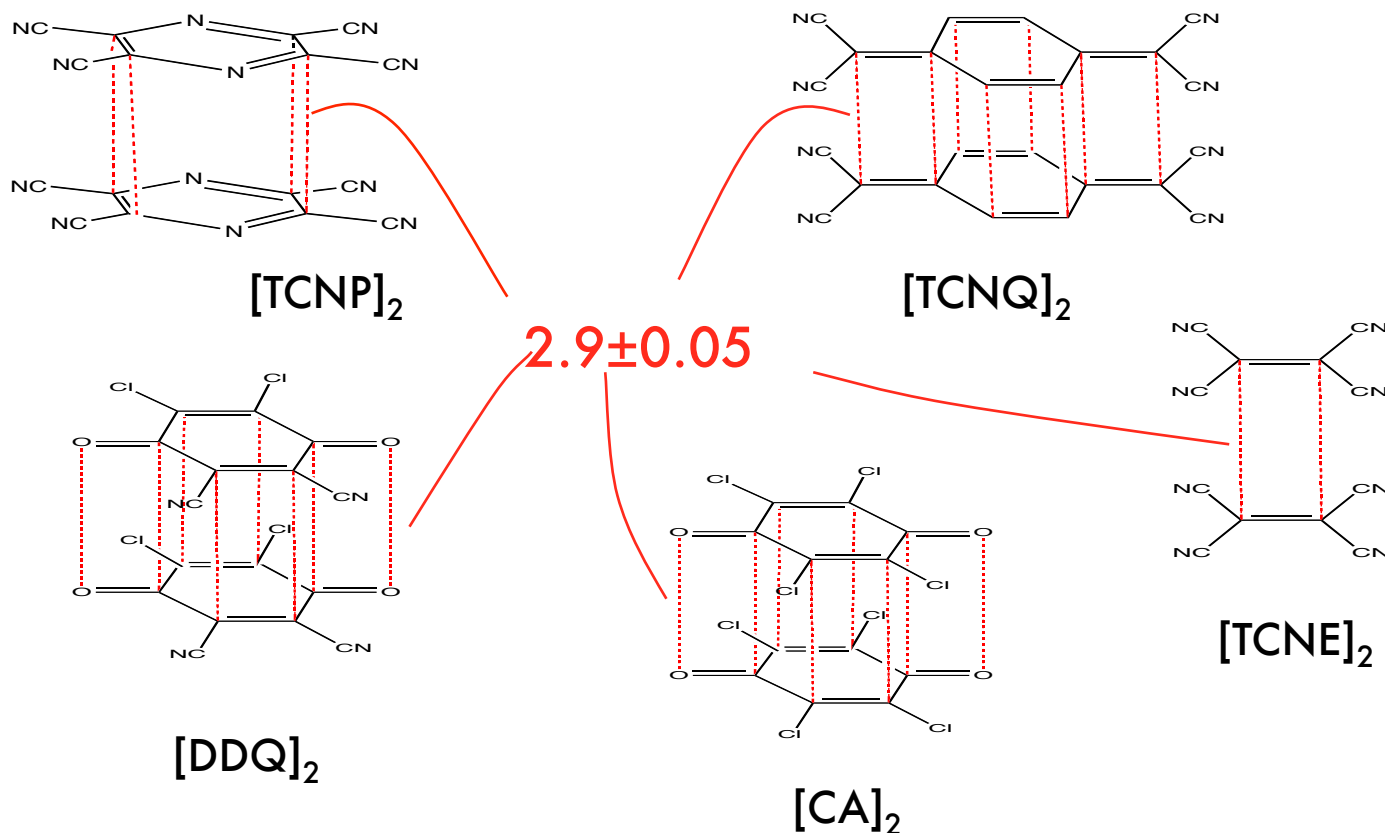


- Same bonding distance whatever the environment !

**$\Rightarrow$  not only electrostatic... also an interfragment bond**

# «Pancake bonding»

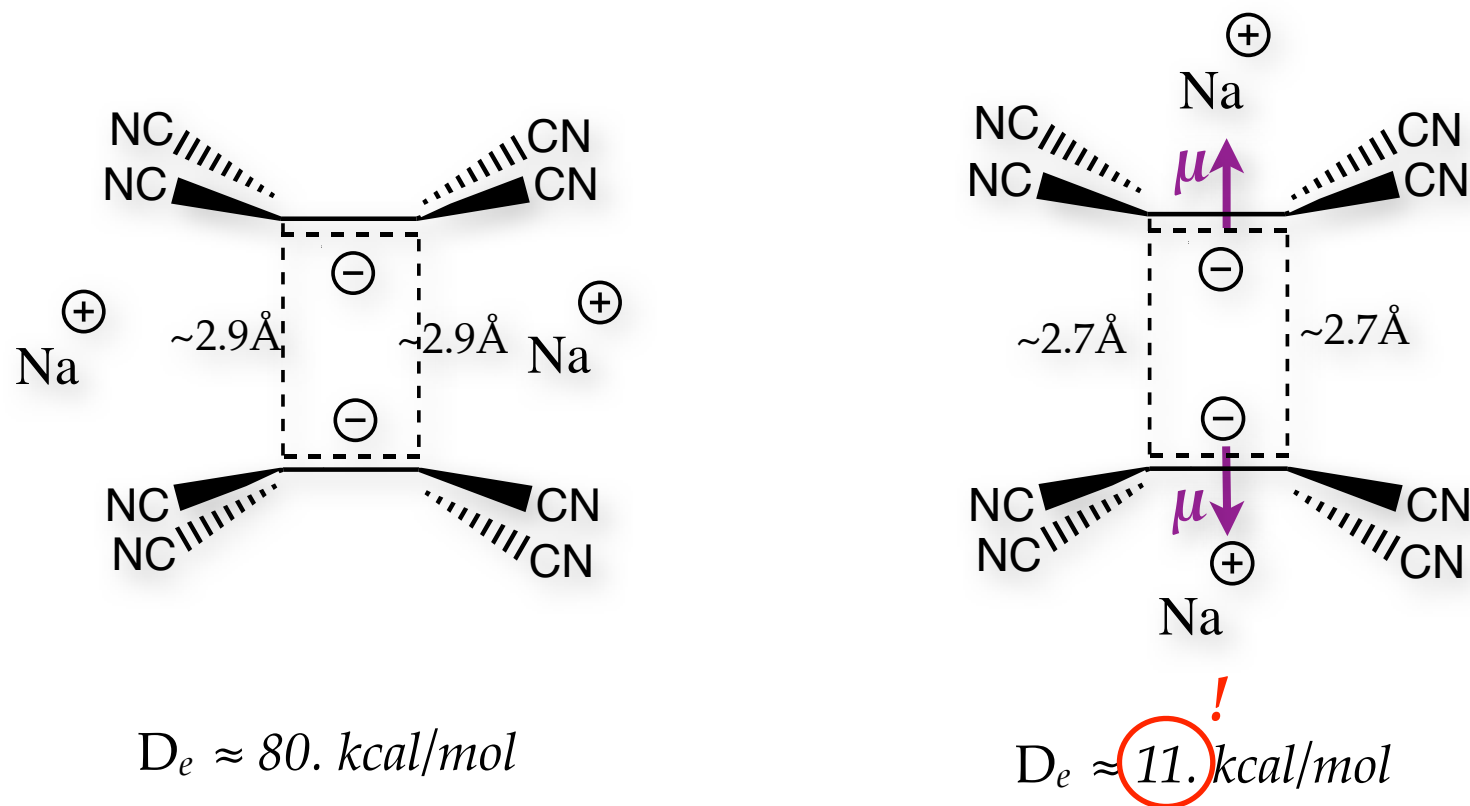
- Other «pancake bonding» systems :



- Same bonding distance whatever the system !
- $d_{C...C} < \text{sum of vdW radii (3.45\text{\AA})}$  , but much longer than C-C 2e bonds (1.54\text{\AA})

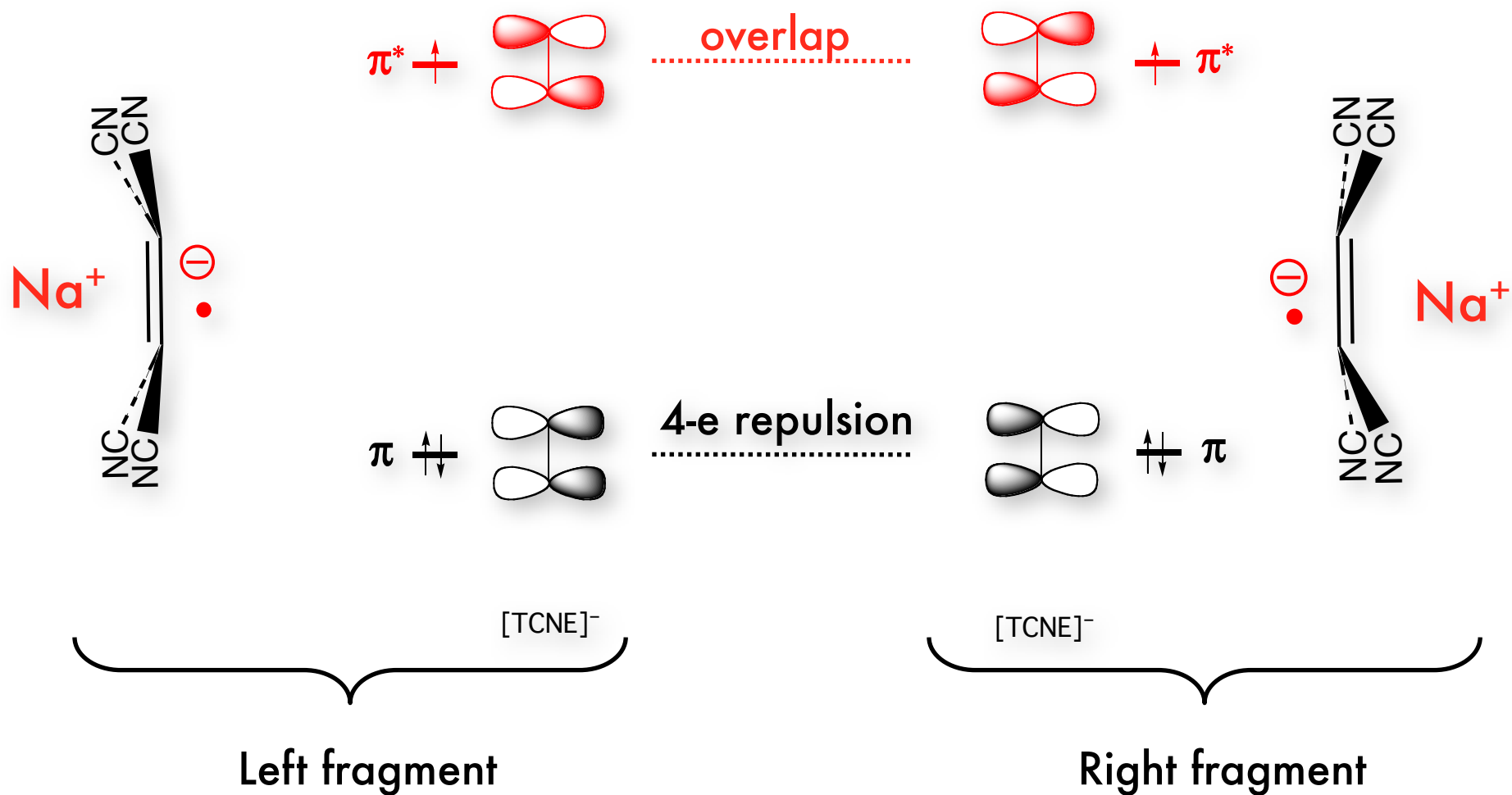
# «Pancake bonding»

- What kind of bonding in  $\text{DTCNE}_2^{2-}$ ?

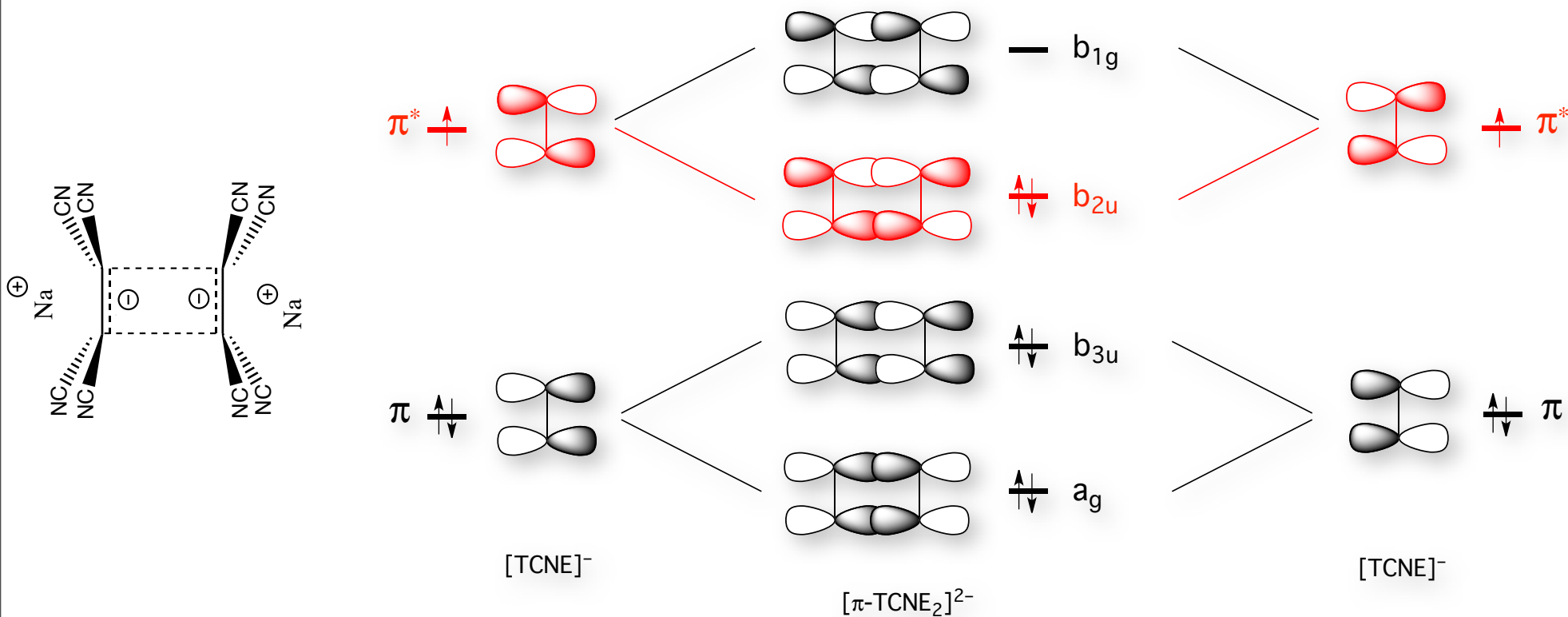


Right geom : significant bonding force overcome repulsive electrostatics !

# Qualitative MO analysis



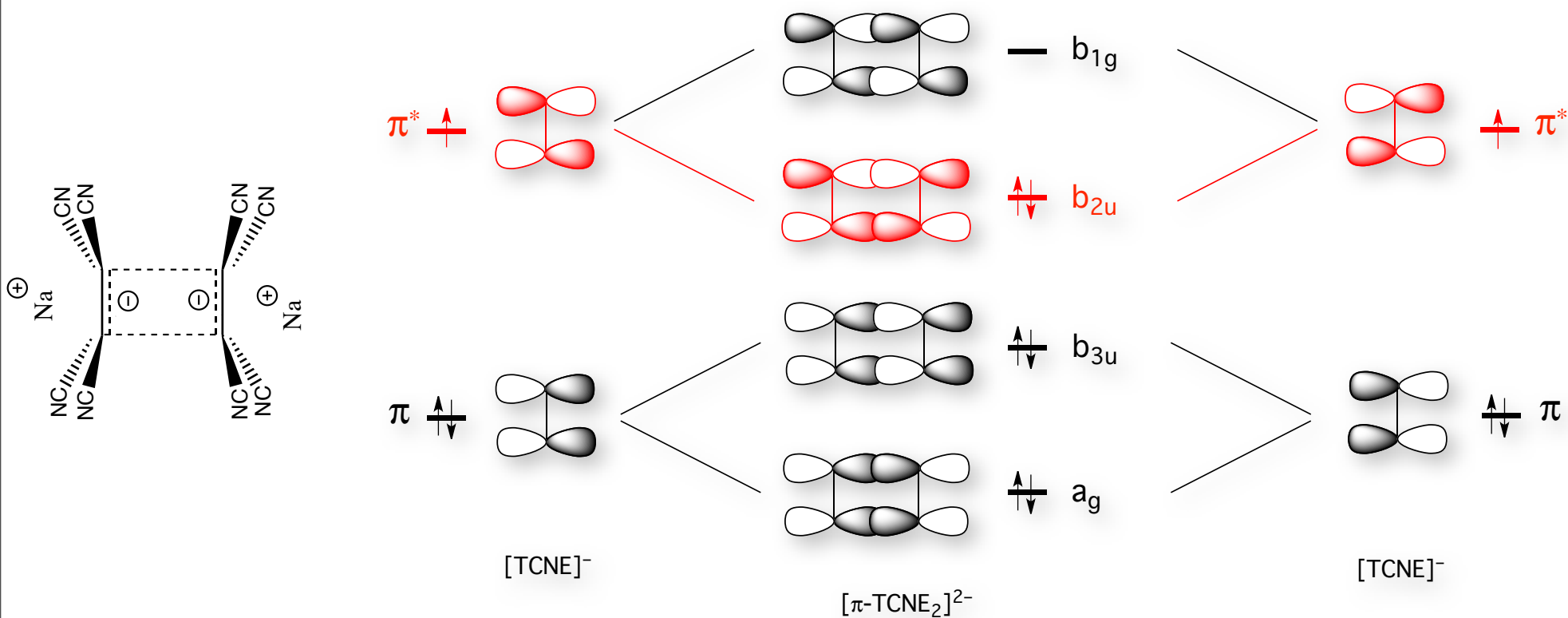
# Qualitative MO analysis



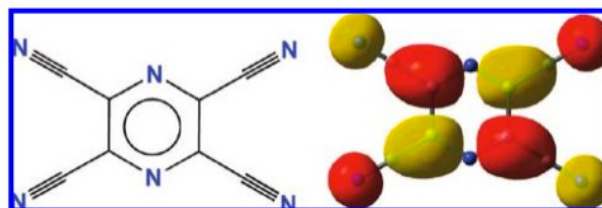
$\Rightarrow$  a simple **4 centers / 2 electrons bond** ?



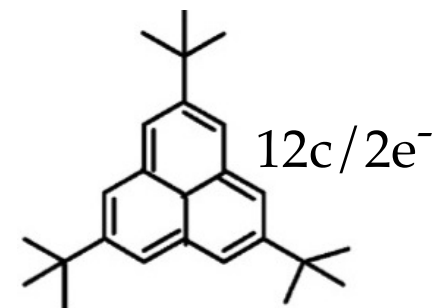
# Qualitative MO analysis



Other «pancake bonding»  
 systems : **n center /  $2e^-$  bonds**

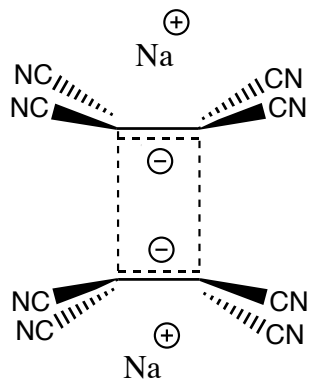


$8c/2e^-$

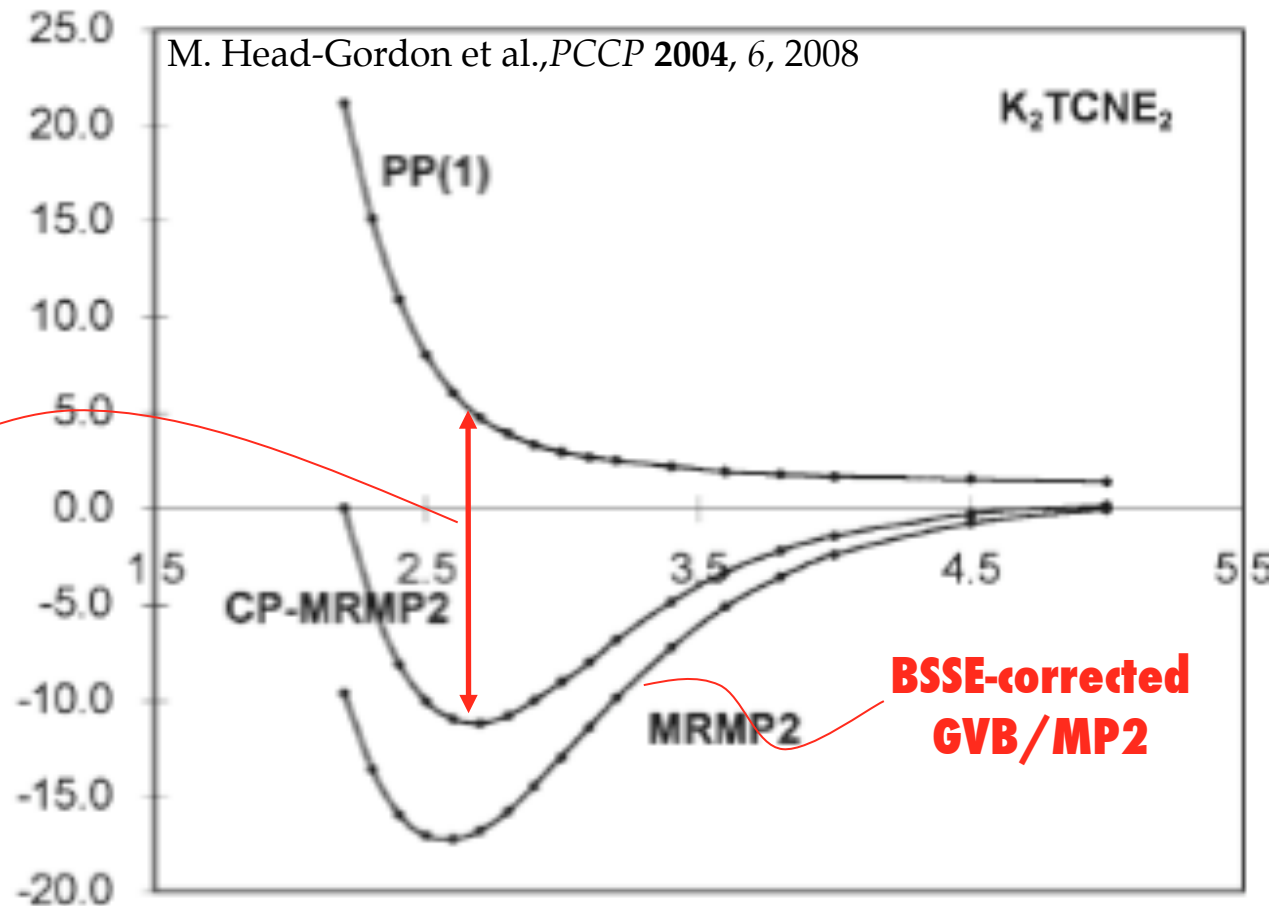


# Quantitative MO computations

- GVB calculations of the axial conformation :



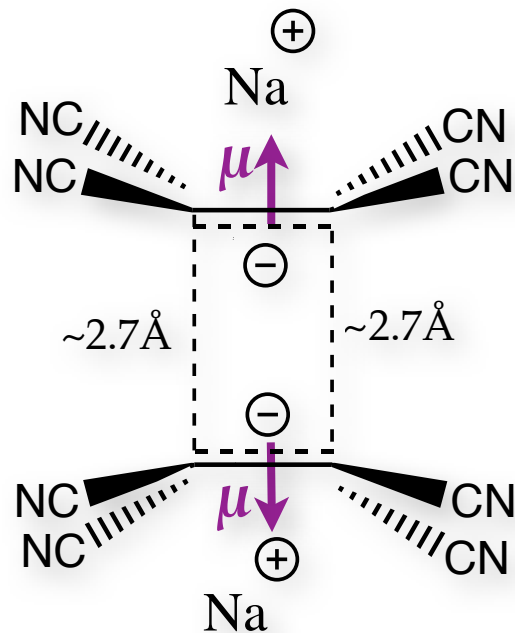
Contribution of  
dynamic correlation  
to bonding:  
**> 23 kcal/mol !**



- GVB-PP describes well 2e bonds (particularly stretched...) : not the case here !
- Contribution of dynamical correlation energy huge !

# Situation

- What kind of bonding in  $\text{DTCNE}_2^{2-}$  ?



$$D_e \approx \textcircled{11.} \text{ kcal/mol} !$$

- Same bonding distance whatever the counter-ion, the environment, the system...
- MO analysis propose a  $4c/2e$  bond, but GVB-PP fails to account for any bonding...
- Distance too short, bond energy too strong to be dispersion...

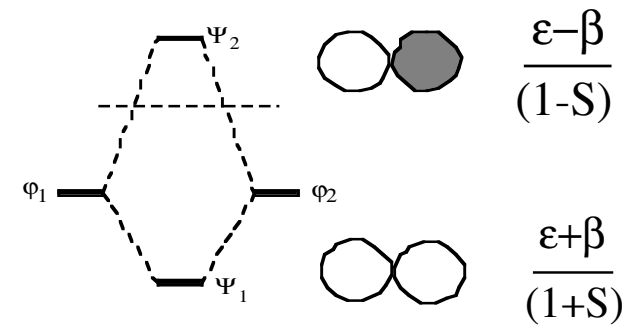
⇒ try a different perspective... with VB theory !

# Qualitative VB

- Let us apply qualitative VB analysis first :

**Effective Hamiltonian :**  $H^{\text{eff}} = (\mathbf{h}(1) + \mathbf{h}(2) + \mathbf{h}(3) + \dots)$

**Parameters:**  $\beta$ ,  $S$ ,  $\mathcal{E}$  (same as in the MO framework)



## 1) Energy of a determinant :

$$\langle D_i | H | D_i \rangle = \frac{-2n\beta S}{1 - S^2} \quad n = \text{N}^{\#} \text{ of neighboring } (\uparrow\uparrow) \text{ pairs}$$

## 2) Off diagonal terms :

- Determinants differ by 2 spinorbitals:

$$\langle (| a \bar{b} \rangle) | H | (| b \bar{a} \rangle) \rangle = 2\beta_{ab} S_{ab}$$

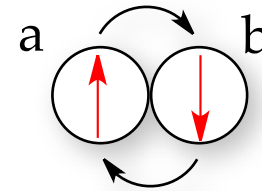
- Determinants differ by + than 2 spinorbitals :

$$\langle D_i | H | D_j \rangle = 0$$

# Qualitative VB

- **2e-bond :**

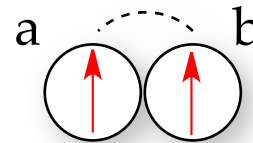
$$\psi_{VB} = \frac{|a\bar{b}| + |b\bar{a}|}{\sqrt{2(1+S^2)}}$$



$$\Rightarrow E = \langle \psi_{VB} | H | \psi_{VB} \rangle = \dots = \frac{2\beta S}{(1+S^2)} = D_e(2e-bond)$$

- **Triplet repulsion :**

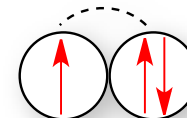
$$\psi_{VB} = \frac{|a\bar{b}| \ominus |b\bar{a}|}{\sqrt{2(1+S^2)}}$$



$$\Rightarrow E = \langle \psi_{VB} | H | \psi_{VB} \rangle = \frac{\ominus 2\beta S}{(1\ominus S^2)}$$

Same as **3e-repulsion :**

$$\psi_{VB} = |a\bar{a}b|$$

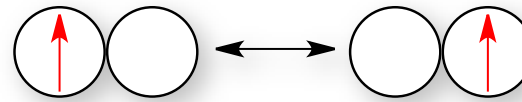




# Qualitative VB

- Elementary interactions :

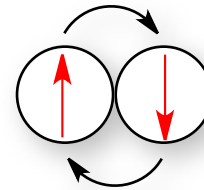
1-e bond ( $A\uparrow B$ ) =



**VB**  
 $\frac{\beta}{1+S}$

**MO**  
 $\frac{\beta}{1+S}$

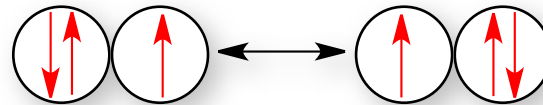
2-e bond ( $A-B$ ) =



$\frac{2\beta S}{1+S^2}$

$\frac{2\beta}{1+S}$

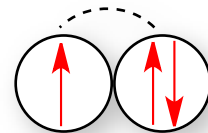
3-e bond ( $A.:B$ ) =



$\frac{\beta(1-3S)}{1-S^2}$

$\frac{\beta(1-3S)}{1-S^2}$

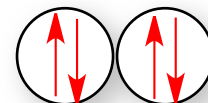
Triplet / 3-e repulsion ( $A\downarrow\uparrow \uparrow B$ ) =



$\frac{-2\beta S}{1-S^2}$

$\frac{-2\beta S}{1-S^2}$

4-e repulsion ( $A\uparrow\downarrow \downarrow\uparrow B$ ) =

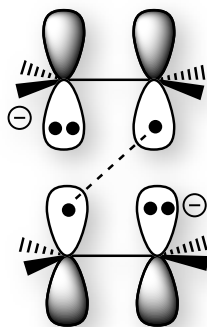
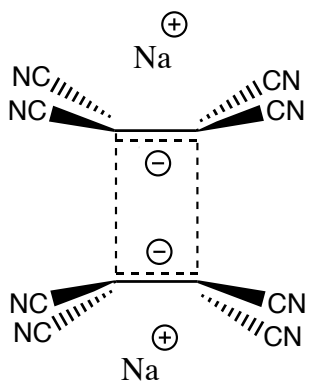


$\frac{-4\beta S}{1-S^2}$

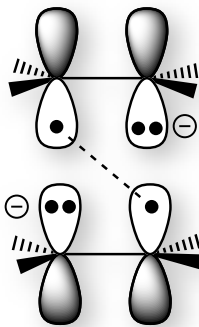
$\frac{-4\beta S}{1-S^2}$

# Qualitative VB analysis

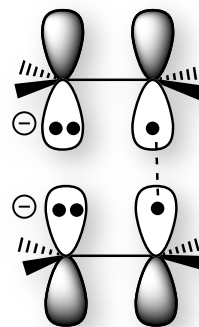
- VB set of structures for  $\text{DTCNE}_2^{2-}$ :



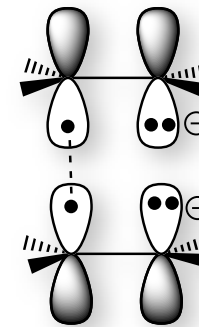
1



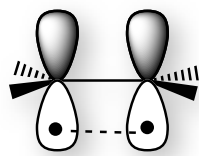
2



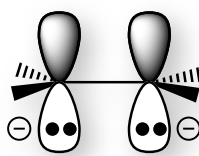
3



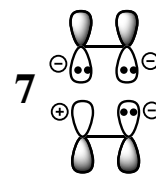
4



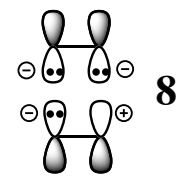
5



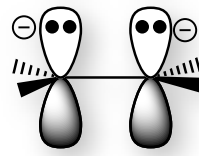
6



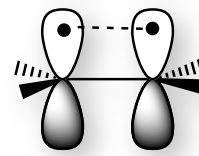
7



8



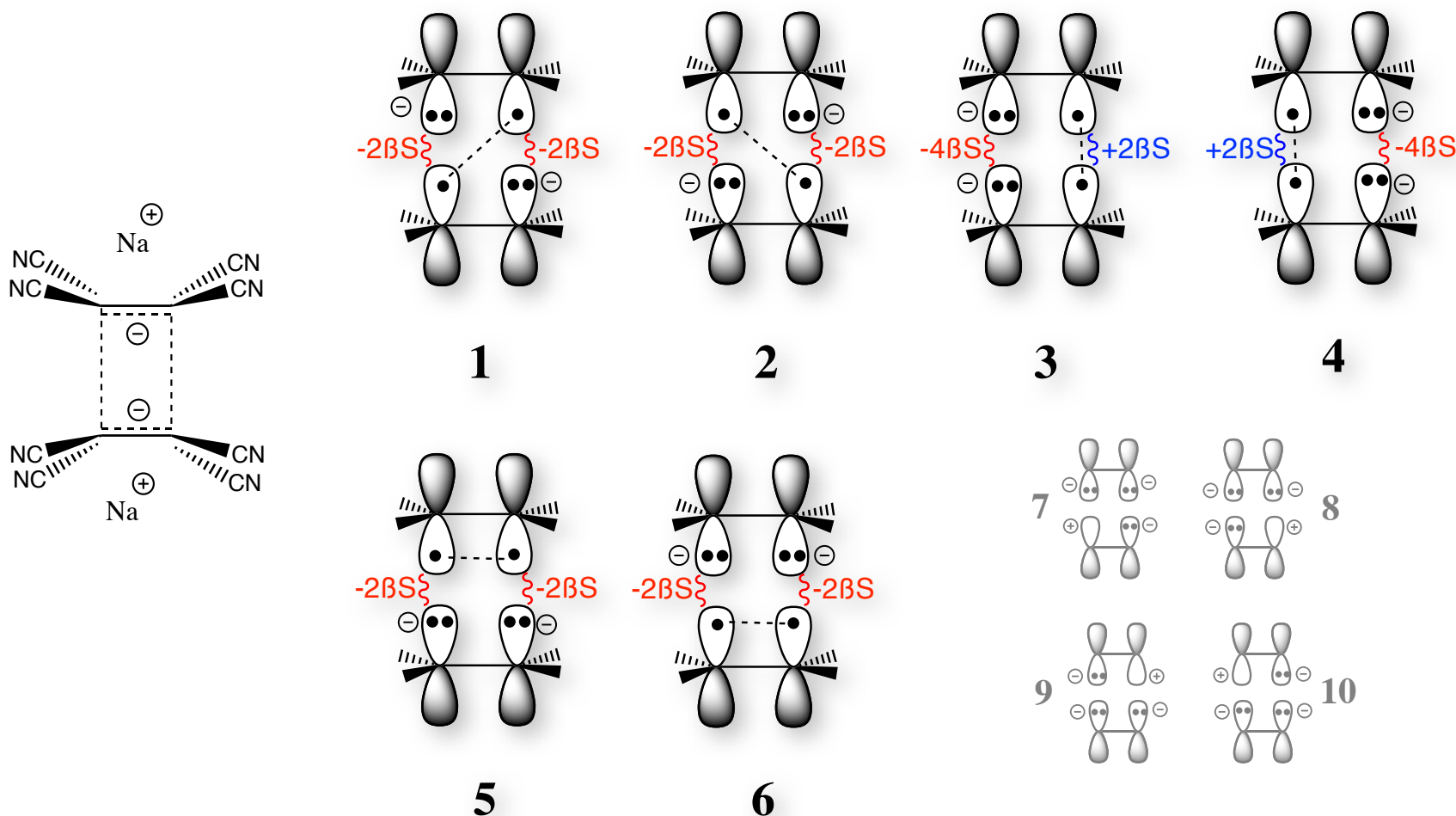
9



10

# Qualitative VB analysis

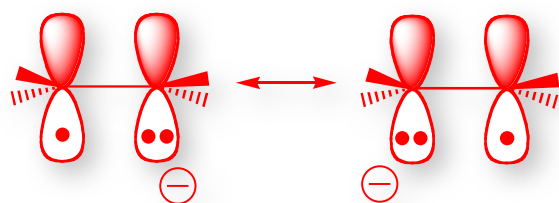
- VB set of structures for  $\text{DTCNE}_2^{2-}$  :



⇒ No structure is bonding by itself, **all the bonding comes from the resonance !**

# Qualitative VB analysis

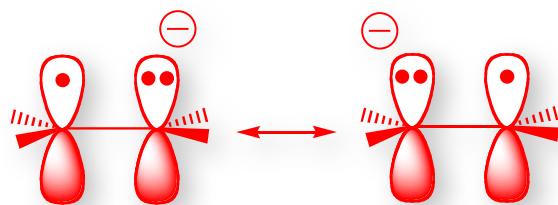
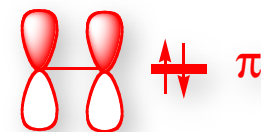
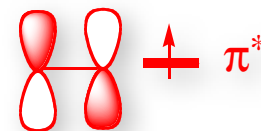
- What about three-electron bonding ?...



Left-right exchange of the negative charge, equivalent to:

At infinite distance, each fragment displays a **3e<sup>-</sup> π bond** :

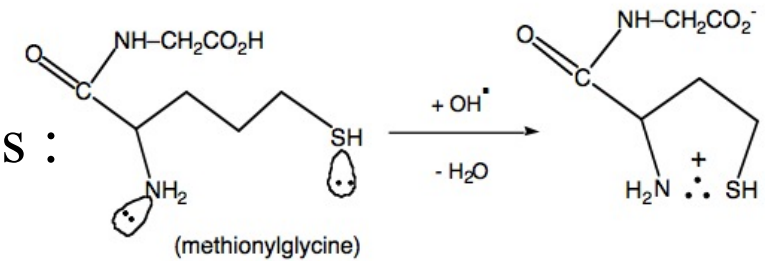
∞



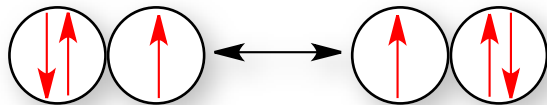
# Qualitative VB analysis

- What is the three-electron bond ?...

Examples :  $\text{He}_2^+$ ,  $\text{RS} \cdot \text{SR}'$ , radical intermediates :

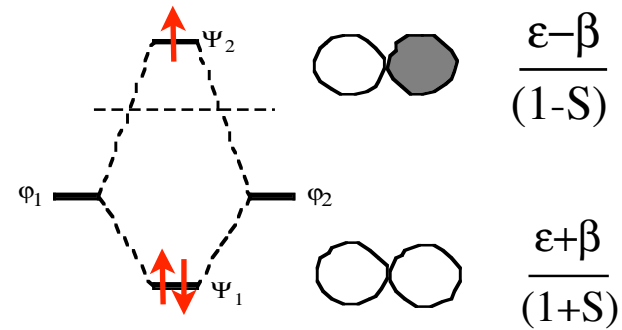


VB description :



$$\Psi_{VB} = |a\bar{a}b| + |b\bar{b}a|$$

MO description :



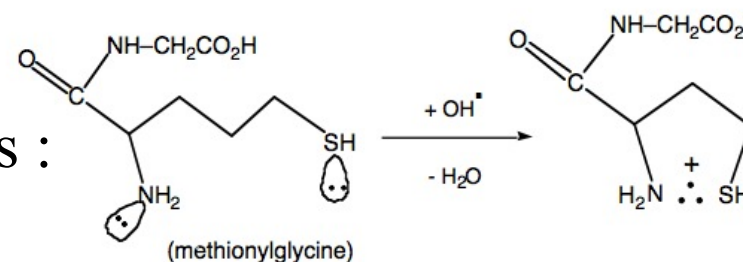
$$\Psi_{MO} = |\sigma\bar{\sigma}\sigma^*|$$



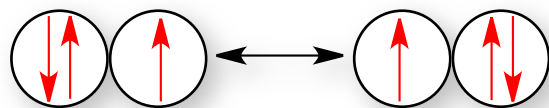
# Qualitative VB analysis

- What is the three-electron bond ?...

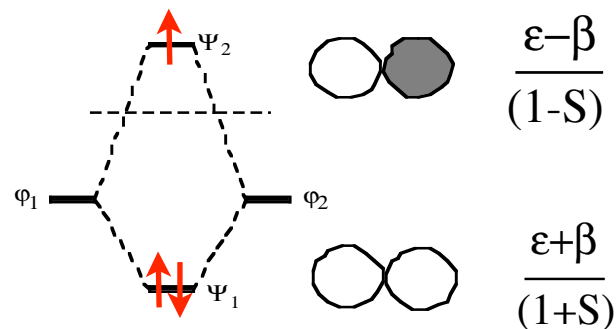
Examples :  $\text{He}_2^+$ ,  $\text{RS} \cdot \text{SR}'$ , radical intermediates :



VB description :



MO description :



$$\Psi_{MO} = |\sigma\bar{\sigma}\sigma^*| = |(a+b)\overline{(a+b)}(a-b)^*| = \dots = -|a\bar{a}b| - |b\bar{b}a| = -\Psi_{VB}$$

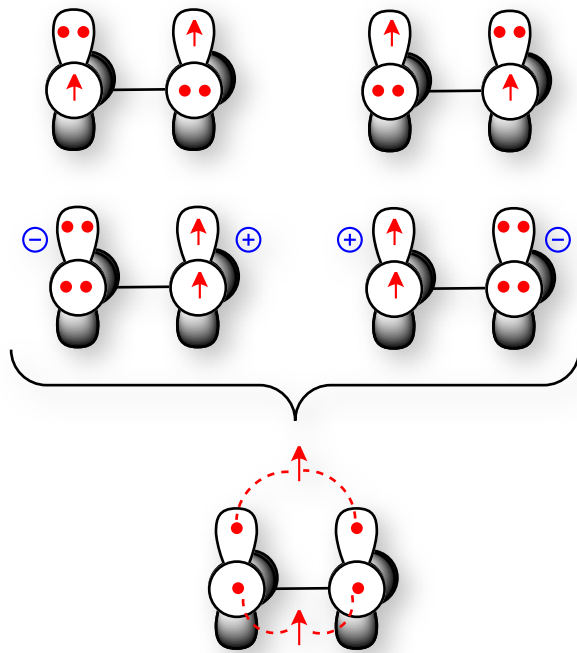
$$\Rightarrow D_e = \frac{\beta(1-3S)}{(1-S^2)} \Rightarrow S_{opt} \approx 0.17$$

# Qualitative VB analysis

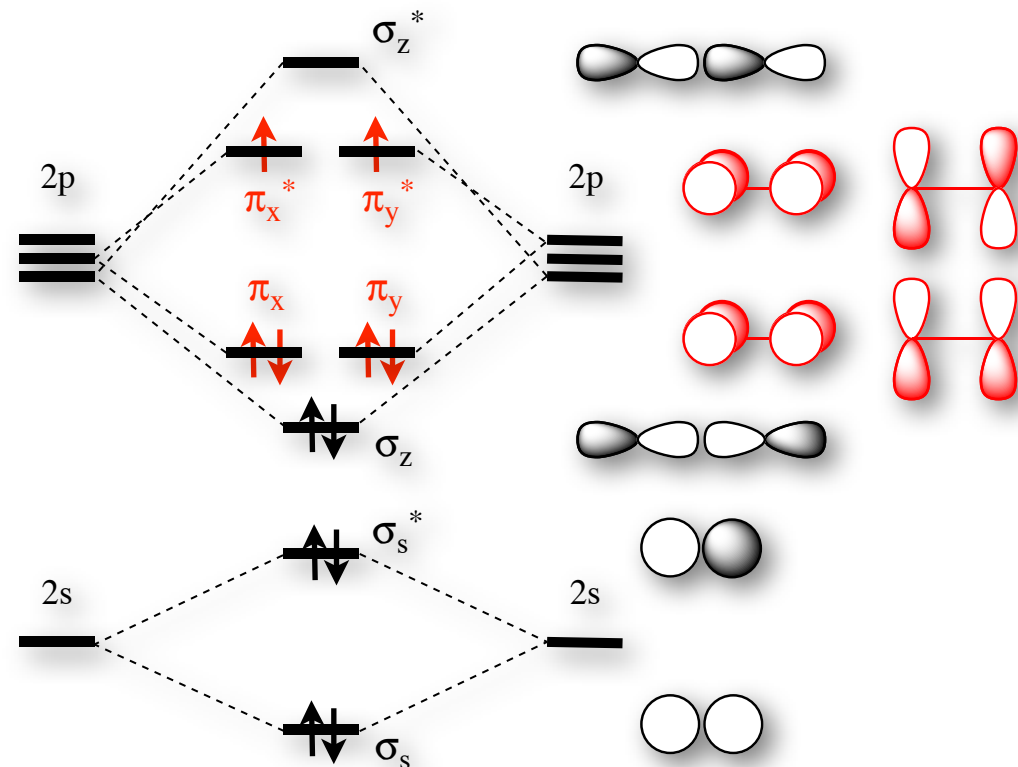
- What is the three-electron bond ?...

The most common 3e bonds : the two  $\pi$  bonds in  $O_2$  ground state :

VB description :

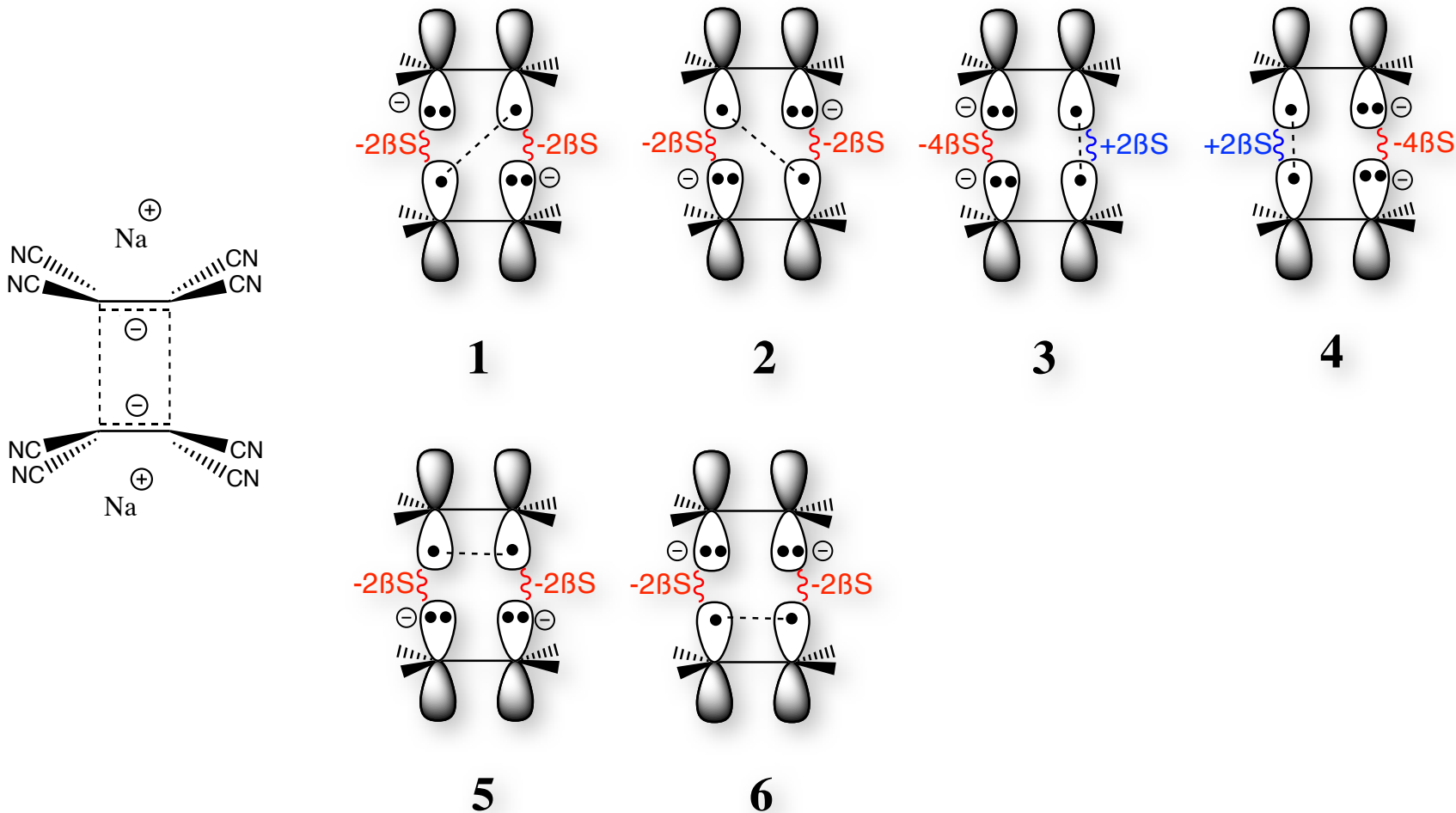


MO description :



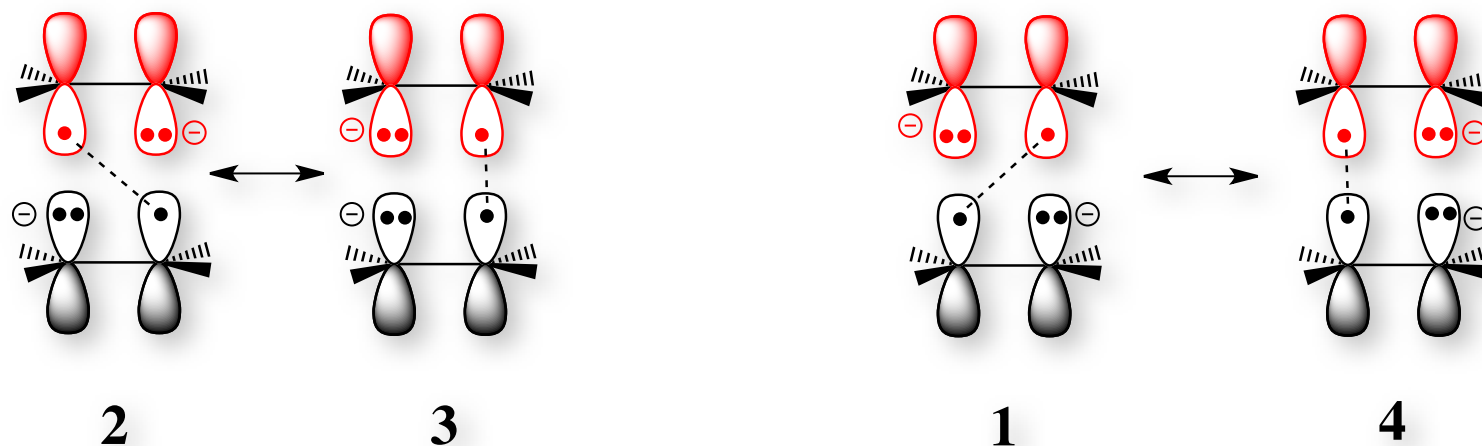
# Qualitative VB analysis

- VB set of structures for  $\text{DTCNE}_2^{2-}$  :



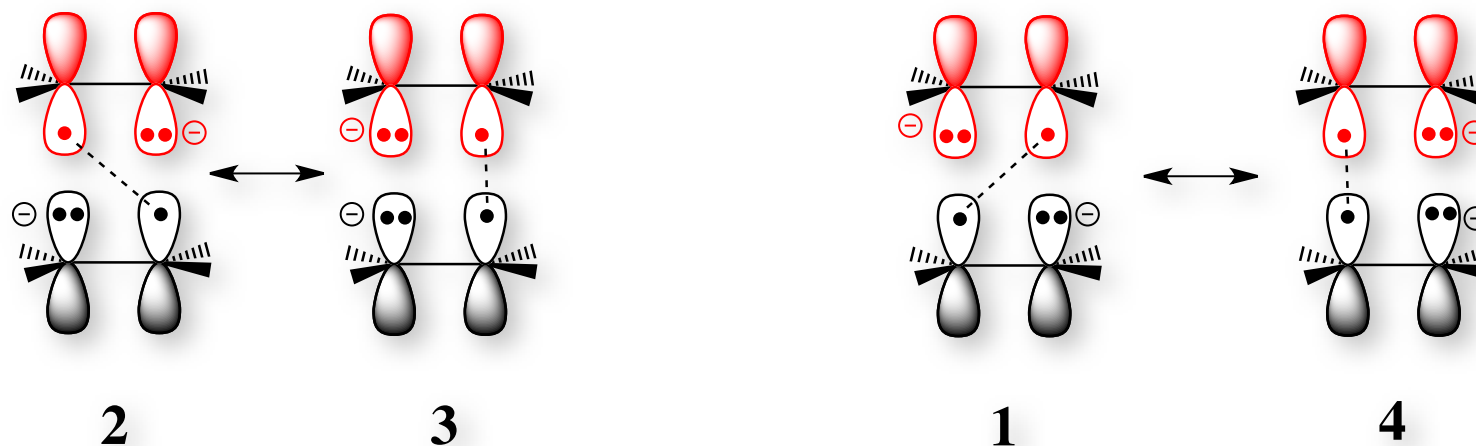
# Qualitative VB analysis

- $2 \leftrightarrow 3$  and  $1 \leftrightarrow 4$  : **intra-fragment  $3e^- \pi$  bond** (upper fragment) :

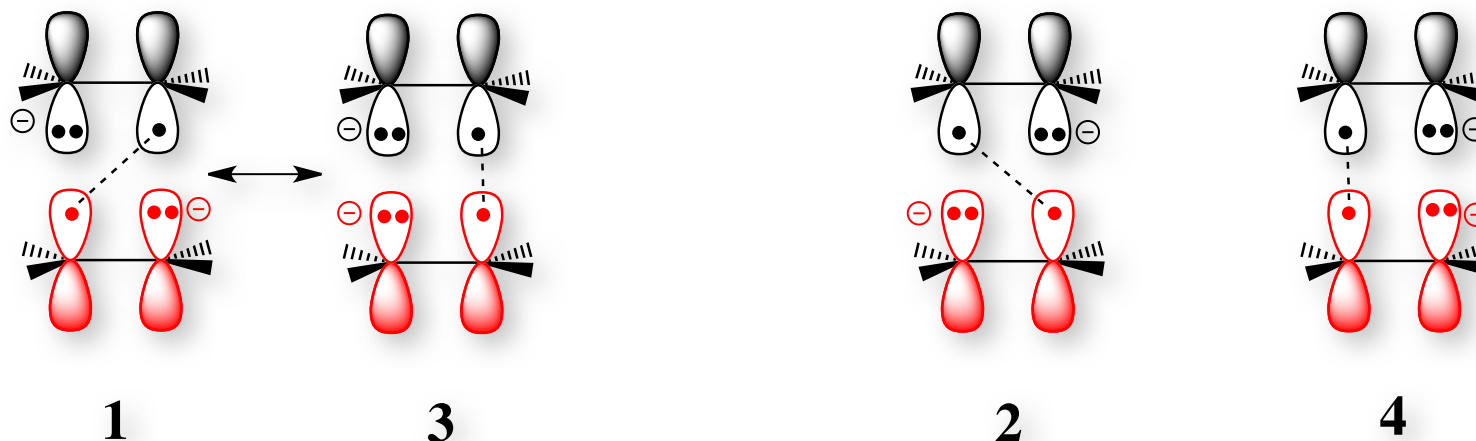


# Qualitative VB analysis

- $2 \leftrightarrow 3$  and  $1 \leftrightarrow 4$  : **intra-fragment  $3e^- \pi$  bond** (upper fragment) :

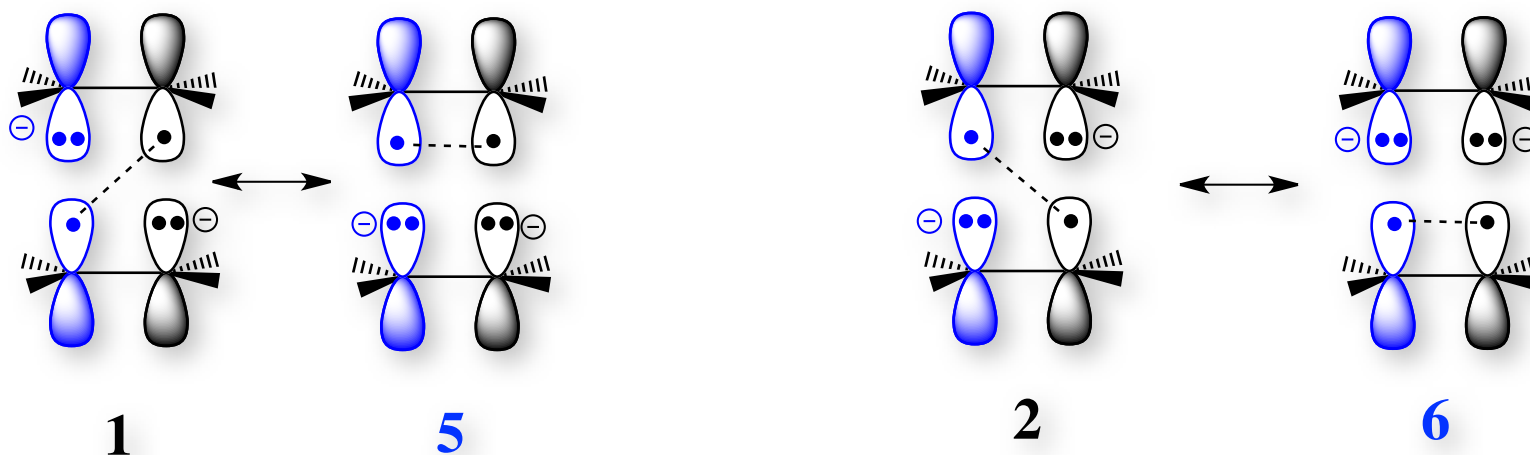


- $1 \leftrightarrow 3$  and  $2 \leftrightarrow 4$  : **intra-fragment  $3e^- \pi$  bond** (lower fragment) :

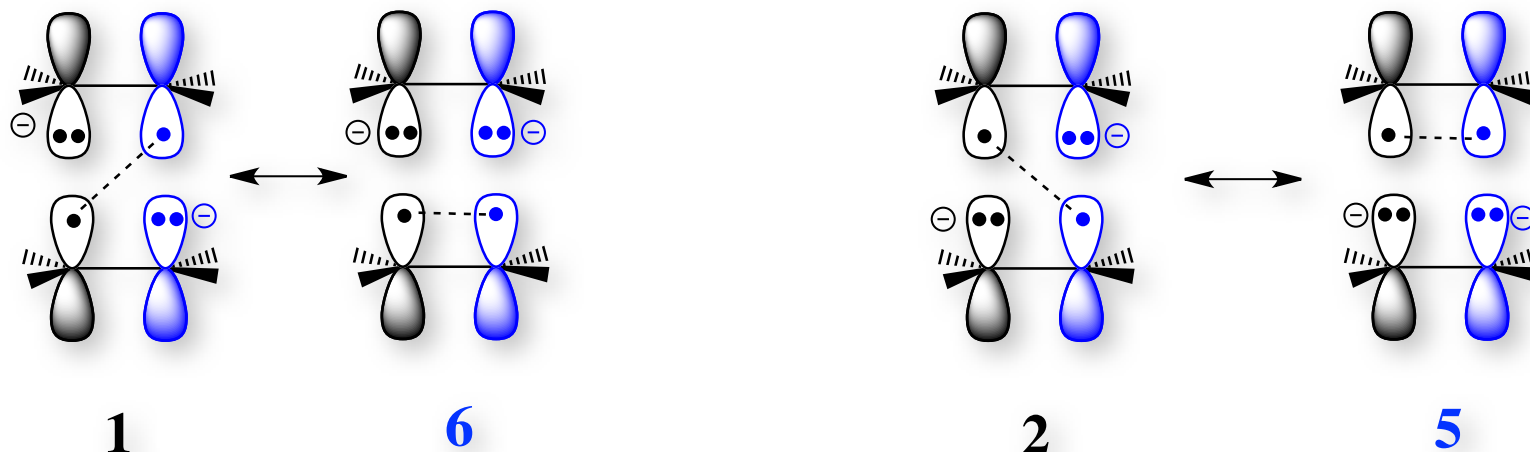


# Qualitative VB analysis

- $1 \leftrightarrow 5$  and  $2 \leftrightarrow 6$  : **inter-fragment  $3e^- \pi$  bond** (left-hand side) :

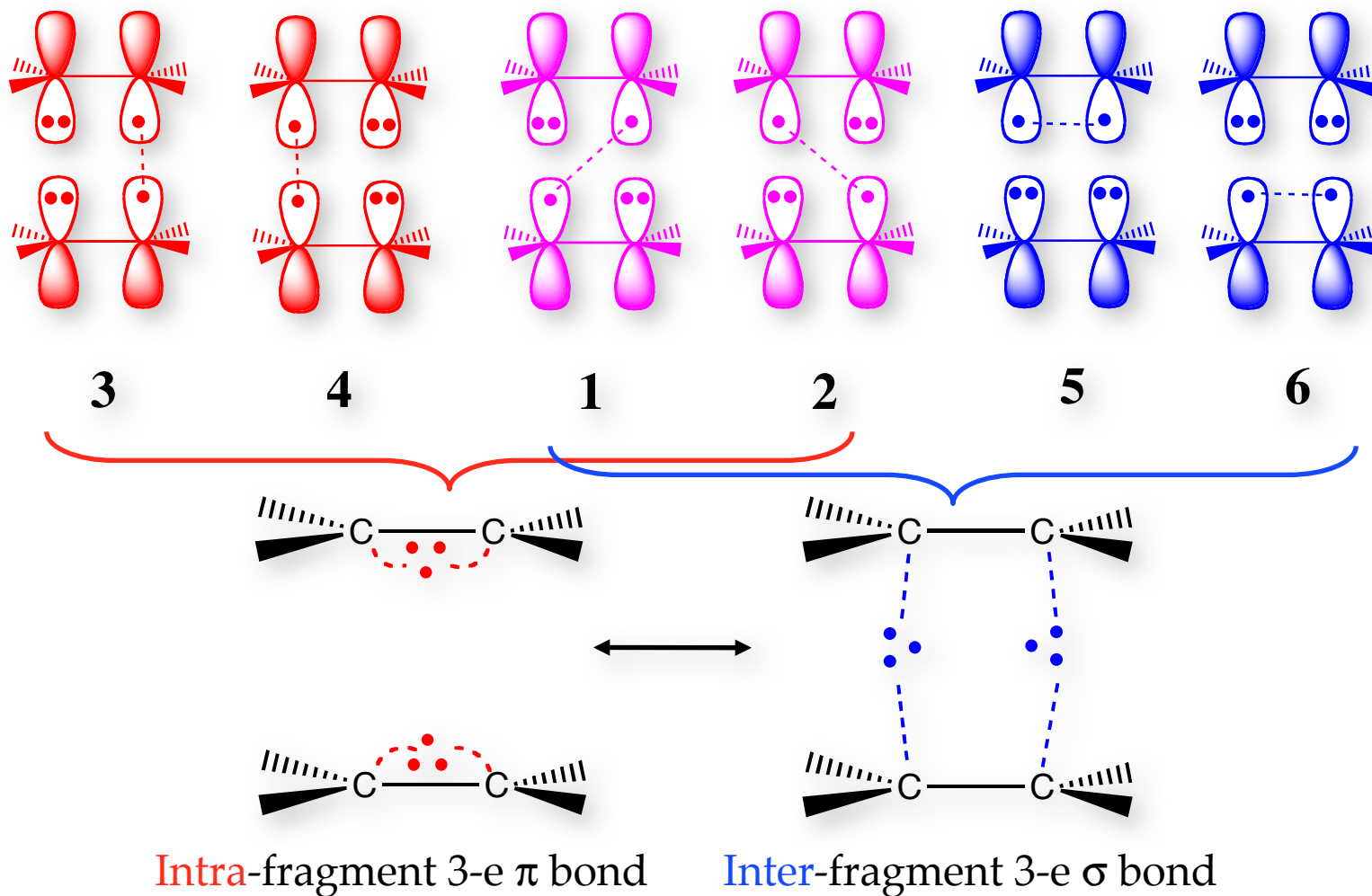


- $2 \leftrightarrow 5$  and  $1 \leftrightarrow 6$  : **inter-fragment  $3e^- \pi$  bond** (right-hand side) :





# Qualitative VB analysis

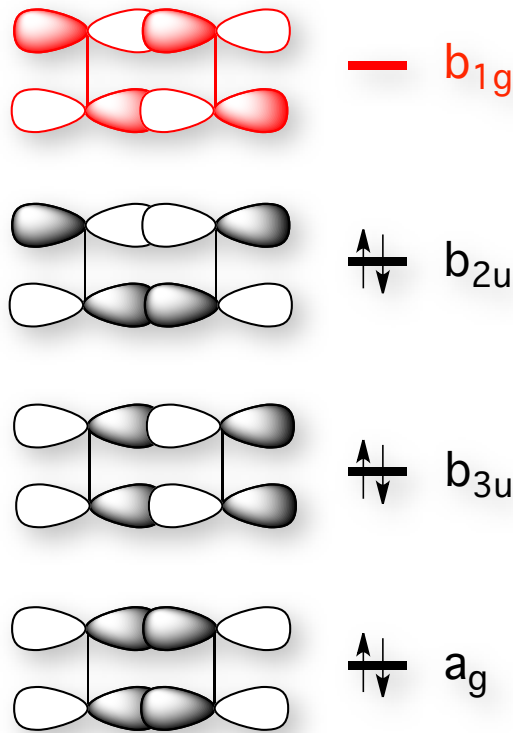


⇒ bonding in DTCNE : two inter-fragment 3e bonds ?

# Qualitative VB analysis

- «VB reading» of MO determinants :

Development of the HF determinant in the VB basis of structures :

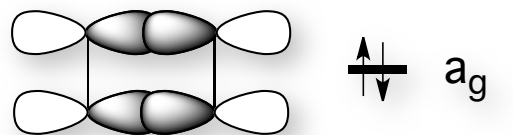
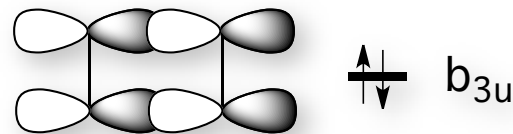
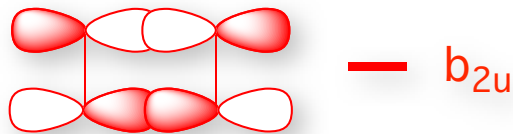
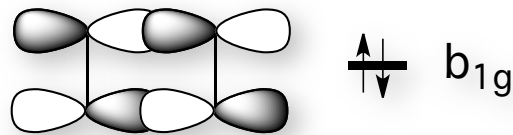


$$\Psi_{HF} = |a_g \bar{a}_g b_{3u} \bar{b}_{3u} b_{2u} \bar{b}_{2u}| = \dots = \ominus \Psi_1^{VB} \ominus \Psi_2^{VB} + \Psi_3^{VB} + \Psi_4^{VB} \ominus \Psi_5^{VB} \ominus \Psi_6^{VB}$$

# Qualitative VB analysis

- «VB reading» of MO determinants :

Development of the 1<sup>st</sup> excited det. in the VB basis of structures :

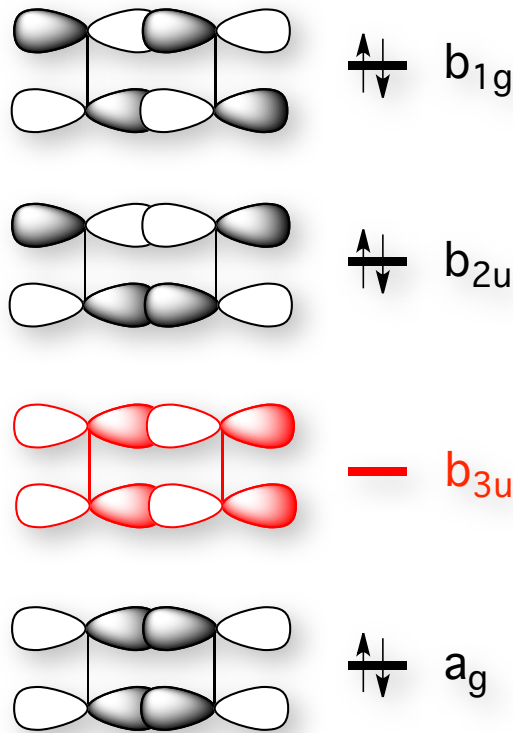


$$\Psi_{1-ext} = |a_g \bar{a}_g b_{3u} \bar{b}_{3u} b_{1g} \bar{b}_{1g}| = \dots = +\Psi_1^{VB} + \Psi_2^{VB} \ominus \Psi_3^{VB} \ominus \Psi_4^{VB} \ominus \Psi_5^{VB} \ominus \Psi_6^{VB}$$

# Qualitative VB analysis

- «VB reading» of MO determinants :

Development of the 2<sup>nd</sup> excited det. in the VB basis of structures :

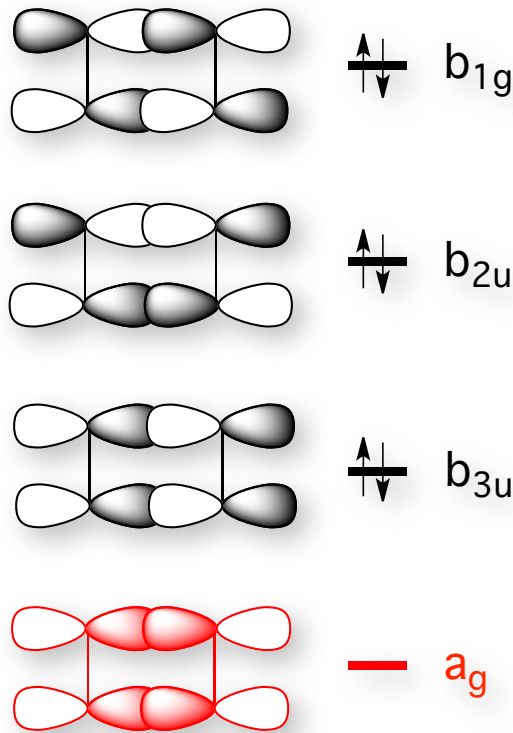


$$\Psi_{2-ext} = |a_g \bar{a}_g b_{2u} \bar{b}_{2u} b_{1g} \bar{b}_{1g}| = \dots = +\Psi_1^{VB} + \Psi_2^{VB} + \Psi_3^{VB} + \Psi_4^{VB} + \Psi_5^{VB} + \Psi_6^{VB}$$

# Qualitative VB analysis

- «VB reading» of MO determinants :

Development of the 3<sup>rd</sup> excited det. in the VB basis of structures :



$$\Psi_{3-ext} = |b_{3u} \bar{b}_{3u} b_{2u} \bar{b}_{2u} b_{1g} \bar{b}_{1g}| = \dots = \ominus \Psi_1^{VB} \ominus \Psi_2^{VB} \ominus \Psi_3^{VB} \ominus \Psi_4^{VB} + \Psi_5^{VB} + \Psi_6^{VB}$$



# Qualitative VB analysis

- «VB reading» of MO determinants :

CAS(4,4)  $\Leftrightarrow$  6 VB structures mixing :

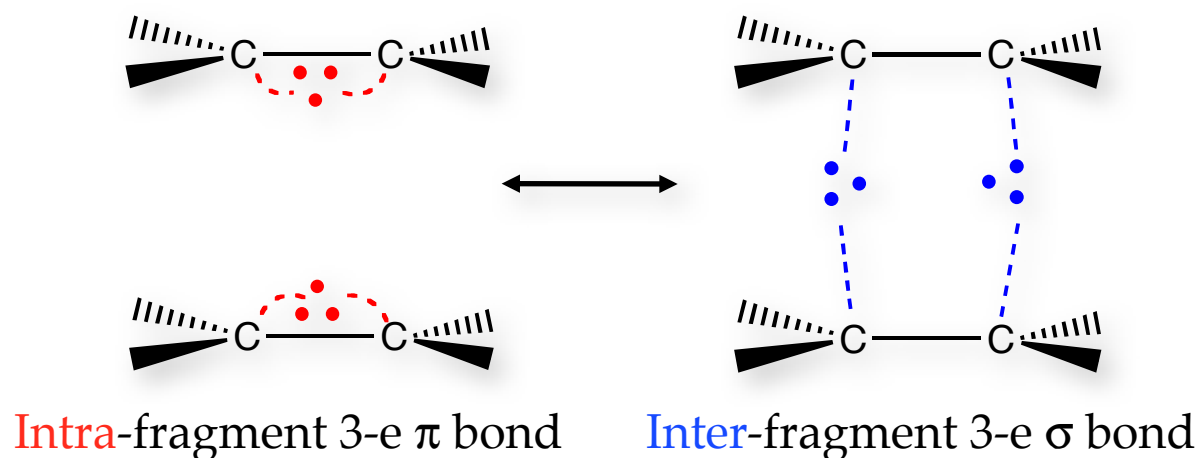
$$\left\{ \begin{array}{l} \Psi_{HF} = |a_g \bar{a}_g b_{3u} \bar{b}_{3u} b_{2u} \bar{b}_{2u}| = \dots = +\Psi_1^{VB} + \Psi_2^{VB} - \Psi_3^{VB} - \Psi_4^{VB} - \Psi_5^{VB} - \Psi_6^{VB} \\ \Psi_{1-ext} = |a_g \bar{a}_g b_{3u} \bar{b}_{3u} b_{1g} \bar{b}_{1g}| = \dots = +\Psi_1^{VB} + \Psi_2^{VB} - \Psi_3^{VB} - \Psi_4^{VB} + \Psi_5^{VB} + \Psi_6^{VB} \\ \Psi_{2-ext} = |a_g \bar{a}_g b_{2u} \bar{b}_{2u} b_{1g} \bar{b}_{1g}| = \dots = +\Psi_1^{VB} + \Psi_2^{VB} + \Psi_3^{VB} + \Psi_4^{VB} - \Psi_5^{VB} - \Psi_6^{VB} \\ \Psi_{3-ext} = |b_{3u} \bar{b}_{3u} b_{2u} \bar{b}_{2u} b_{1g} \bar{b}_{1g}| = \dots = +\Psi_1^{VB} + \Psi_2^{VB} + \Psi_3^{VB} + \Psi_4^{VB} + \Psi_5^{VB} + \Psi_6^{VB} \end{array} \right.$$

MO (4x4) description = VB description, but...

**the VB analysis which reveals the 3e-bond nature**

# Qualitative VB analysis

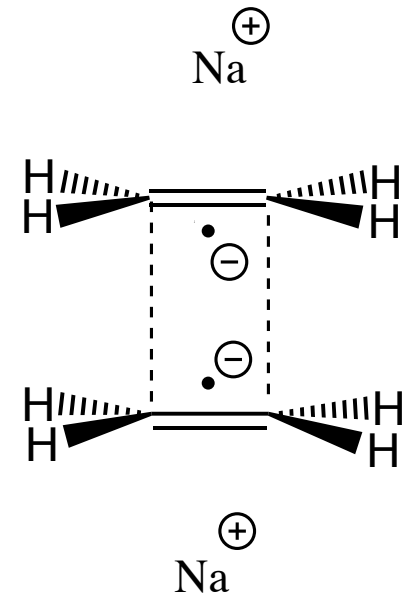
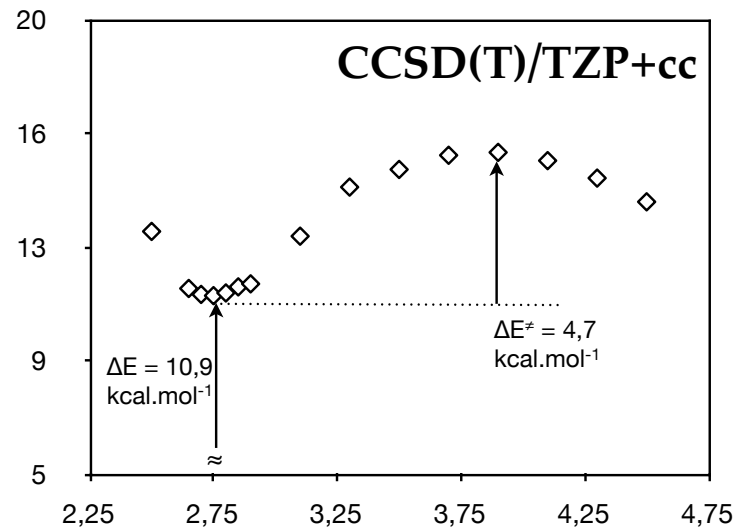
- Conclusion of the qualitative VB analysis :



Bonding in DTCNE :  
two inter-fragment 3e bonds ?...

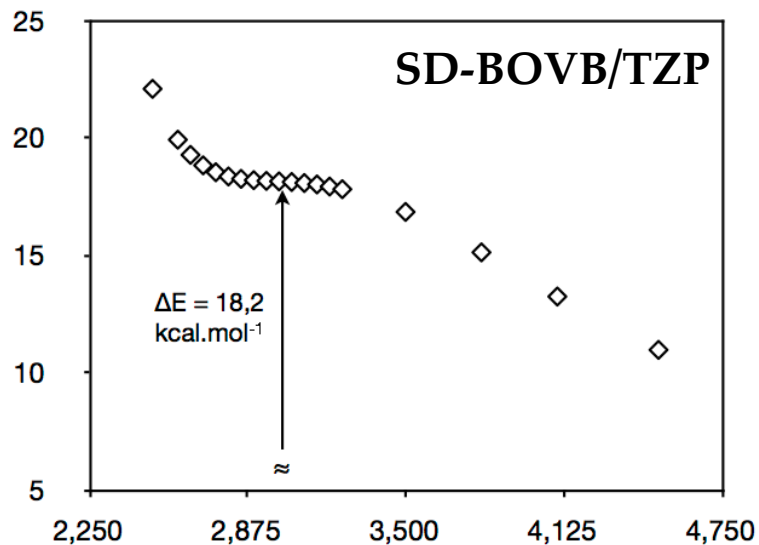
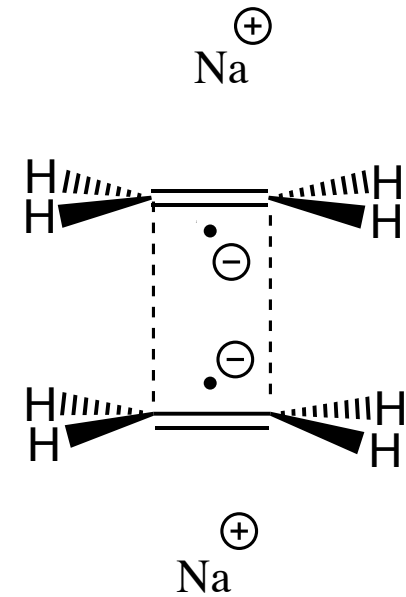
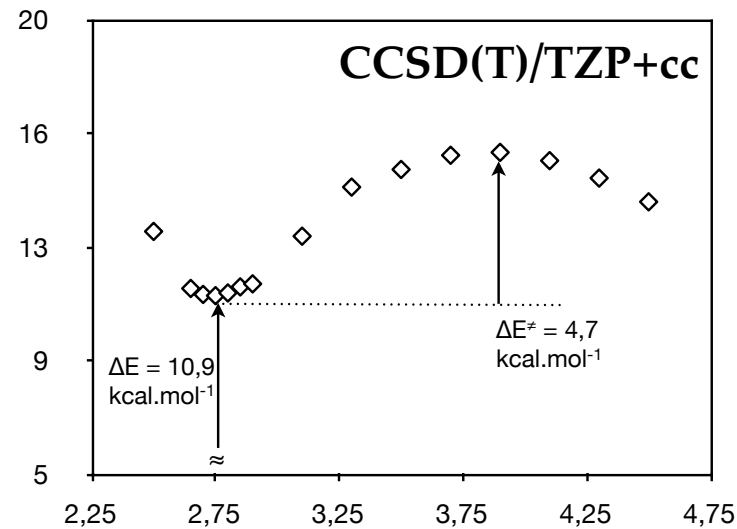
# Quantitative calculations

- Diethylene dianion system test :



# Quantitative calculations

- Diethylene dianion system test :

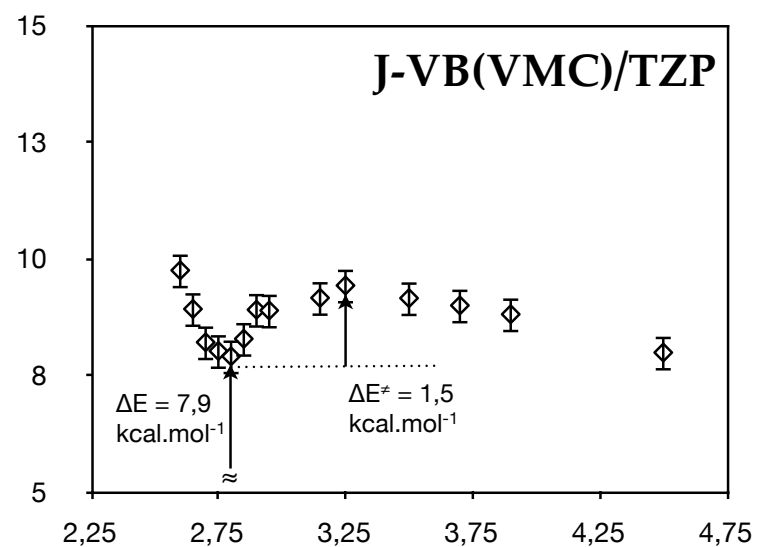
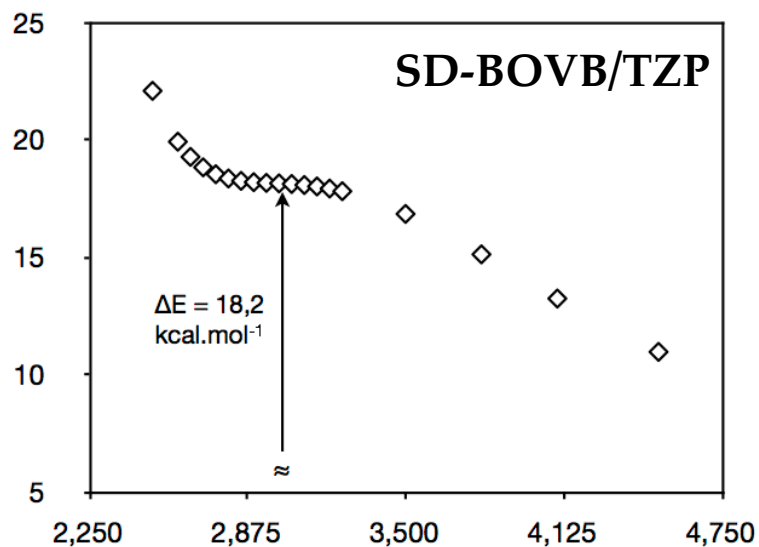
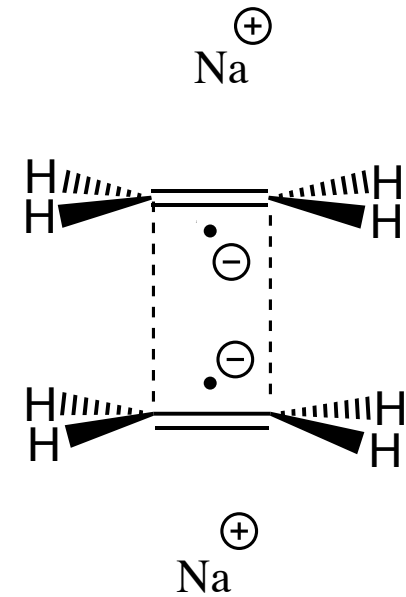
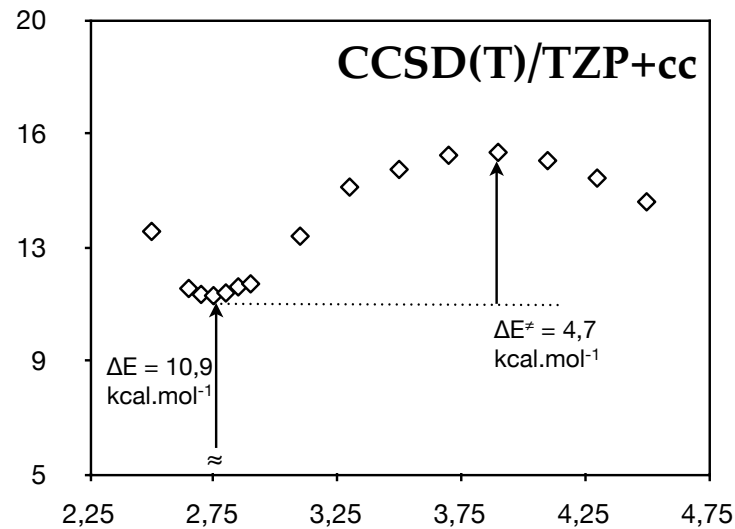


SD-BOVB disappointing :

- Not accurate enough (fails to find a minimum) ;
- Too long : ~20 hours to converge (4 heavy atoms, 26 electrons)

# Quantitative calculations

- Diethylene dianion system test :

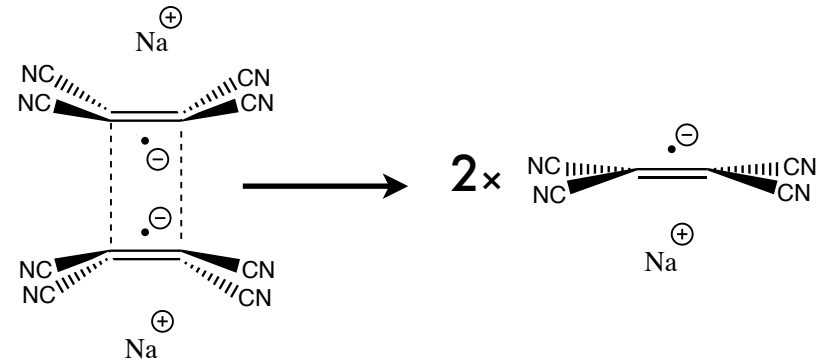




# Quantitative calculations

- DTCNE<sub>2</sub><sup>2-</sup> dissociation energy :

	De (kal/mol)	Re (Å)
CCSD(T)/VTZ+cc	<b>11.6</b>	<b>2.56</b>
GVB+PT2/VTZ+cc	<b>11.2</b>	<b>2.7</b>
VBSCF	<b>-24.5</b>	
J-VB(VMC)/VTZ★	<b>5.2(9)</b>	
J-VB(DMC)/VTZ★	<b>9.7(9)</b>	



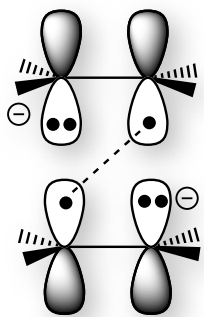
← No dynamic correlation

- VB/QMC quantitatively reliable (even when dynamical correlation is strong)
- VB/QMC can treat large systems (22 heavy atom / 90 electrons here)

★ *CHAMP program by C. J. UMRIGAR, C. FILIPPI and J. TOULOUSE*

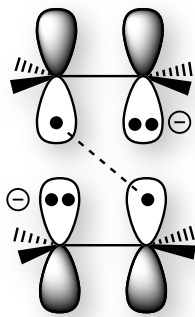
# VB/QMC calculations

- DTCNE<sub>2</sub><sup>2-</sup> : computed weights (J-VB) :



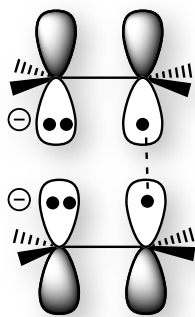
1

20.2%



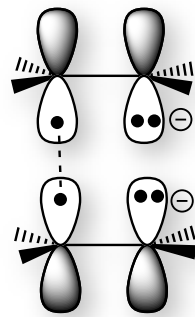
2

20.2%



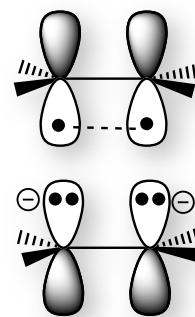
3

16.1%



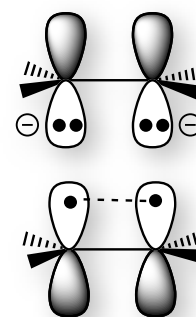
4

16.1%



5

10.1%

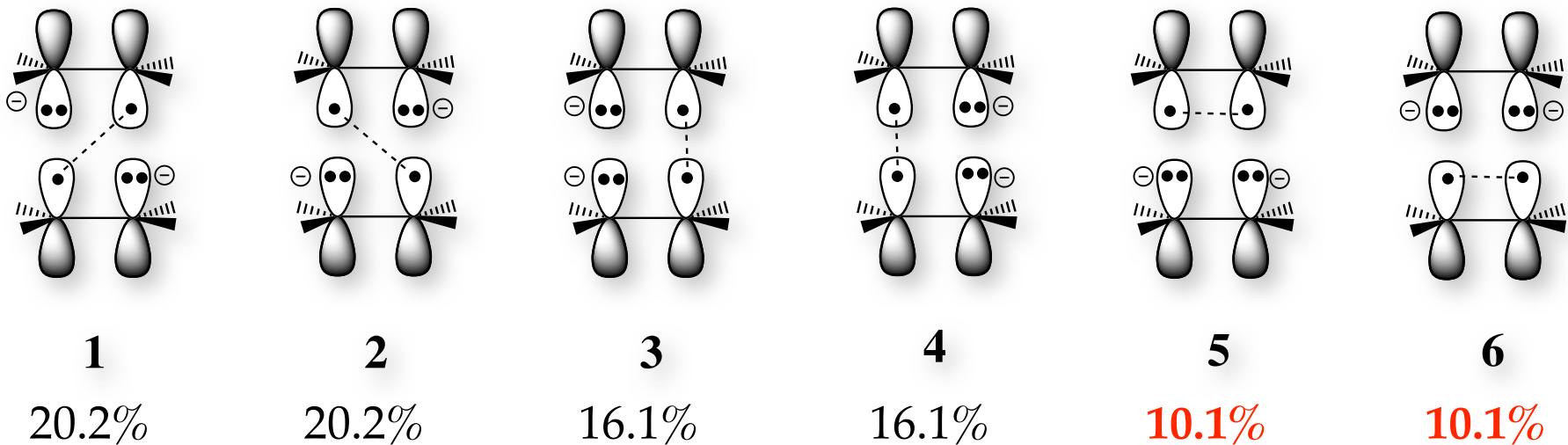


6

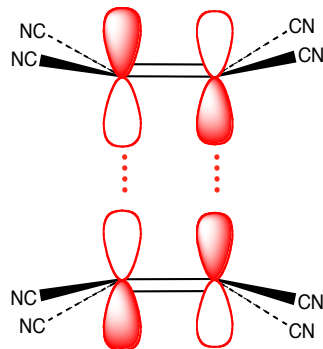
10.1%

# VB/QMC calculations

- DTCNE<sub>2</sub><sup>2-</sup> : computed weights (J-VB) :



- Interfragment 2e bond ?

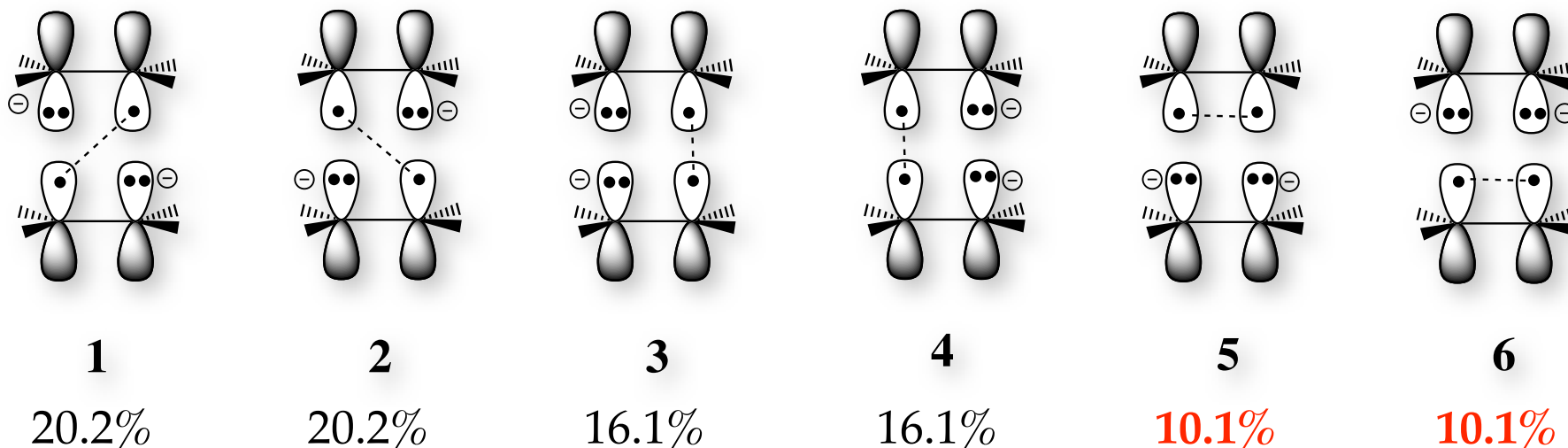


MO/VB mapping :

Purely covalent  $\pi^*-\pi^*$  2-e bond =  
**1 + 2 + 3 + 4** (equal weights)

# VB/QMC calculations

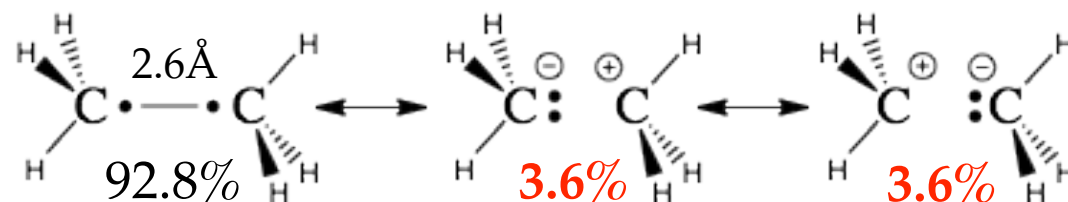
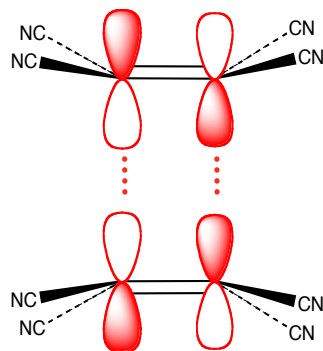
- DTCNE<sub>2</sub><sup>2-</sup> : computed weights (J-VB) :



- Interfragment 2e bond ? (covalent +  $\epsilon$  ionic)  $\pi$ - $\pi^*$  2-e bond =

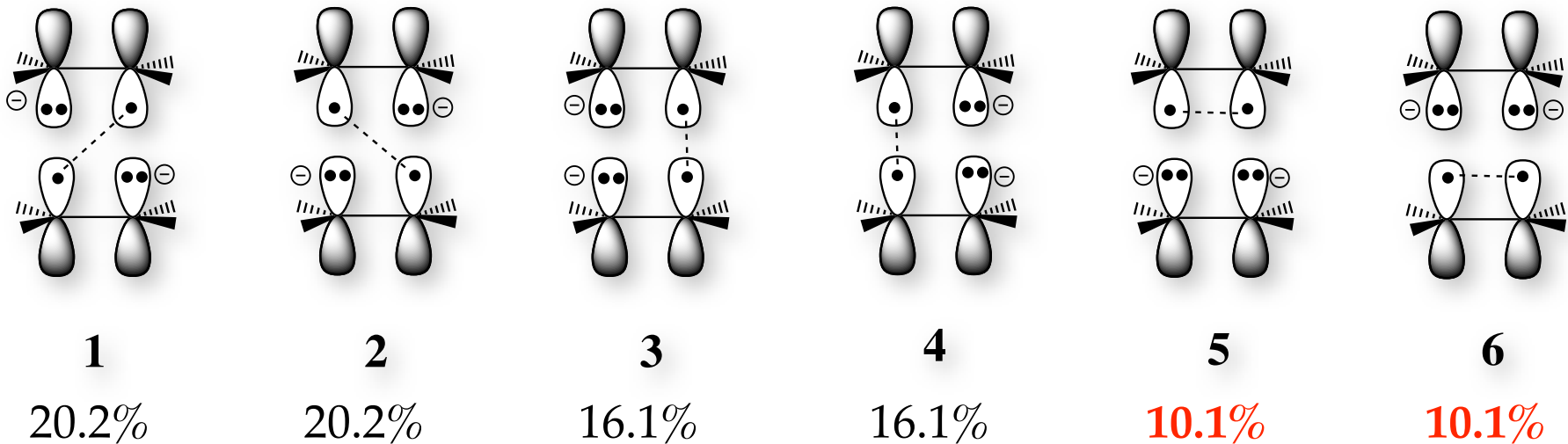
$$1 + 2 + 3 + 4 + \epsilon (5 + 6)$$

However,  $\epsilon = 3.6\%$  in a stretched C-C bond:



# VB/QMC calculations

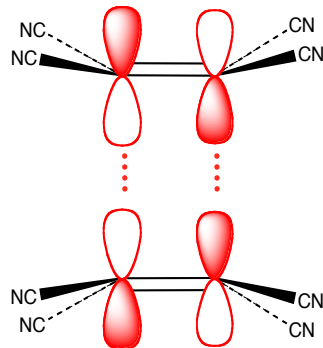
- DTCNE<sub>2</sub><sup>2-</sup> : computed weights (J-VB) :



- Interfragment 2e bond ? (covalent +  $\epsilon$  ionic)  $\pi$ - $\pi^*$  2-e bond =

$$1 + 2 + 3 + 4 + \epsilon (5 + 6)$$

However,  $\epsilon = 3.6\%$  in a stretched C-C bond:

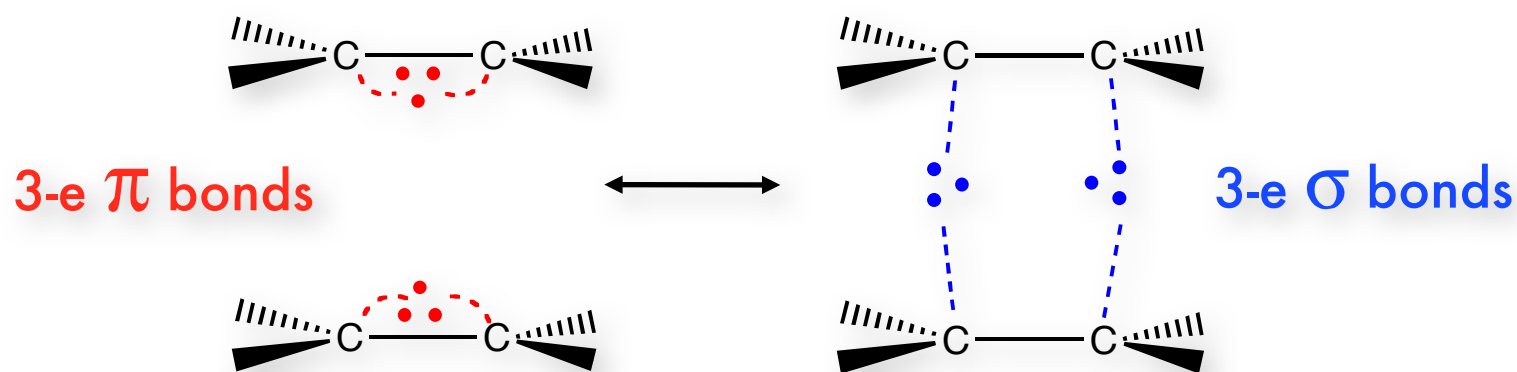


$\Rightarrow$  The inter-fragment bond cannot be a simple  $\pi$ - $\pi^*$  2-e bond



# VB/QMC calculations

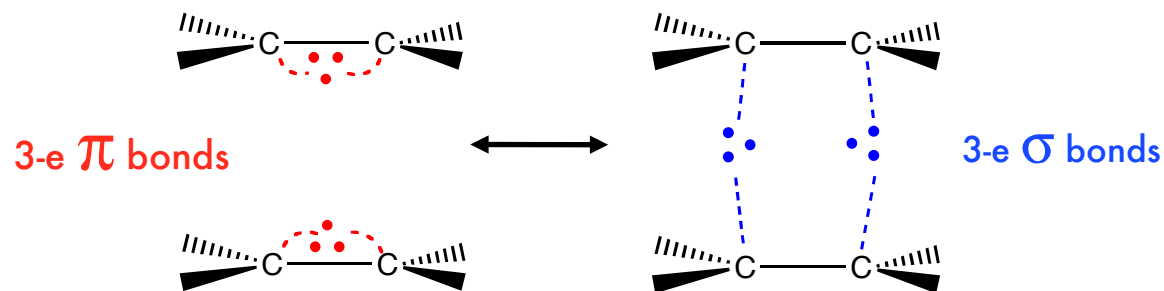
- Some tests of credibility :



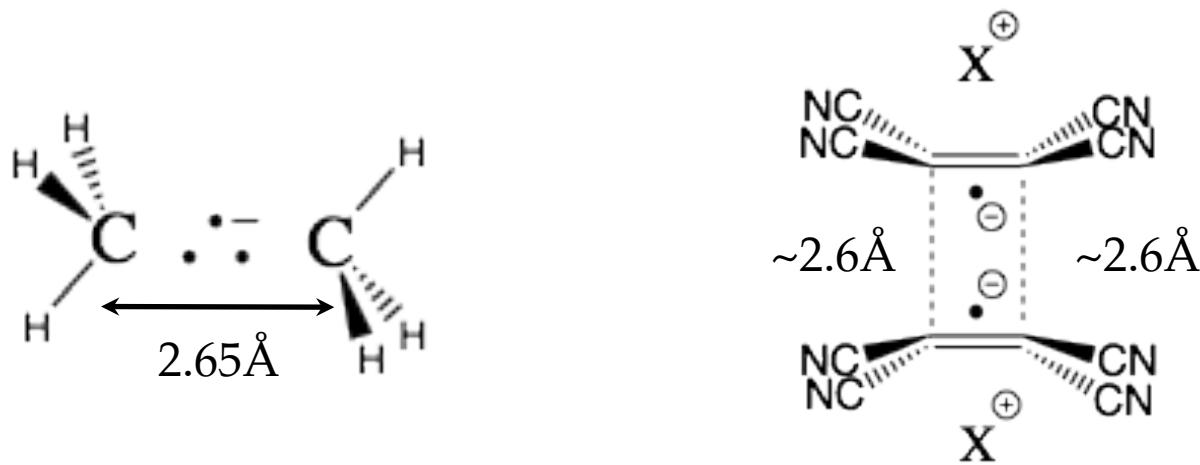
Characteristics of 3e bonds :

- 1) Large equilibrium distances :  $d(\text{S—S}) \approx 2.0 \text{ \AA}$  ;  $d(\text{S} \cdot\cdot\text{S}^-) \approx 2.8 \text{ \AA}$
- 2) Small overlap ( $S_{opt} \approx 0.17$ )
- 3) Importance of dynamical correlation

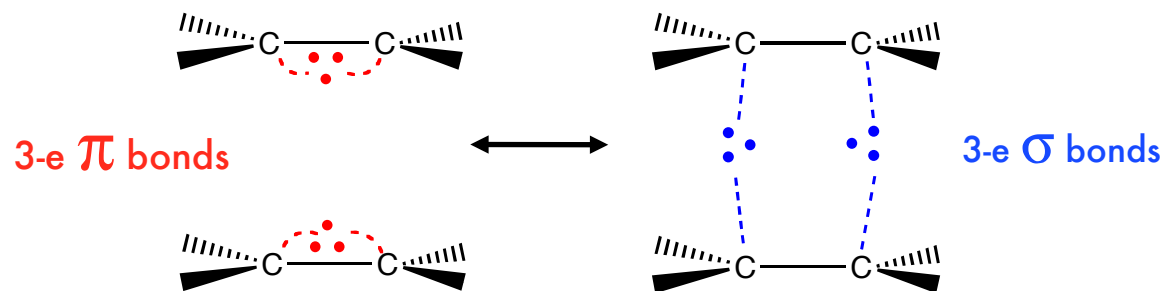
# VB/QMC calculations



1) Interfragment bond length close to  $3e^-$  bonded ethane anion :



# VB/QMC calculations



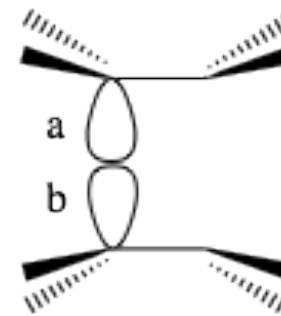
2) Interfragment orbital overlaps close to optimal 3e<sup>-</sup> bond value :

$$S_{opt} \approx 0.17$$

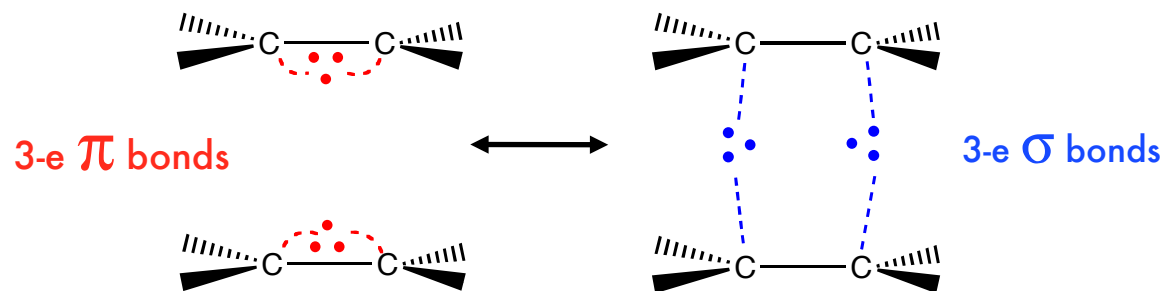
for any 3-e bond  
(demonstrated in  
qualitative VB theory  
as well as Extended Hückel theory)

$$S_{ab} = 0.15$$

(computed)



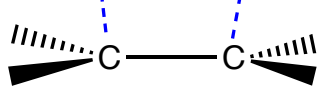
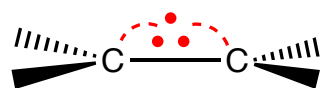
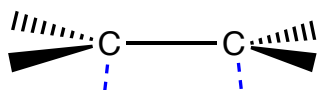
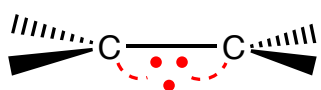
# VB/QMC calculations



### 3) Contribution of dynamic correlation to bonding:

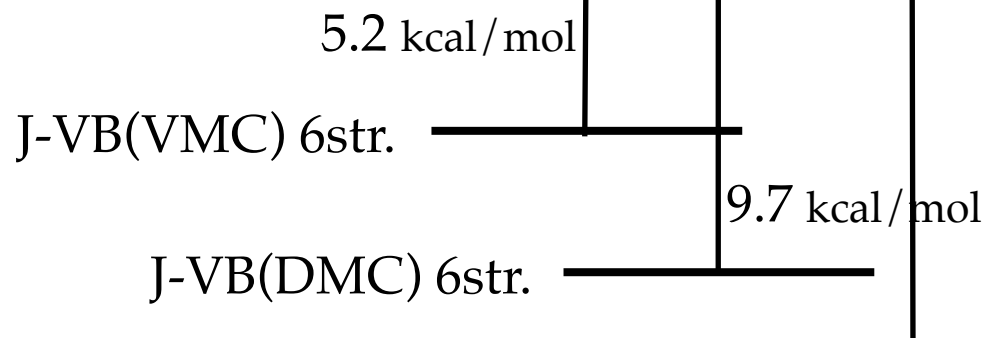
- In  $[\text{F} \cdot \cdot \text{F}]^-$ , dynamic correlation contributes  $\sim 30$  kcal/mol to bonding
- In  $[\text{TCNE}]_2^{2-}$ , dynamic correlation contributes  $> 30$  kcal/mol to bonding

# VB/QMC calculations



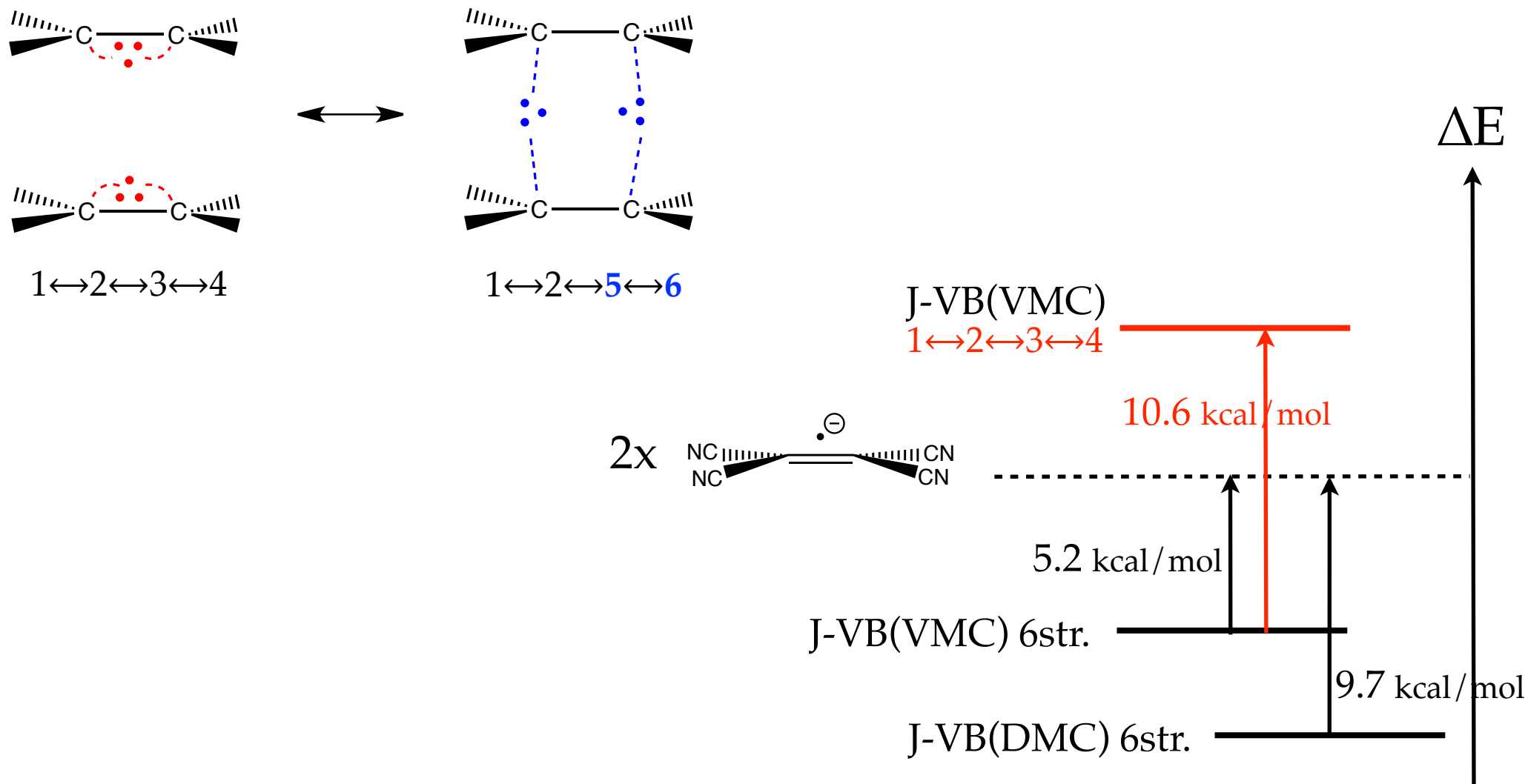
1 ↔ 2 ↔ 3 ↔ 4

1 ↔ 2 ↔ 5 ↔ 6



$\Delta E(\llcorner\text{reference}\llcorner) \approx 11. \text{ kcal/mol}$

# VB/QMC calculations

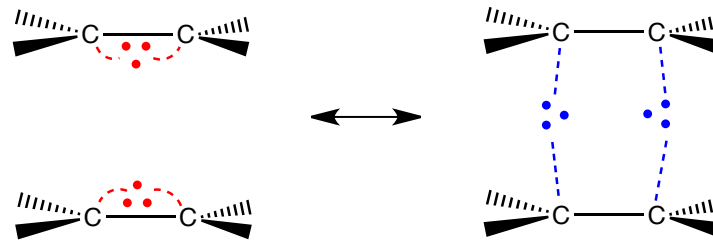


⇒ Without str. 5-6, DTCNE becomes repulsive !



# Conclusion

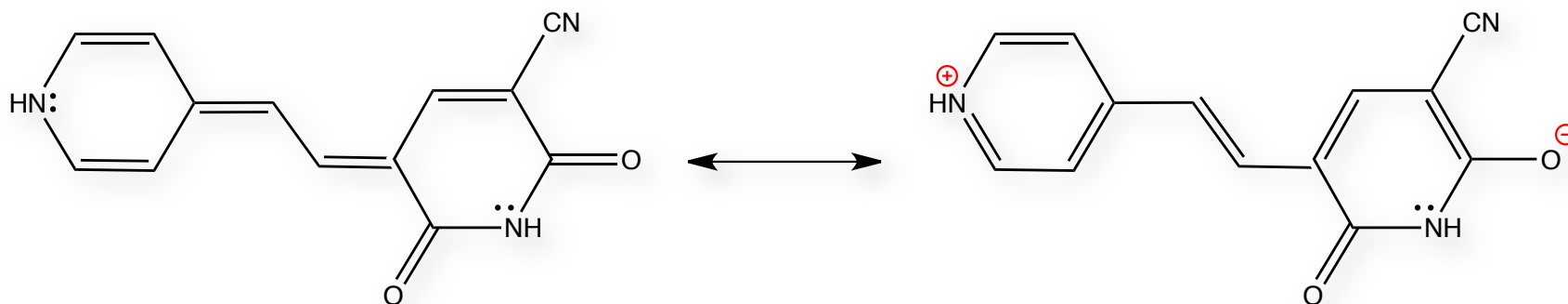
- About  $[\text{TCNE}]_2^{2-}$  :



- A simple VB wave function in terms of only 6 VB structures accurately describes the electronic structure of  $[\text{TCNE}]_2^{2-}$
- Even when electrostatic interaction is repulsive (axial conformation), the two fragment anions are directly bonded by **a pair of 3-e bonds**
- This simple picture explains at once :
  - 1) the unusual bond length,
  - 2) the interfragment overlap,
  - 3) the importance of dynamic correlation

# Conclusion

- About VB-QMC:



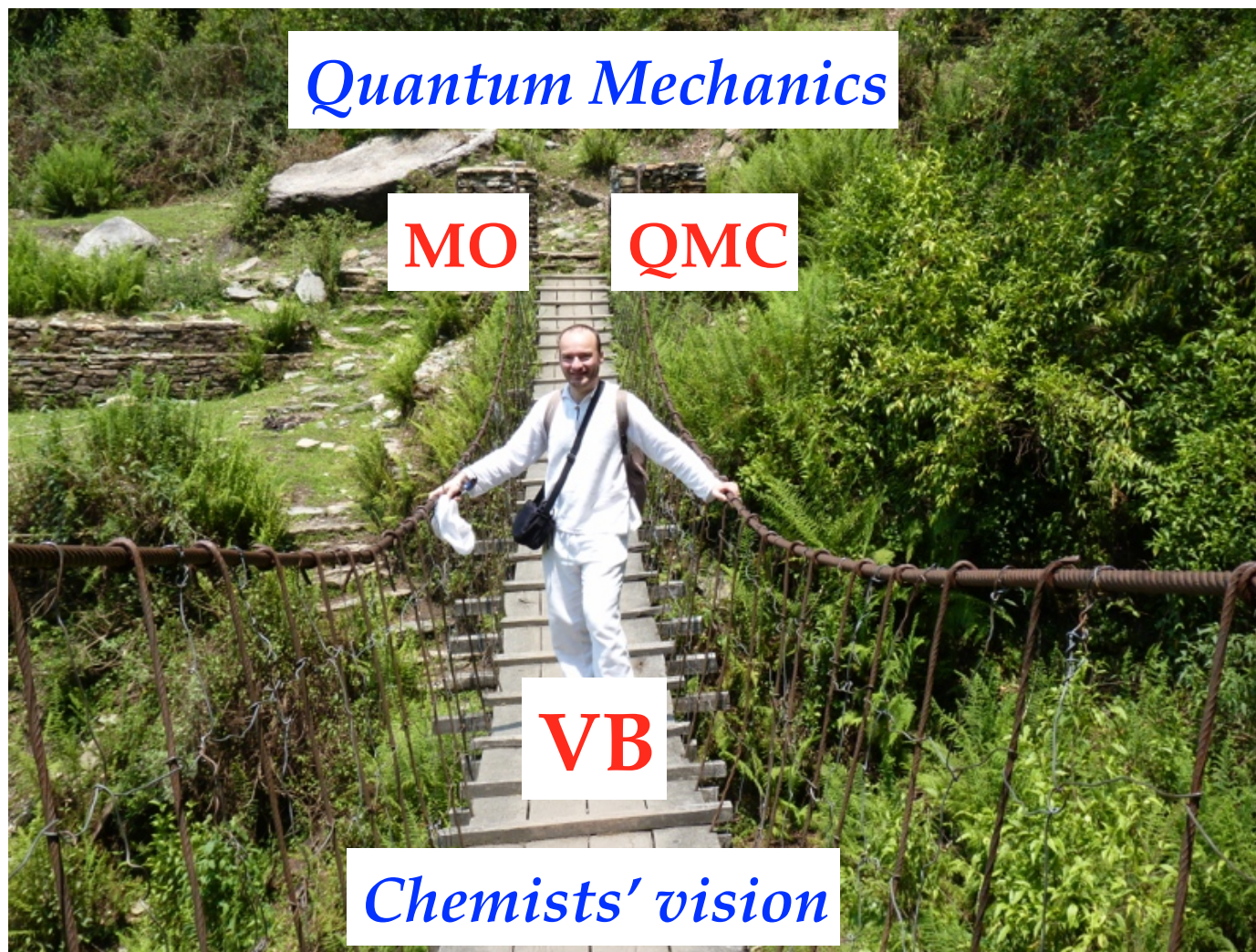
- up to : ~30 heavy atoms
- up to : ~100 valence electrons
- accuracy matching CASPT2 or CCSD(T) on large basis sets

It does more than answering Coulson's request:

*Give me insight and numbers ! (Coulson)*

# Conclusion

But all this is finally about building bridges...



# Conclusion

... and to look at things from different perspectives !





# Conclusion

... and to look at things from different perspectives !

(Prof. Keating, Dead Poets Society)



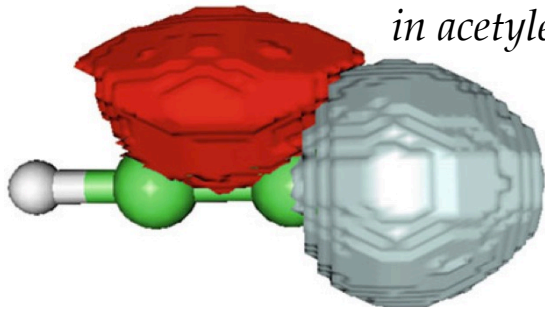
Thanks :  
Philippe HIBERTY (Orsay)  
Kévin HENDRICKS



# Opening post-doc positions !

- 1) Maximum Probability Domains :

*Banana C-C bonds and C-H bonds  
in acetylene*



- With : A. SAVIN
- Col. : Oviedo (Pendas), Napoli (Causa),  
Gent (Bultinck), Aachen (Lüchow)

➔ Optimisation algorithms (with CERMIS and lab. J. J. Lions),  
implementation of new ideas (multi-domains opt., softness,...)

- 2) VB-QMC :

- With : P. HIBERTY, J. TOULOUSE
- Col. : Jerusalem (Shaik), Xiamen (Wu),  
Cornell (Umrigar), Aachen (Lüchow)

➔ New Jastrows, excited states, w.f. optimization, applications