# On Low Energy Excited Spectra of Super and Hyper-deformed Shape Isomers in the Preactinide Region of Nuclei

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Abstract

Using the macroscopic-microscopic model with the macroscopic energy determined by the Lublin-Strasbourg-Drop and the microscopic, shell plus pairing, energy corrections evaluated using the Yukawa-folded mean-field potential, a certain number of yet unknown super and hyper-deformed shape isomers in even-even Pt, Hg, Pb, Po, Rn, and Ra isotopes are predicted. Quadrupole moments in the local minima of the potential-energy surfaces were evaluated and turned out to be in good agreement with the available experimental data in the ground states.

## Introduction

Very broad fission barriers that are found in the Platinum - Plutonium region of nuclei [1, 2], indicate that the inclusion of shell effects will give a good chance to produce local minima in the potential energy surface of these nuclei corresponding to super- and hyper-deformed shapes. Extended calculations for long isotopic chains of even-even nuclei in that region have therefore been performed within the macroscopic-microscopic model. The Lublin-Strasbourg Drop (LSD) formula [1] has been used for the macroscopic part of the potential energy surface, microscopic effects were evaluated with the Yukawa-folded single-particle potential [3] by the Strutinsky shell correction method [4] and the BCS [5] theory for pairing correlations. New, Fourier expansion technique [6] is used to describe nuclear deformations. Several shape isomers could be identified in our calculations. There is a hope that some of them can produced experimentally using e.g. proton beam which allows to produce compound systems with not too large angular momentum. Quadrupole moments of the most pronounced shape isomeric minima were evaluated.

## Model

In order to describe shape isomeric states in general, and fission isomers in particular, one needs the energy of a given nucleus as a function of its deformation. Such a calculation relies on two essential ingredients. On one hand a description that is able to determine the energy of the studied nucleus as realistically as ever possible. It is clear that the energy that one is speaking about needs to include the quantum-mechanical structure as well as the pairing correlations, and this for any nuclear deformation. Such a description could rely on some mean-field approach using nucleon-nucleon interaction rooted (more or less directly) in the meson fields or elementary nucleon nucleon diffusion cross sections. Instead, and this is the method that is used in our present investigation, one could use the macroscopic-microscopic model to obtain this nuclear energy. On the other hand one also needs to describe the vast variety of nuclear shapes that are encountered in the course of the deformation process. In the case of the nuclear fission , this is particularly demanding since very elongated and necked-in shapes have to be taken into account. The new Fourier expansion [6], described below, is used in our calculation.

### Fourier expansion of deformed shapes

The shape-profile function $\rho_s^2(z)$ (in cylindrical coordinates) of a fissioning nucleus is expanded in a Fourier series:

$$\frac{\rho_s^2(z)}{R_0^2} = \sum_{n=1}^{\infty}\left[a_{2n}\cos\left(\frac{(2n-1)\pi}{2}\frac{z-z_{sh}}{z_0}\right) + a_{2n+1}\sin\left(\frac{2n\pi}{2}\frac{z-z_{sh}}{z_0}\right)\right] , \quad (1)$$

The condition $\rho_s(z_{sh}-z_0) = \rho(z_{sh}+z_0) = 0$ is automatically satisfied by Eq. 1. The shift coordinate $z_{sh}$ insures that the centre of the nucleus is located at the origin of the coordinate system. One introduces an elongation parameter $c$ through $z_0 = cR_0$, which is equal to unity for the sphere, smaller than one for oblate, and larger than one for prolate deformations. The parameters $a_2$, $a_3$, $a_4$ describe respectively quadrupole, octupole and hexadecapole type of deformations, or better to say, elongation, reflection asymmetry and neck degree of freedom. As the path to fission corresponds to decreasing values $a_2$ and growing negative values of $a_4$ we introduce new optimal coordinates, ensuring better and more intuitive presentation of potential energy landscapes:

$$q_2 = a_2^{(0)}/a_2 - a_2/a_2^{(0)}; \quad q_3 = a_3; \quad q_4 = a_4 + \sqrt{(q_2/9)^2 + (a_4^{(0)})^2} \quad (2)$$

with $a_n^{(0)}$ being the value of the coordinate $a_n$ for a spherical shape [6]. The non-axial deformation parameter $q_1 = \eta$ was found to be zero in all investigated nuclei.

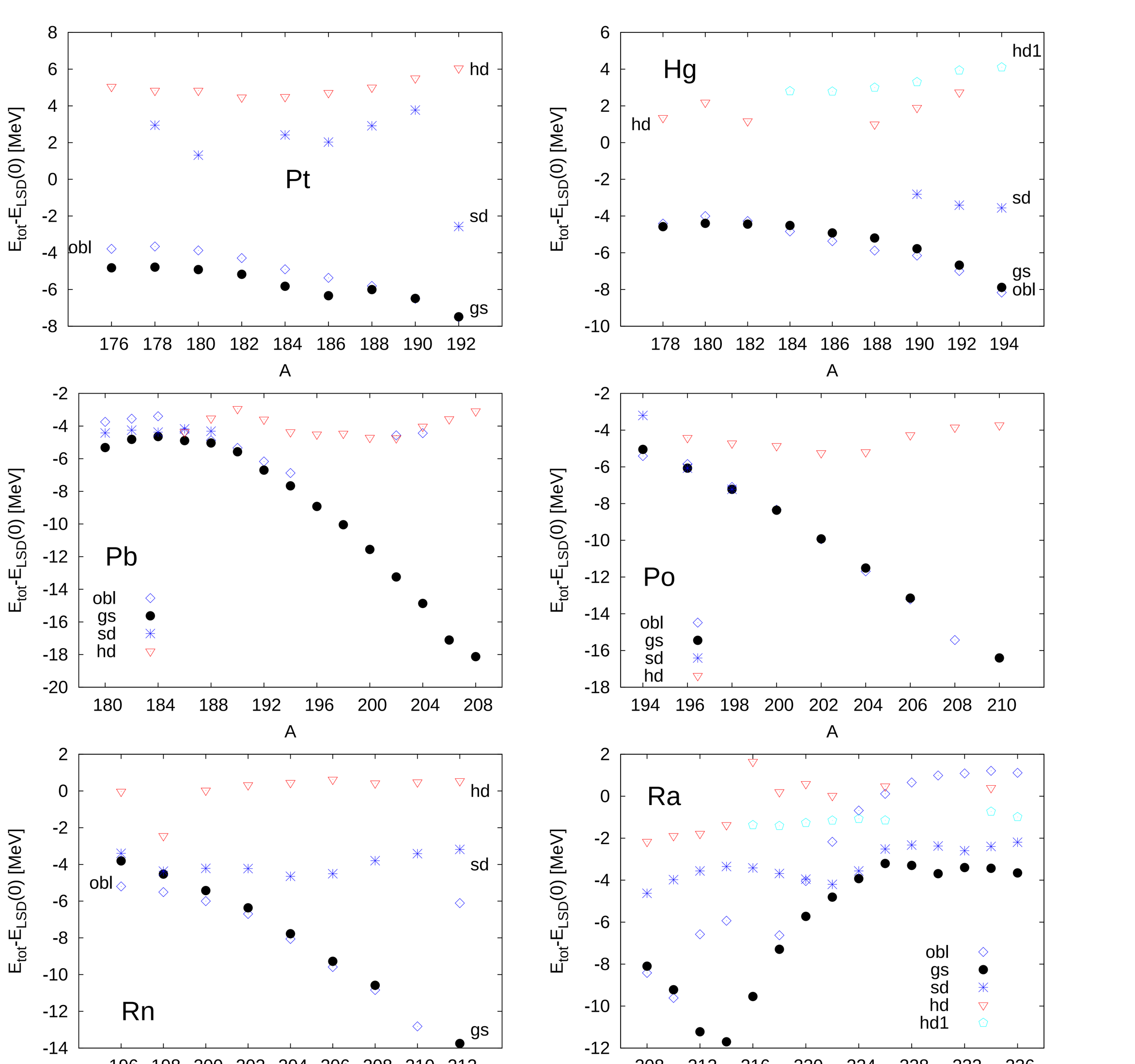

FIGURE 1:Shape Isomers potential energy (relied to spherical LSD energy) of Pt - Ra even-even isotopes minimised with respect to $q_2$ (elongation), $q_4$ (neck parameter), and octupole $q_3$ Fourier deformation parameters (see Eq. 2) [6].

## Deformation energies

The calculation was done for the Pt - Pu even-even isotopes. Examples of the potential energy (relied to the spherical LSD energy) landscapes on $q_2, q_3$ plane (after minimization with respect to $q_4$) for the Pt, Pb, Rn isotopes were shown in Ref. [7]. The maps for Hg, Po, Ra were already published [8],[9], so, here we repeat only the conclusions concerning those nuclei. One can get these results analysing the potential energy landscapes in the Fourier deformation space described in [6]. It appears interesting to follow the evolution (appearance and disappearance) of the different *super − deformed*, *hyper − deformed*, and even *ultra − deformed* local minima.
In Pt isotopes one observes two energy minima: oblate and deeper prolate one. Above $^{186}$Pt the both minima become comparable in energy and less deformed, going with $N$ towards spherical shape. The isotope $^{192}$Pt is not deformed, as $N$=114 is the quasi =-magic number.
In the Hg chain [8] one can see local minima in the potential energy at large elongations ($q_2 > 2$) which correspond to a reflection asymmetric partition, with a heavy-fragment mass around 100-110 depending in the Hg isotope. This is in a good agreement with the experimental observation made in Ref. [10]. Nevertheless, it is important to emphasize that a clear evolution of the static fission path occurs with increasing Hg mass. Indeed, for the lightest $^{178-182}$Hg isotopes, an asymmetric valley appears already at $q_2 \approx 1$, while the minimum energy path evolves through a symmetric splitting for a sizeable part, and to an asymmetric partition only at the last stage beyond $q_2 \approx 1.7$. This result corroborates previous conjectures [11].
In the Pb isotopes the magic $Z = 82$ number gives the zero deformation in the ground states, but the octupole minimum appears for $^{190}$Pb. The super-deformed isomer arrives there up to $^{202}$Pb, but the octupole one disappears. Then the $^{204}$Pb - $^{208}$Pb get the deep spherical ground state.

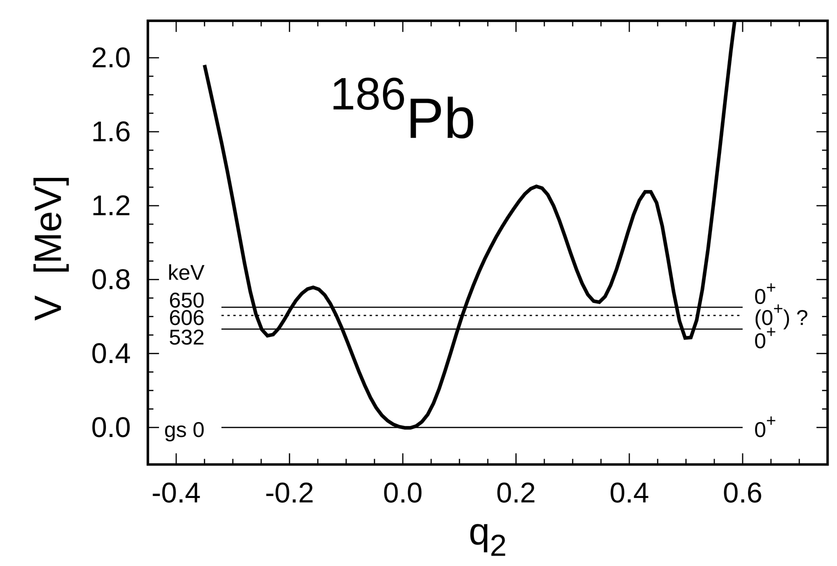


FIGURE 2: Potential energy of $^{186}$Pb minimized with respect the deformations $q_3$ and $q_4$ as function of the elongation parameter $q_2$. The horizontal (thin solid) lines show the energies of the ground and the excited $0^+$ states [12], while the dashed-line corresponds to the position of the $0^+$ rotational band head which we have deduced from the rotational series observed in [12].

For the Pb isotopic chain shown in Fig. 1, the Z = 82 magic number leads to a ground state of zero deformation throughout the chain. Shape isomers corresponding to an elongation $q_2 \approx 0.6$ have a good chance to be found in Pb isotopes with mass number $A = 192 - 208$. Pronounced hyper-deformed minima are found at $q_2 \approx 1.75$ in the PES of $^{192-204}$Pb, whereas sd shape isomers (with $q_2 \approx 1.3$) are only visible in the heaviest four Pb isotopes $^{202-208}$Pb. All candidates for Pb shape isomers are reflection symmetric ($q_3 = 0$). In heavier Pb isotopes with $A \geq 202$ the previously present mass asymmetric fission path disappears. It is worth mentioning in this connection that the PES of some of these Pb isotopes is often very *rich*, showing several shape isomeric states on the oblate, but more often on the prolate side. This is in particular the case for the neutron deficient isotopes between $A = 180$ and $A = 190$. The triple shape coexistence in $^{186}$Pb advocated in Ref. [12], turns out, according to us, to be a quadruple shape coexistence with two prolate local minima at $q_2 \approx 0.35$ and $q_2 \approx 0.5$ respectively, as shown on Fig. 2. Similarly to Andreyev [12], we find the coexistence of an oblate and a prolate minimum at the energies very close to the measured ones. In addition, we have found a second prolate minimum that appears at an only slightly lower energy than two others shape-isomeric states. What is shown in Fig. 2 is a one-dimensional cross-section of the PES of $^{186}$Pb (energy as function of the only quadrupole $q_2$ parameter), where, in addition, horizontal lines indicate the position of the ground state and both the excited $0^+$ states observed in Ref. [12]. Andreyev et al. have also observed a rotational band running from the $2^+$ to the $14^+$ state, and which decays by gamma emission and $e^-$ conversion directly to the $0^+$ ground-state, while the bottom of this band was not directly observed in Ref. [12]]. Using the rotational model [13] we have estimated the position of the (non-observed) $0^+$ state at which the band ends, and have found it located at approximately 606 keV (dotted line in Fig. . This value differs from the energies of both $0^+$ reported in Ref. [12]. This fact can be used as certain confirmation of quadruple shape coexistence in $^{186}$Pb. We have found a similar kind of coexistence other neutron deficient Pb isotopes, as can be seen from Fig. 1.
In the Po isotopic chain [8],[13] the potential energy landscapes suggest a coexistence of symmetric and asymmetric fission for the most neutron deficient isotopes, whereas only symmetric fission is expected for the heavier isotopes. Such a prediction is in line with recent experimental data [14].
In the Rn isotopes even 4 octupole symmetric isomers can be observed but above $^{208}$Rn they slowly go towards spherical, very deep in $^{210}$Rn, minimum.
The survey of the Ra chain has shown [8] that the path to fission corresponding to the minimum energy is reflection-symmetric for all of the light isotopes. An asymmetric valley progressively develops, however, with increasing Ra mass which competes with a symmetric splitting around $^{222}$Ra, and finally becomes the favoured fission configuration for still heavier Ra. This evolution is again in good agreement with experimental data (see [15] and Refs. therein).

## Electric quadrupole moments

The quadrupole moments associated with most pronounced energy minima were evaluated. The corresponding calculated electric quadrupole moments are presented in Fig. 3 for the Pt - Ra isotopic chains in the shape isomers visible in the potential energy surfaces. They agree rather well with the available data for the ground state (gs), what allows to trust the predicted ones. Experimental efforts to look for exotic shapes (super-deformed - sd, hyper-deformed - hd,hd1) in this region are strongly encouraged.

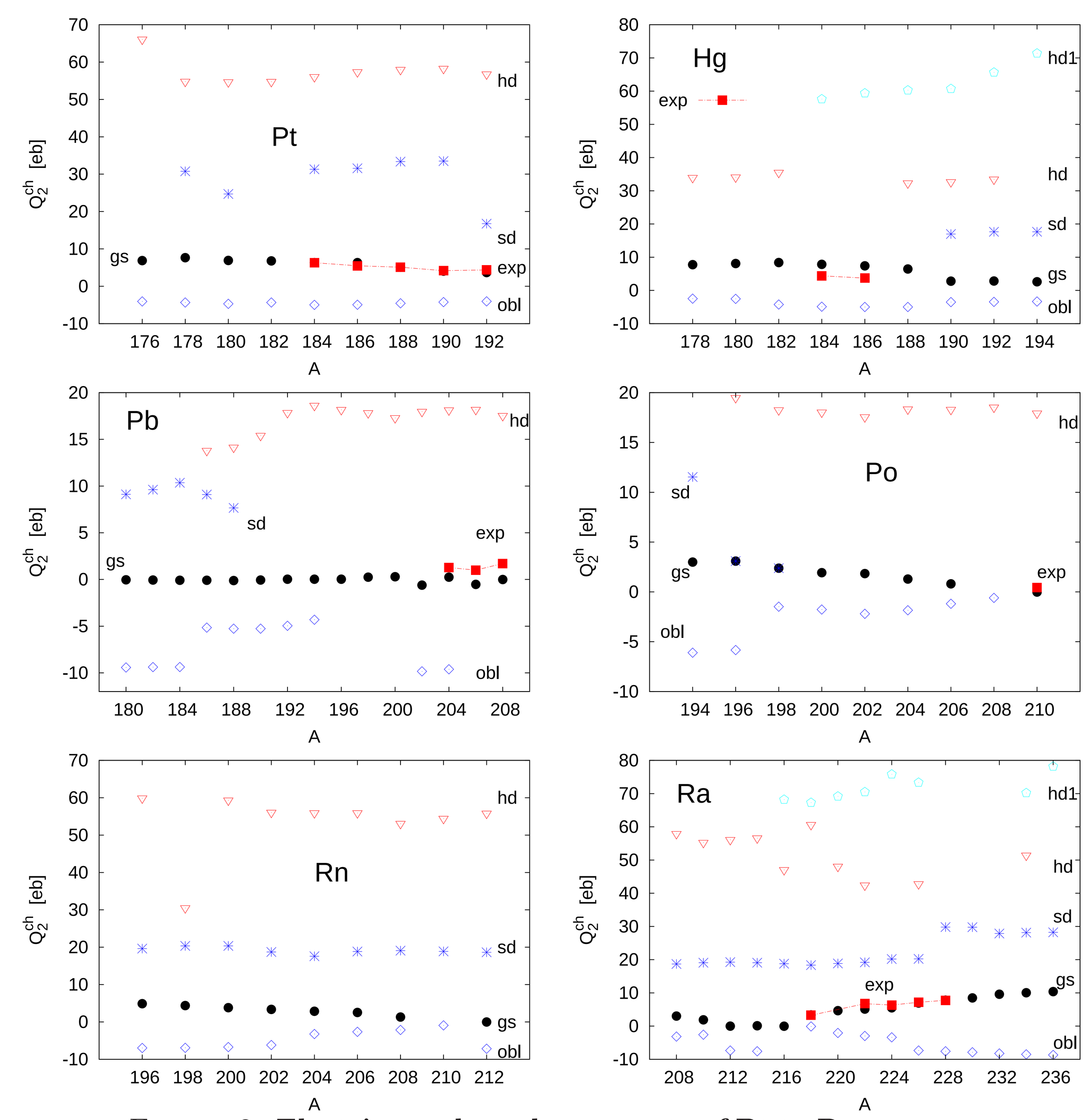

FIGURE 3: Electric quadrupole moments of Pt to Ra even-even isotopes for the ground states (gs), super-deformed (sd), and hyper-deformed (hd, hd1) shape-isomers. The data for ground states are shown, where available.

## Summary

Our investigations of the liquid-drop fission barriers heights and their shapes performed in Ref. [2] in a variational calculation have shown that the fission barriers of medium-heavy nuclei are very broad and do not decrease rapidly when going from the saddle to the scission point. This property of the macroscopic energy offers a chance that in this region of nuclei shell effects may produce local minima corresponding to a large nuclear elongation. As first candidates, we have chosen the chains of Polonium , Radium and Thorium [8] isotopes, and we have shown that the microscopic energy corrections can, indeed, produce pronounced minima in the rather flat macroscopic potential-energy surfaces corresponding to Pt - Pu isotopes. The electric quadrupole moments for all the ground states and isomers were found and they reproduce rather well the data in the ground states. In the case of very flat potential energy valleys the minima are not precise, so the agreement can be worse. The large $B(E2)$ transition probability corresponding to an electric quadrupole moment could be a fingerprint for such ultra-deformed isomers. We hope that in the near future this new island of super-, hyper- and ultra-deformed shape isomers will be discovered in the experimental analysis.

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