# KIDS – A new Energy-Density Functional for exotic nuclei

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## Abstract

Inspired by an effective field theory for dilute Fermi systems, a new nuclear energy density functional, **KIDS** (Korea: IBS-Daegu-Sungkyunkwan), was recently proposed by [P. Papakonstantinou, T. Park, Y. Lim, and C. H. Hyun, arXiv:1606.04219]. The model consists of a systematic expansion in powers of the Fermi momentum. The parameters are fitted to the empirical properties of symmetric nuclear matter and the equation of state of pure neutron matter calculated in [A. Akmal, V. R. Pandharipande, and D. G. Ravelhall, Phys. Rev. C 58 (1998) 1804]. In order to apply the KIDS model to nuclei, we extract equivalent Skyrme-type interactions. Solving Hartree-Fock equations, we obtain the energies per particle, charge radii of closed nuclei and neutron skins of magic nuclei, namely, $^{16}$O, $^{28}$O, $^{40}$Ca, $^{48}$Ca, $^{60}$Ca, $^{90}$Zr, $^{132}$Sn and $^{208}$Pb. This model is found to successfully reproduce the experimental data for stable nuclei and we present predictions for $^{28}$O and $^{60}$Ca with the optimized parameters of the model. Next, we plan to apply the model to an RPA calculation of giant resonance and exotic modes of nuclear excitations.

## KIDS

- From **Brueckner theory** with a realistic potential and **Effective field theory** with a dilute Fermi system, the nuclear energy density functional is described by expansion in term of $k_F^2$, $k_F^3$, $k_F^4$, $k_F^5$, $k_F^6$, … , and $ln\ k_F$ at low density, in homogeneous nuclear matter.

- KIDS nuclear energy density functional is defined by [1]

$$\mathcal{E}(\rho,\delta) = \mathcal{T}(\rho,\delta) + \sum_{i=0}^{3} c_i(\delta)\rho^{1+i/3}$$

where the total density $\rho = \rho_p + \rho_n$ and asymmetry $\delta = (\rho_n - \rho_p)/\rho$.

- Kinetic energy is defined as

$$\mathcal{T}(\rho,\delta) = \frac{3}{5}\frac{\hbar^2}{2m_p}\left(\frac{1-\delta}{2}\right)^{5/3}(3\pi\rho)^{3/2} + \frac{3}{5}\frac{\hbar^2}{2m_n}\left(\frac{1+\delta}{2}\right)^{5/3}(3\pi\rho)^{3/2}$$

- Parameters in the nuclear potential are defined as

$$c_i(\delta) = \alpha_i + \delta^2\beta_i$$

and determined by fitting to the empirical data and the APR pseudo-data which are the results of a microscopic calculation [2].

## Application to nuclei

- For applying to nuclei, the **Skyrme Hartree-Fock** model is used in our study.
- For spin-saturated homogeneous infinite nuclear matter, the Skyrme energy density functional (EDF) is given as

$$\mathcal{E}(\rho,\delta) = \mathcal{T}(\rho,\delta) + \frac{1}{8}\{3t_0 - (t_0+2y_0)\delta^2\}\rho + \frac{1}{48}\sum_{i=1}^{3}\left\{3t_{3i} - (t_{3i}+2y_{3i})\delta^2\right\}\rho^{i/3+1}$$
$$+ \frac{1}{16}\left[(3t_1+5t_2+4y_2) - \{(t_1+2y_1)-(t_2+2y_2)\}\delta^2\right]\tau$$

- **KIDS parameter in nuclei**

Parameters of symmetry nuclear matter (SNM) are obtained by using properties at saturation point and pure neutron matter (PNM) parameters are determined by fitting the APR data at the weight factor β ~ 0.97 [3].

| $\delta$ | $c_0$ [MeV·fm$^3$] | $c_1$ [MeV·fm$^4$] | $c_2$ [MeV·fm$^5$] | $c_3$ [MeV·fm$^6$] |
|---|---|---|---|---|
| 0 | −664.52 | 763.55 | 40.13 | 0.0 |
| 1 | −411.13 | 1007.78 | −1354.64 | 956.47 |

- Comparison of KIDS EDF with Skyrme EDF

$$c_0(\delta) = \frac{3}{8}t_0 - \frac{1}{8}(t_0+2y_0)\delta^2$$
$$c_1(\delta) = \frac{1}{16}t_{31} - \frac{1}{48}(t_{31}+2y_{31})\delta^2$$
$$c_2(\delta) = \frac{1}{16}t_{32} - \frac{1}{48}(t_{32}+2y_{32})\delta^2 + \frac{3}{5}\left(\frac{6\pi^2}{\nu}\right)^{2/3}\frac{1}{16}\left[(3t_1+5t_2+4y_2) - \{(t_1+2y_1)-(t_2+2y_2)\}\delta^2\right]$$
$$c_3(\delta) = \frac{1}{16}t_{33} - \frac{1}{48}(t_{33}+2y_{33})\delta^2$$

**Simple trick !!**

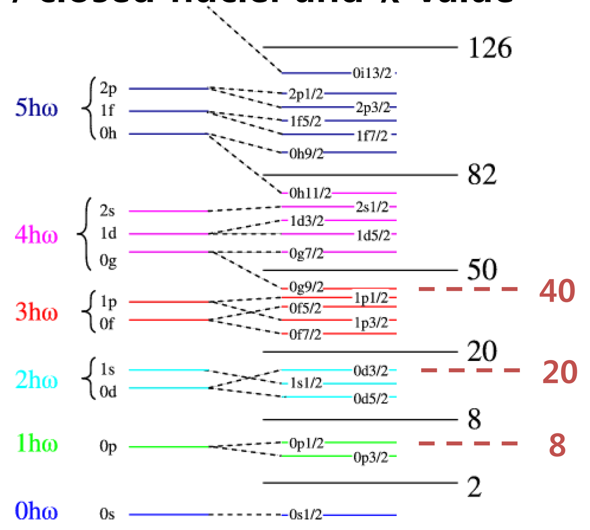
$$c_2^{t_1,t_2}(\delta) = k \times c_2(\delta),$$
$$c_2^{t_{32},y_{32}}(\delta) = (1-k) \times c_2(\delta)$$

- **Skyrme parameters from KIDS**

| $t_0$ | $t_1$ | $t_2$ | $t_{31}$ | $t_{32}$ | $t_{33}$ |
|---|---|---|---|---|---|
| −1772.044 | 2492.112 × $k$ | −1459.767 × $k$ | 12216.732 | 642.115 × (1 − $k$) | 0.000 |

| $y_0$ | $y_1$ | $y_2$ | $y_{31}$ | $y_{32}$ | $y_{33}$ |
|---|---|---|---|---|---|
| −127.524 | 0.000 | 0.000 | −11969.990 | 33153.477 × (1 − $k$) | −22955.280 |

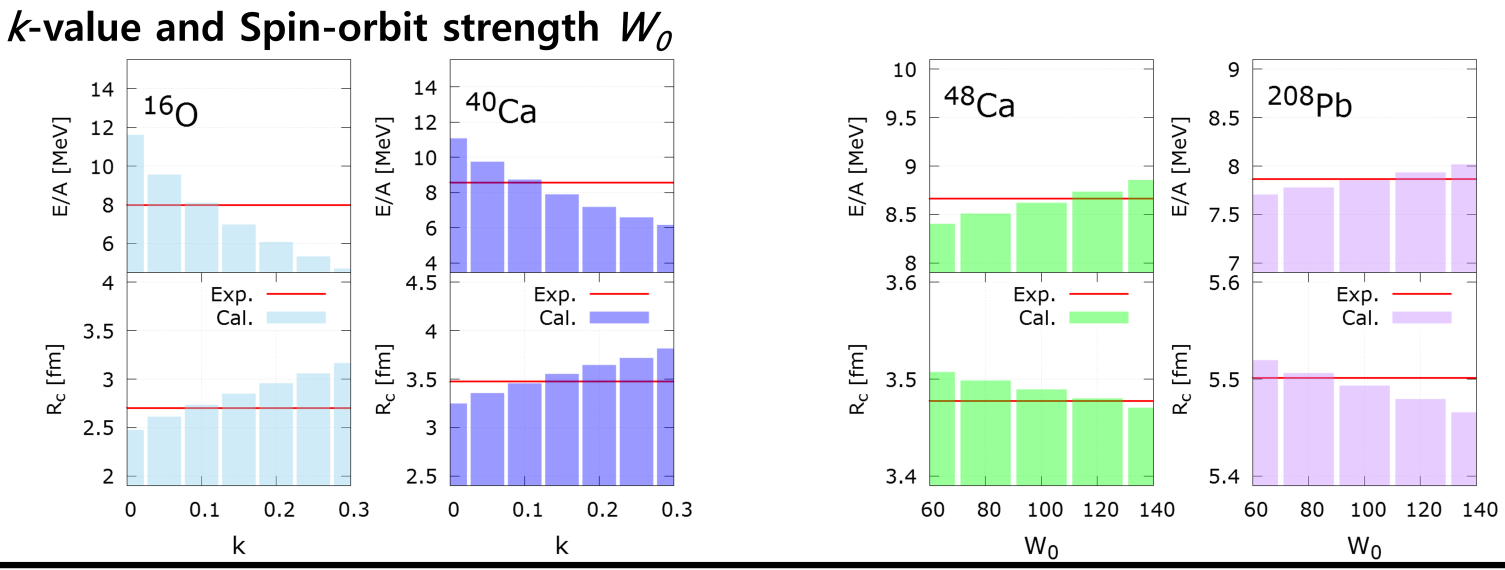
- ***l*-closed nuclei and *k*-value**




Merit : We do not need to consider $V_{LS}$ .
Shortcoming : Application is limited to *N* or *Z* = 8, 20, 40 nuclei.

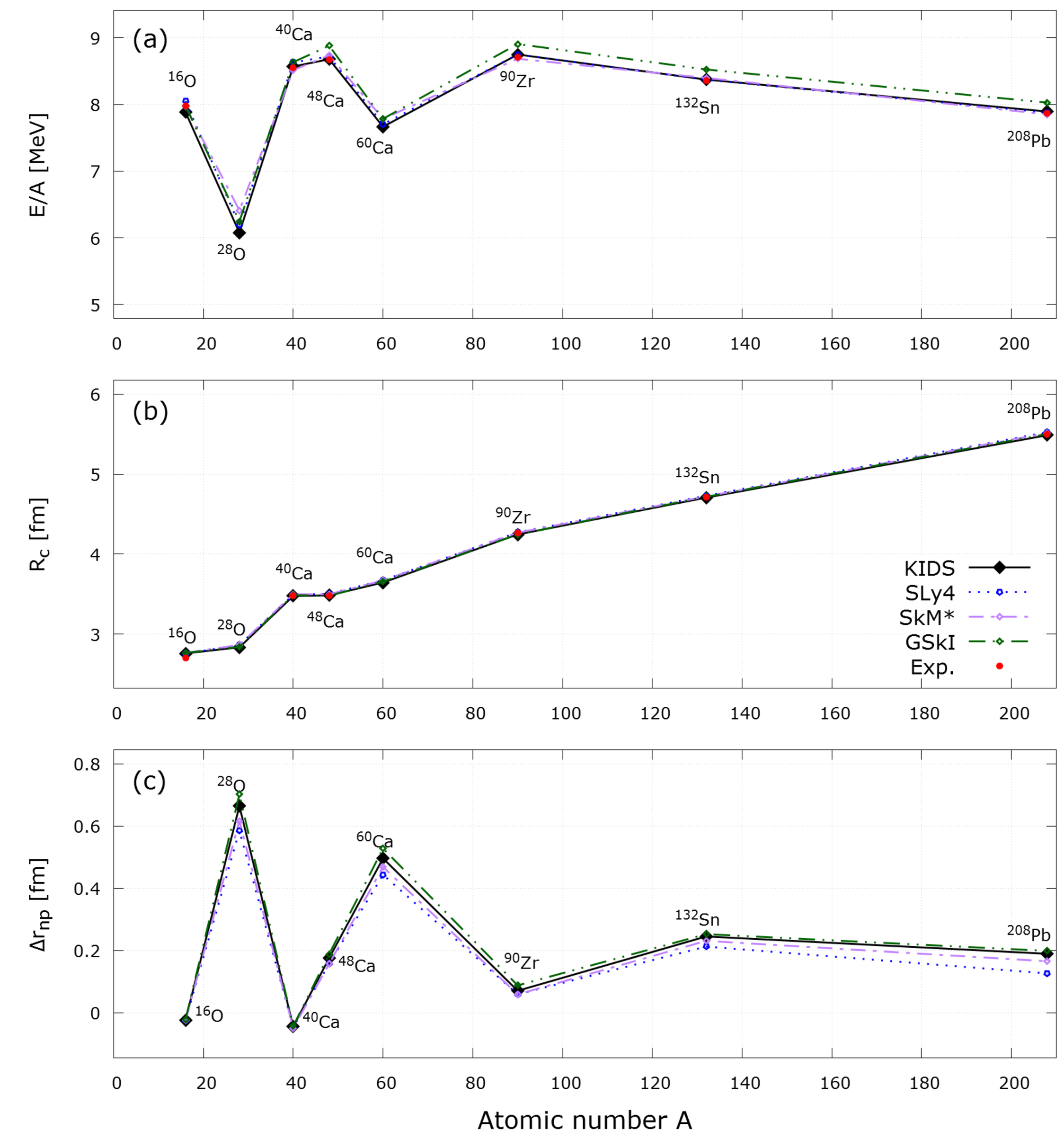
*k* is determined by comparing calculated energies per particle and charge radii of *l*-closed nuclei with experimental data, which gives ***k* = 0.11**.
The spin-orbit strength is determined as **$W_0$ = 110** by the properties of $^{48}$Ca and $^{208}$Pb.

- ***k*-value and Spin-orbit strength $W_0$**

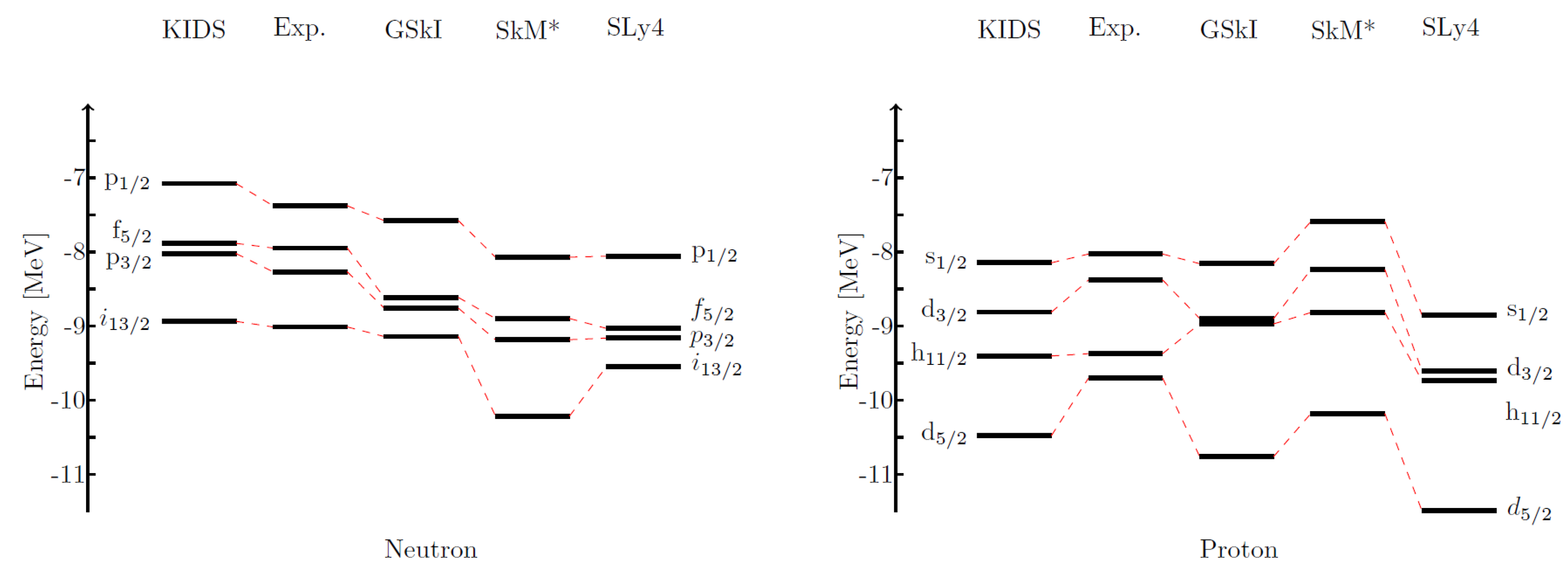



## Results

- The **energy per particle**, **charge radius** and **neutron skin** are calculated by using KIDS and are compared with other Skyrme force model results and experimental data [4,5].




- **Level schemes of $^{208}$Pb**




- **Prediction of $^{28}$O and $^{60}$Ca**

| Model | $^{28}$O $E/A$ [MeV] | $^{28}$O $R_c$ [fm] | $^{28}$O $\Delta r_{np}$ [fm] | $^{60}$Ca $E/A$ [MeV] | $^{60}$Ca $R_c$ [fm] | $^{60}$Ca $\Delta r_{np}$ [fm] |
|---|---|---|---|---|---|---|
| SLy4 | 6.1925 | 2.8656 | 0.58476 | 7.703 | 3.6734 | 0.4435 |
| SkM* | 6.4114 | 2.8646 | 0.61631 | 7.7857 | 3.6713 | 0.4685 |
| KIDS | 6.0757 | 2.8353 | 0.66398 | 7.6652 | 3.6452 | 0.4960 |
| AME2012 | 5.9883 | | | | | |

## Conclusion

- We applied the newly developed KIDS nuclear energy density functional to calculate the properties of magic nuclei : $^{16}$O, $^{28}$O, $^{40}$Ca, $^{48}$Ca, $^{60}$Ca, $^{90}$Zr, $^{132}$Sn, and $^{208}$Pb.
- The parameter *k*, which was introduced to match the parameters of the KIDS and Skyrme force model, was determined by the energies per nucleon and charge radii of $^{16}$O and $^{40}$Ca.
- The spin-orbit strength $W_0$ was determined by the energies per nucleon and charge radii of $^{48}$Ca and $^{208}$Pb.
- Known data are found to be described successfully.
- We obtain predictions on the same physical quantities for $^{28}$O and $^{60}$Ca.
- Further works: we plan to apply the model to an RPA calculation of giant resonance and exotic modes of nuclear excitations.

## Reference

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