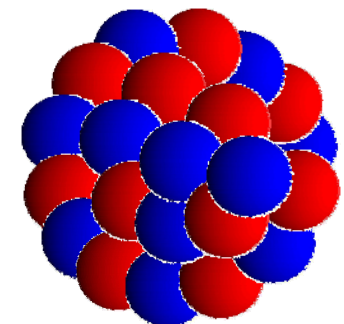
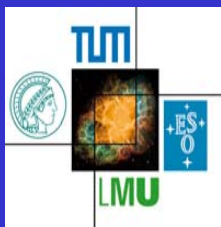


# RPA for Superfluid Nuclei and Extensions

Paris, Jan. 28, 2010

**Peter Ring**

Technical University Munich



- **Static density functionals in nuclei**
- **Time dependent density functionals in nuclei**
- **Quasiparticle RPA**
- **Applications:**
  - relativistic RPA
  - continuum RPA
  - deformed RPA
  - pn - RPA
- **Extensions**
  - energy dependent KS-fields
  - Mixing of deformed configurations
  - Symmetry restoration before variation
- **Conclusions and outlook**

## ab initio: 3 scales

- **QCD**: non-linear gauge theory, **quarks, gluons**
- confinement
- running coupling constant
- at low energies **non-perturbative**
- effective degrees of freedom: nucleon, pion

**> 1 GeV**

- effective Lagrangian in **nucleons and pions**
- parameters (LEC) so far phenomenological
- non renormalizable
- at low momenta: **chiral perturbation theory:  $\chi$ PT**
- → bare nucleon-nucleon interaction: **NNL N3LO**

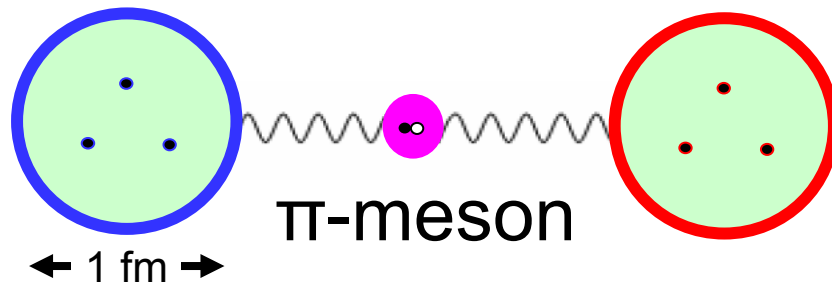
**~ 100 MeV**

- effective forces within the nucleon
- density dependent
- configuration mixing, **density functional theory**

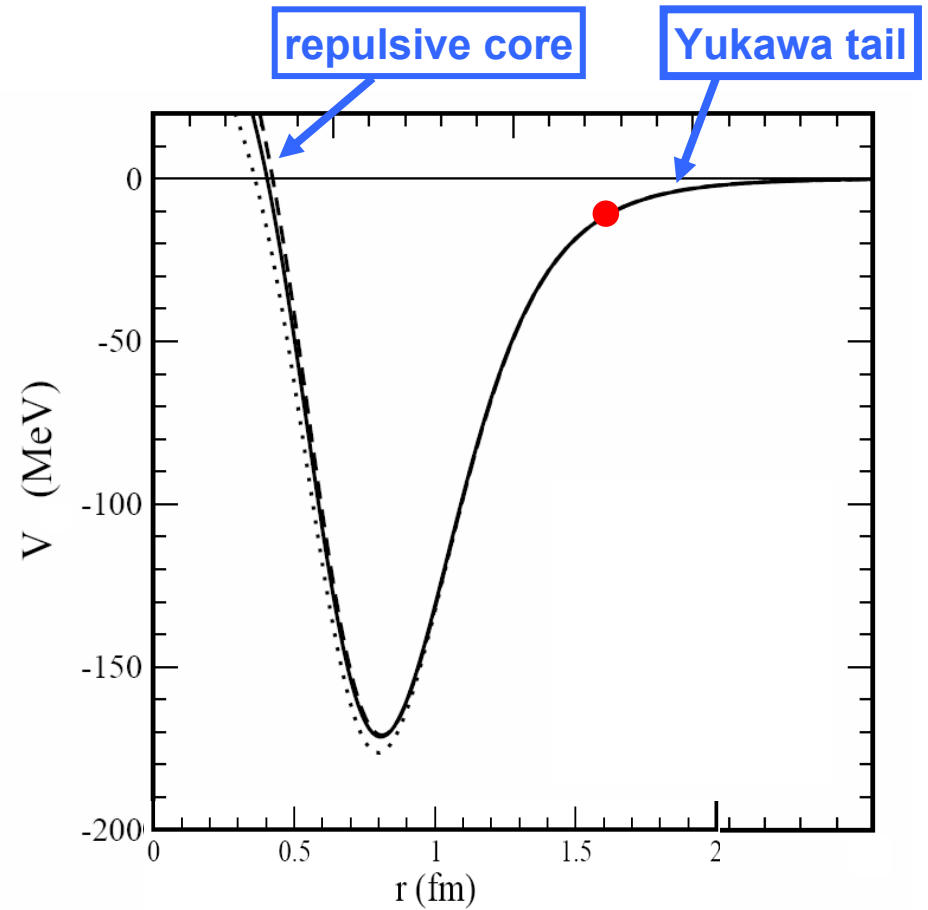
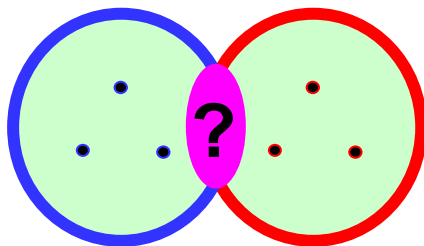
**~ 100 keV**

# bare nucleon-nucleon interaction:

distance  $> 1$  fm: attractive



distance  $< 0.8$  fm: repulsive



three-body forces ?

# Density functional theory in nuclei

D.Brink  
D.Vauterin

**Skyrme**

$$E[\rho] = \langle \Psi | H | \Psi \rangle \approx \langle \Phi | H_{eff}(\rho) | \Phi \rangle$$

$|\Phi\rangle$  Slater determinant  $\iff \bar{\rho}$  density matrix

$$|\Phi\rangle = \mathcal{A}\{\varphi_1(\mathbf{r}_1) \dots \varphi_A(\mathbf{r}_A)\} \iff \bar{\rho}(\mathbf{r}, \mathbf{r}') = \sum_{i=1}^A |\varphi_i(\mathbf{r})\rangle \langle \varphi_i(\mathbf{r}')|$$

**Mean field:**

$$\hat{h} = \frac{\delta E}{\delta \bar{\rho}}$$

**Eigenfunctions:**

$$\hat{h}|\varphi_i\rangle = \varepsilon_i|\varphi_i\rangle$$

**Interaction:**

$$\hat{V} = \frac{\delta^2 E}{\delta \bar{\rho} \delta \bar{\rho}}$$

Extensions: Pairing correlations, Covariance  
Relativistic Hartree Bogoliubov (RHB)

## Density functionals in nuclei:

- they are based on **density dependent** two-body interactions  $H(\rho)$ 
  - on the mean field level they contain also **three-body** forces
- they are **completely phenomenological**
  - few parameters are adjusted to binding energies and radii
- they can be represented by **zero range forces** (and gradient corrections)
  - on the mean field level one needs only momenta  $k < k_F$
- for zero range forces **Fock terms** can be absorbed in the parameters
- there are large **spin-orbit** terms
- strong pp-correlations lead in open shell nuclei to **superfluidity**
  - is treated on the mean field level by **Hartree-Fock-Bogoliubov** methods

### 3 types of functionals are presently used:

- non relativistic **zero range forces** (Skyrme)
- finite range forces of **Gaussian shape** (Gogny)
- **relativistic** density functionals (RMF)

# Density functionals in superfluid systems:

$$E[\rho, \kappa] = E_{\text{KS}}[\rho] + E_{\text{pair}}[\kappa]$$

where

$$\rho = \langle a^\dagger a \rangle \quad \kappa = \langle a^\dagger a^\dagger \rangle$$

$$\mathcal{R} = \begin{pmatrix} \rho & \kappa \\ -\kappa^* & -\rho^* \end{pmatrix}$$

and the Kohn-Sham equations are of the form

Valatin density

$$\begin{pmatrix} h_{\text{KS}} - \mu & \Delta \\ -\Delta^* & -h_{\text{KS}}^* + \mu \end{pmatrix} \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix} = E_k \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix}$$

with

$$h_{\text{KS}} = \frac{\delta E}{\delta \rho}, \quad \Delta = \frac{\delta E}{\delta \kappa}$$

# Relativistic Kohn-Sham equations:

$$\begin{pmatrix} m - S + V & \vec{\sigma}\vec{p} \\ \vec{\sigma}\vec{p} & -m + S + V \end{pmatrix} \begin{pmatrix} f_i \\ g_i \end{pmatrix} = \varepsilon_i \begin{pmatrix} f_i \\ g_i \end{pmatrix}$$

scalar potential:

$$S(\mathbf{r}) = G_\sigma \rho_s(\mathbf{r}) = G_\sigma \sum_{i=1}^A (|f_i(\mathbf{r})|^2 - |g_i(\mathbf{r})|^2) \approx 400 \text{ MeV}$$

vector potential:

$$V(\mathbf{r}) = G_\omega \rho(\mathbf{r}) = G_\omega \sum_{i=1}^A (|f_i(\mathbf{r})|^2 + |g_i(\mathbf{r})|^2) \approx 350 \text{ MeV}$$

No-sea approximation

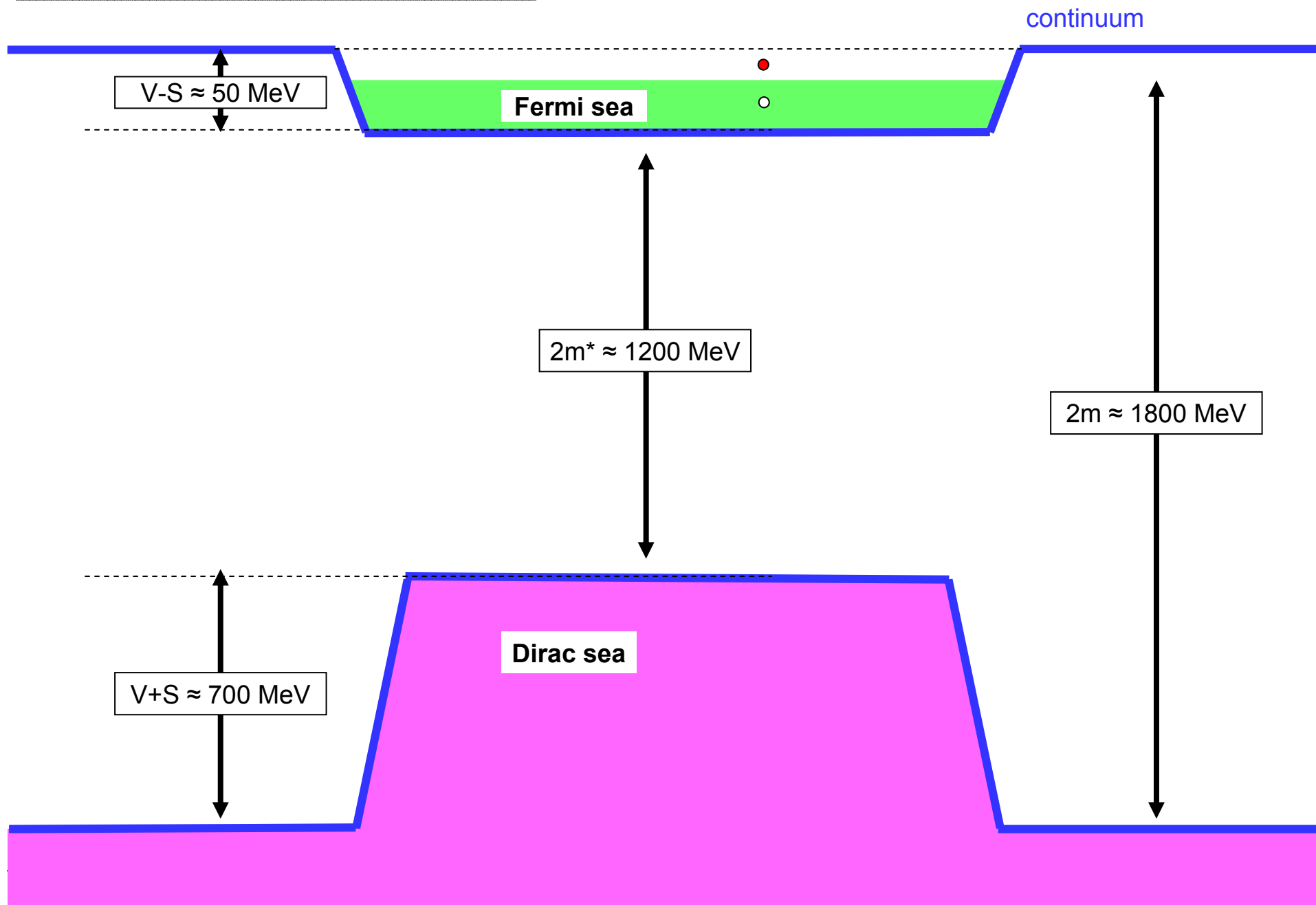
density dependent couplings:  $G = G(\rho)$



rearrangement terms!

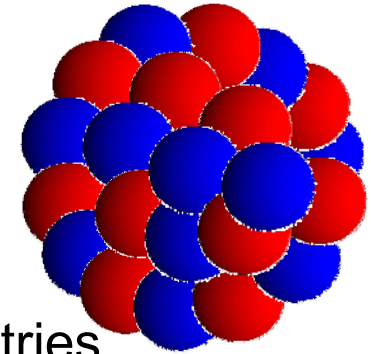


# Relativistic potentials

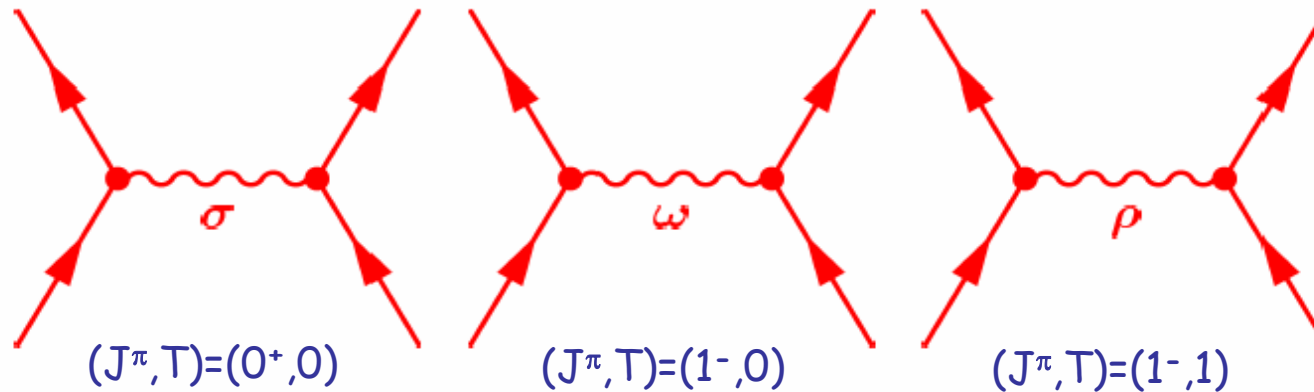


$$E[\rho]$$

## Walecka model



- the basis is an **effective Lagrangian** with all relativistic symmetries
- it is used in a **mean field concept** (Hartree-level)
- with the **no-sea approximation**



$$S(\mathbf{r}) = g_\sigma \sigma(\mathbf{r})$$

Sigma-meson:  
attractive scalar field

$$V(\mathbf{r}) = g_\omega \omega(\mathbf{r}) + g_\rho \vec{\tau} \vec{\rho}(\mathbf{r}) + eA(\mathbf{r})$$

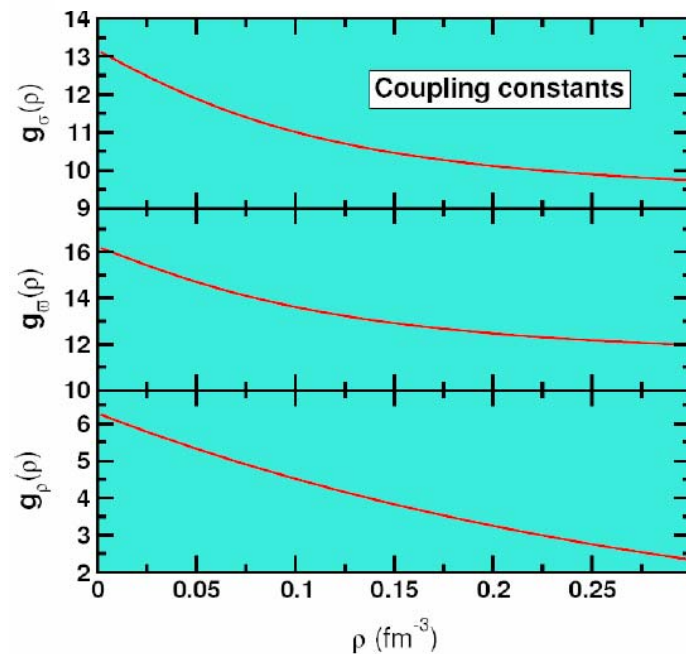
Omega-meson:  
short-range repulsive

Rho-meson:  
isovector field

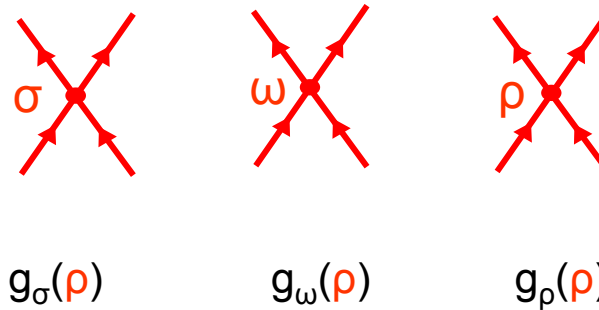
# Effective density dependence:

The basic idea comes from **ab initio calculations**  
 density dependent coupling constants include **Brueckner correlations**  
 and **threebody forces**

non-linear meson coupling: **NL3**



Point-coupling models  
 with derivative terms:



adjusted to ground state properties of finite nuclei

Manakos and Mannel, Z.Phys. **330**, 223 (1988)

Bürvenich, Madland, Maruhn, Reinhard, PRC **65**, 044308 (2002):

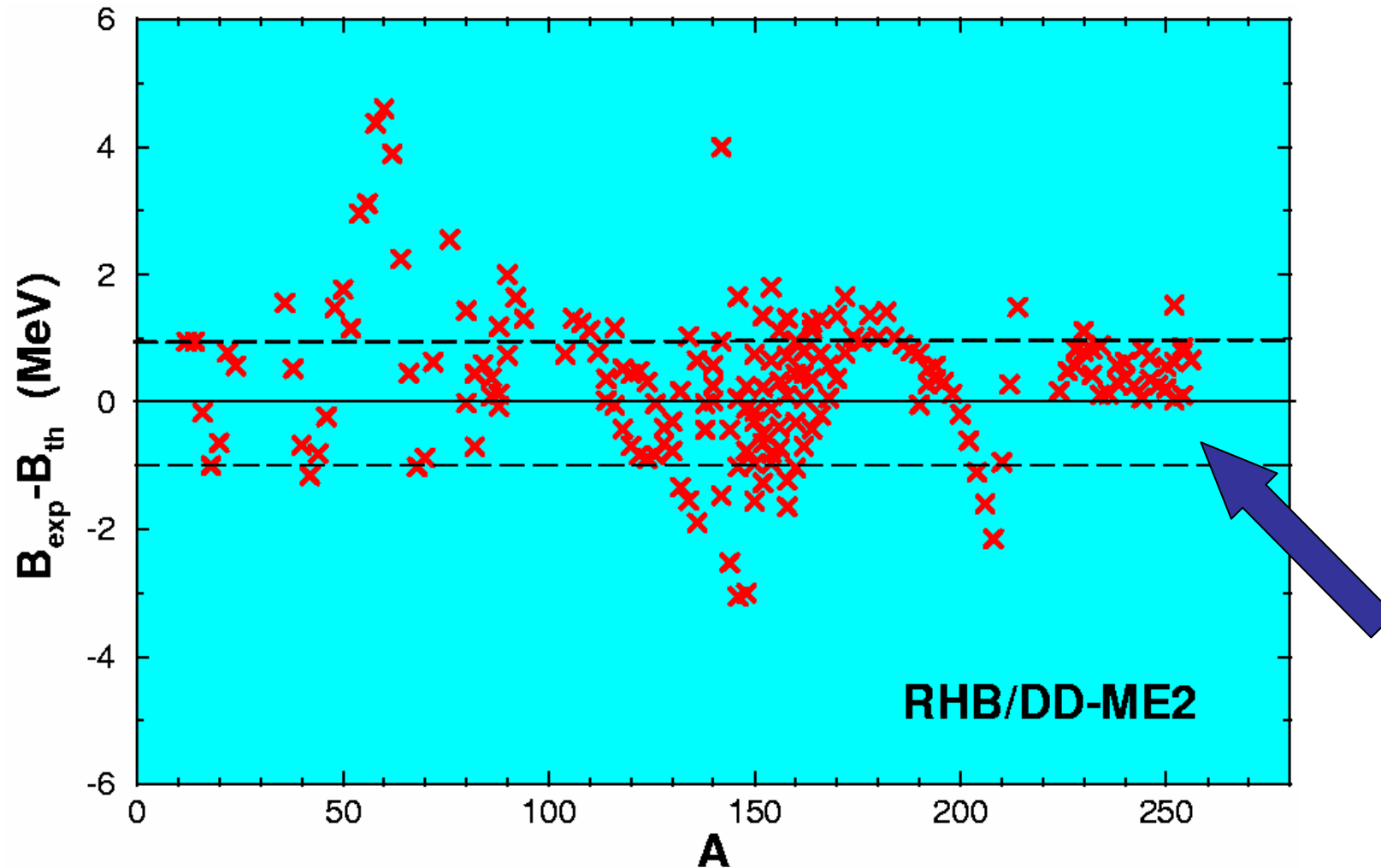
Niksic, Vretenar, P.R., PRC **78**, 034318 (2008):

**PC-F1**

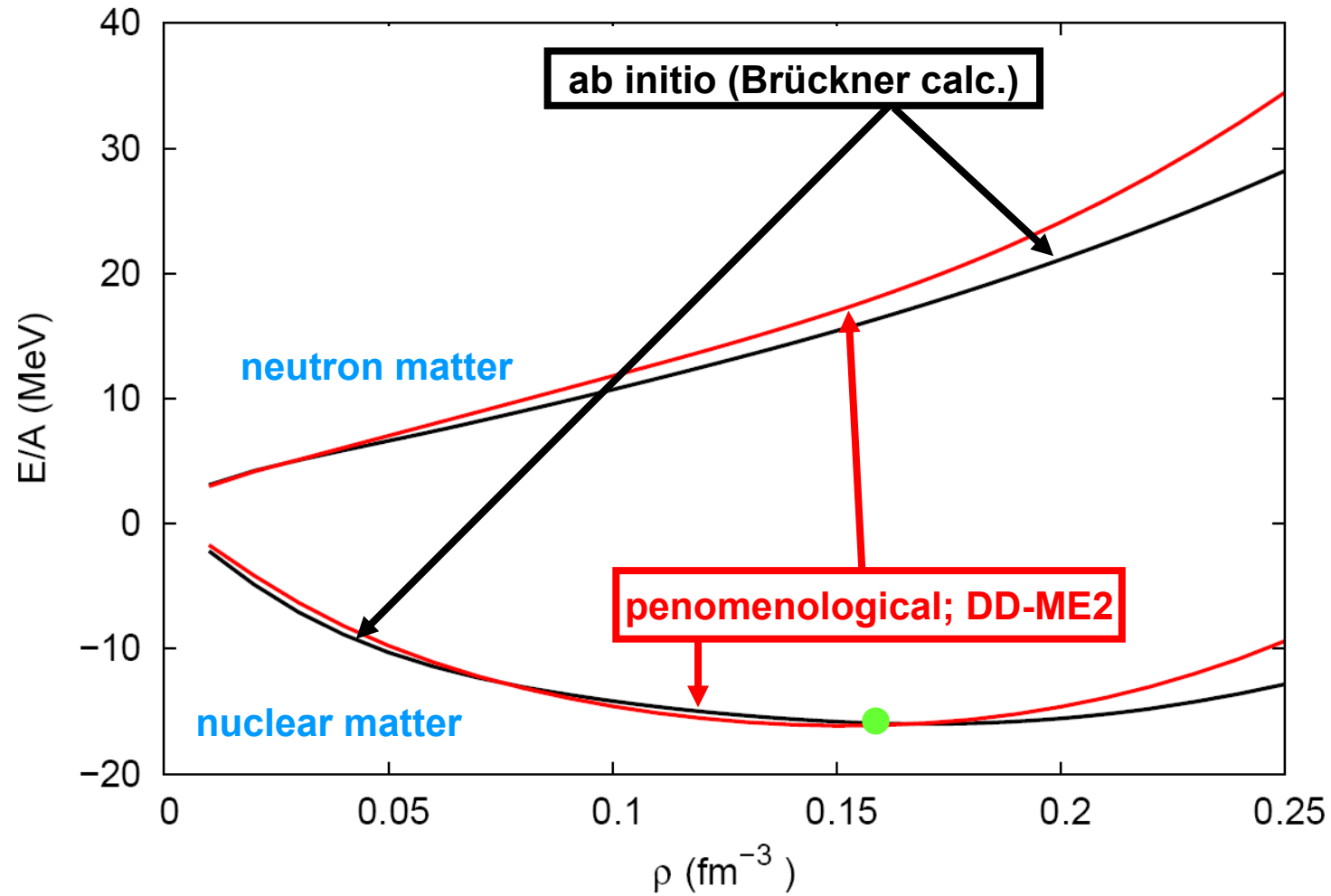
**DD-PC1**

**rms-deviations: masses:  $\Delta m = 900$  keV**  
**radii:  $\Delta r = 0.015$  fm**

Lalazissis, Niksic, Vretenar, Ring, PRC 71, 024312 (2005)



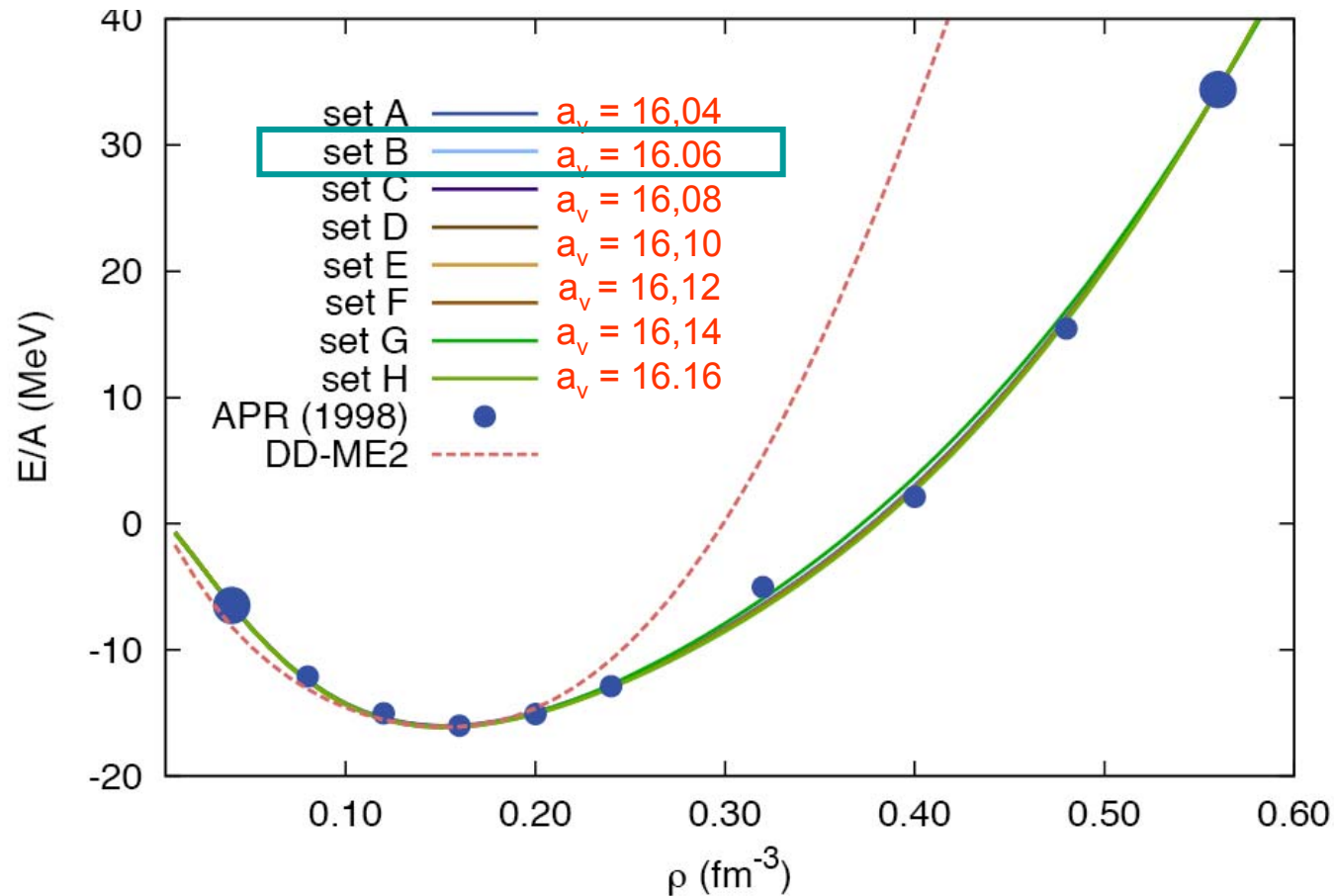
# Comparison with ab initio calculations:



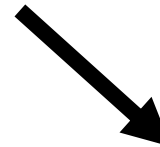
we find excellent agreement with ab initio calculations of Baldo et al.

# Fit to ab-initio results

point coupling model is fitted to microscopic nuclear matter:



$\rho_{\text{sat}} = 0.152 \text{ fm}^{-3}$   
 $m^* = 0.58m$   
 $K_{\text{nm}} = 230 \text{ MeV}$



DD-PC1

● A. Akmal, V.R. Pandharipande, and D.G. Ravenhall, PRC. 58, 1804 (1998).

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## Time dependent mean field theory:

$$\int dt \{ \langle \Phi(t) | i\partial_t | \Phi(t) \rangle - E[\hat{\rho}(t)] \} = 0$$



$$i\partial_t \hat{\rho} = [\hat{h}(\hat{\rho}) + \hat{f}, \hat{\rho}]$$

$$i\partial_t \psi(t) = [(\vec{\alpha}(\vec{p} - \vec{V}(t)) + V(t) + \beta(m - S(t)))\psi(t)]$$

We neglect retardation  
and find for the fields  
at each time-step:

$$S(t) = G_{\sigma} \rho_s(t)$$

$$V(t) = G_{\omega} \rho(t)$$

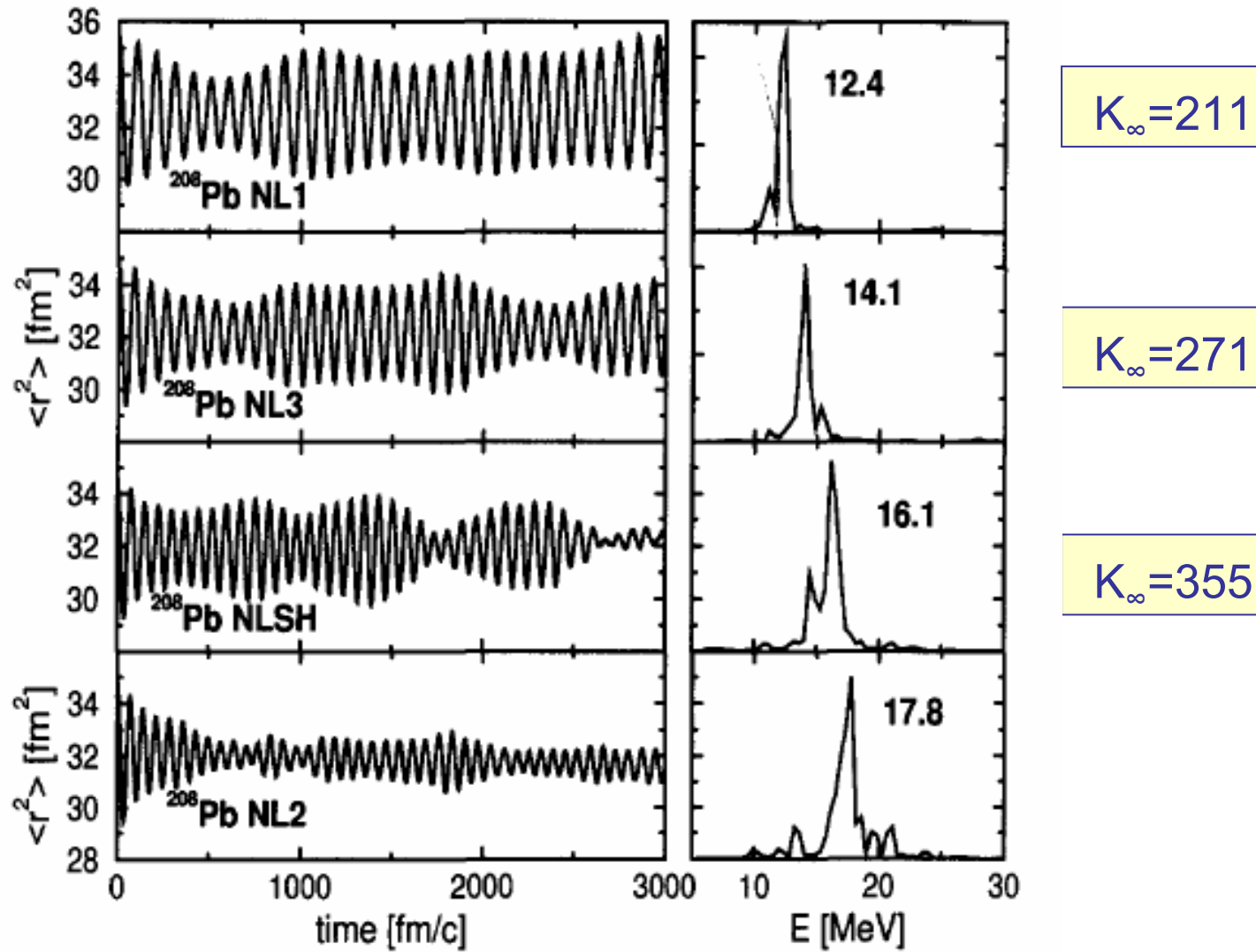
$$\vec{V}(t) = G_{\omega} \vec{j}(t)$$

and similar equations for the isovector and electromagnetic-fields



$$\langle \Phi(t) | r^2 | \Phi(t) \rangle$$

## Breathing mode: $^{208}\text{Pb}$



- **Static density functionals in nuclei**
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# Small amplitude limit gives RPA:

Small amplitude limit:

$$\hat{\rho}(t) = \hat{\rho}^{(0)} + \delta\hat{\rho}(t)$$

ground-state density

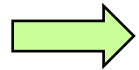
$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \hbar\omega \begin{pmatrix} X \\ Y \end{pmatrix}$$

$\delta\rho_{ph}, \delta\rho_{\alpha h}$

$\delta\rho_{hp}, \delta\rho_{h\alpha}$

RPA matrices:

$$A_{minj} = (\epsilon_n - \epsilon_i)\delta_{mn}\delta_{ij} + \frac{\partial h_{mi}}{\partial \rho_{nj}}, \quad B_{minj} = \frac{\partial h_{mi}}{\partial \rho_{jn}}$$

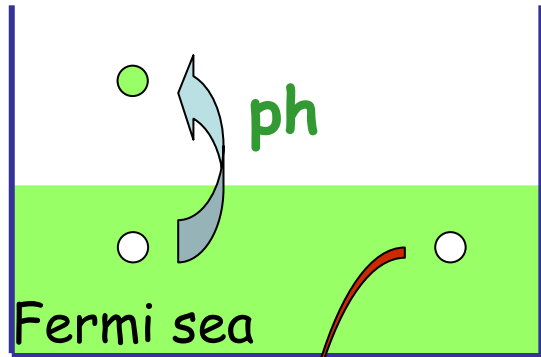


the same effective interaction determines the Dirac-Hartree single-particle spectrum and the residual interaction

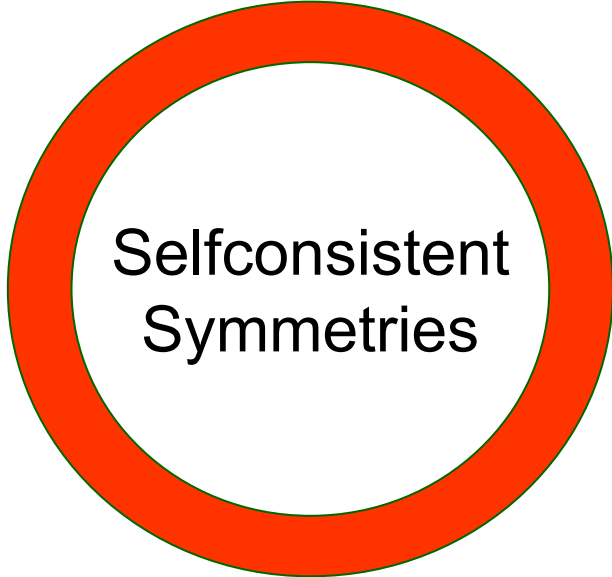
Interaction:

$$\hat{V} = \frac{\delta^2 E}{\delta \hat{\rho} \delta \hat{\rho}}$$

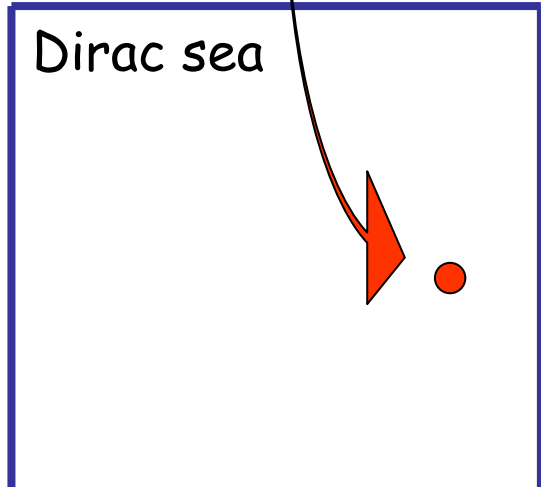
In superfluid systems: quasiparticle RPA



$$E_{ph} \approx 10 \text{ MeV}$$



$$E_{ah} \approx -1200 \text{ MeV}$$



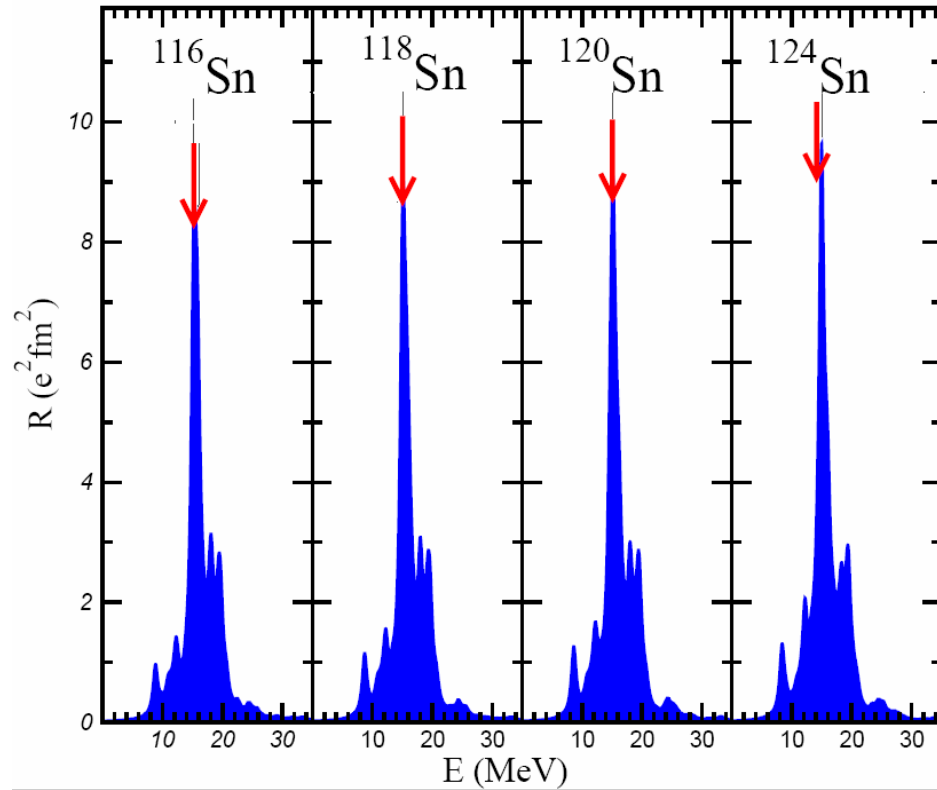
$$\langle ph | V_{\omega} | ah \rangle \approx 0 \quad \text{for vector mesons}$$

$$\langle ph | V_{\sigma} | ah \rangle \neq 0 \quad \text{for scalar mesons}$$

1. size of matrix.element
2. the same quantum numbers possible
3. large degeneracy

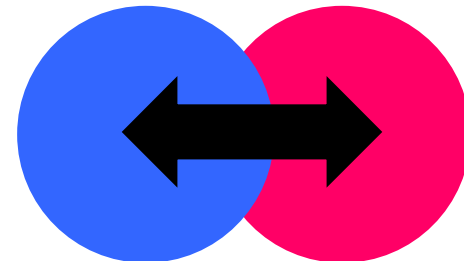
P. R. et al., NPA 694 (2001) 249

# Relativistic (Q)RPA calculations of giant resonances

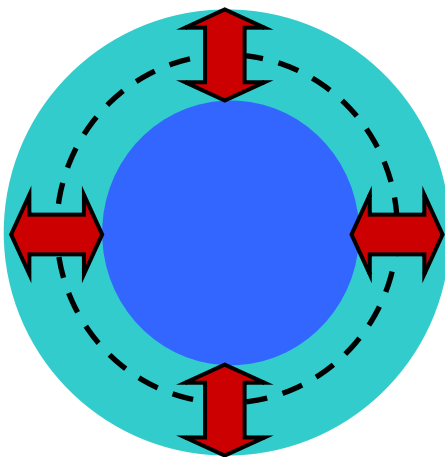


Isovector dipole response

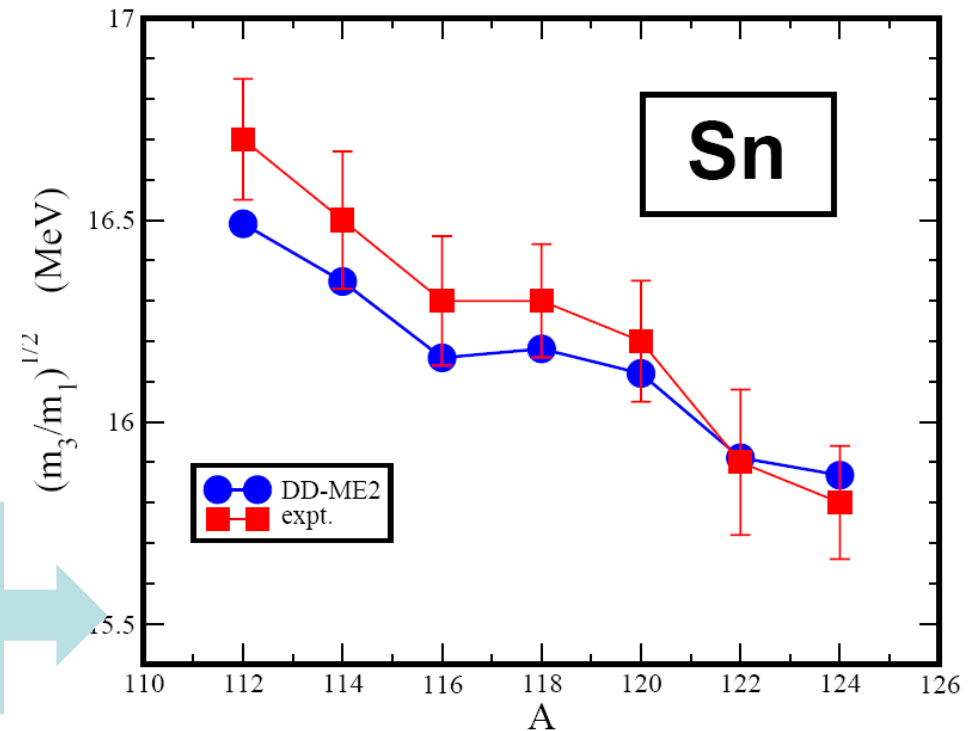
protons



neutrons



Isoscalar  
monopole  
response



# Linear response theory:

Strength function:  $S(\omega) = -\frac{1}{\pi} \text{Im} R_{FF}(\omega),$

Response function:  $R_{cc'}(\omega) = \sum_{\mu>0} \frac{\langle 0|Q_c^\dagger|\mu\rangle\langle\mu|Q_{c'}|0\rangle}{\omega - \Omega_\mu + i\eta} - \frac{\langle\mu|Q_c^\dagger|0\rangle\langle 0|Q_{c'}|\mu\rangle}{\omega + \Omega_\mu + i\eta}.$

without interaction:  $|\mu\rangle \rightarrow |ph\rangle, \quad \Omega_\mu \rightarrow \epsilon_p - \epsilon_h$

$$R_{cc'}^0(\omega) = \sum_n \frac{\langle h|Q_c^\dagger|p\rangle\langle p|Q_{c'}|h\rangle}{\omega - \epsilon_n + \epsilon_h} - \dots$$

separable interaction:  $V^{ph}(1,2) = \sum_c \int_0^\infty dr Q_c^{(1)}(r) v_c(r) Q_c^{\dagger(2)}(r)$

lin. Bethe Salpeter Eq:  $R(\omega) = R^0(\omega) + R^0(\omega)V^{ph}R(\omega)$

solution by inversion:

$$R(\omega) = \frac{1}{1 - R^0 V} R^0 = \frac{1}{R^{0-1} - V}$$

in the ph-basis:

$$= \frac{1}{\omega - \begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}}$$

Treatment of the continuum:

Bertsch 1974

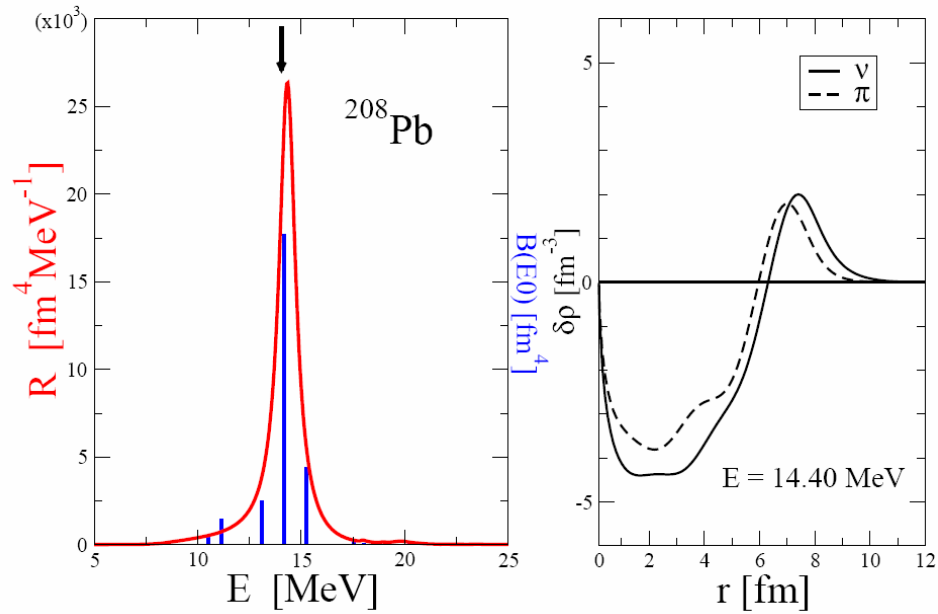
$$\begin{aligned} R_{cc'}^0(\omega) &= \sum_{ph} \frac{\langle h | Q_c^\dagger | p \rangle \langle p | Q_{c'} | h \rangle}{\omega - \epsilon_p + \epsilon_h} - \dots \\ &= \sum_h \langle h | Q_c^\dagger \frac{1}{\omega + \epsilon_h - \hat{h}_D} Q_{c'} | h \rangle - \dots \\ &= \sum_h \langle h | Q_c^\dagger G(\omega + \epsilon_h) Q_{c'} | h \rangle - \dots \end{aligned}$$

single particle Greens function:

$$G(E) = \frac{1}{E - \hat{h}_D} \quad \left( E - \hat{h}_\kappa(r) \right) G_\kappa(r, r'; E) = \delta(r - r'),$$

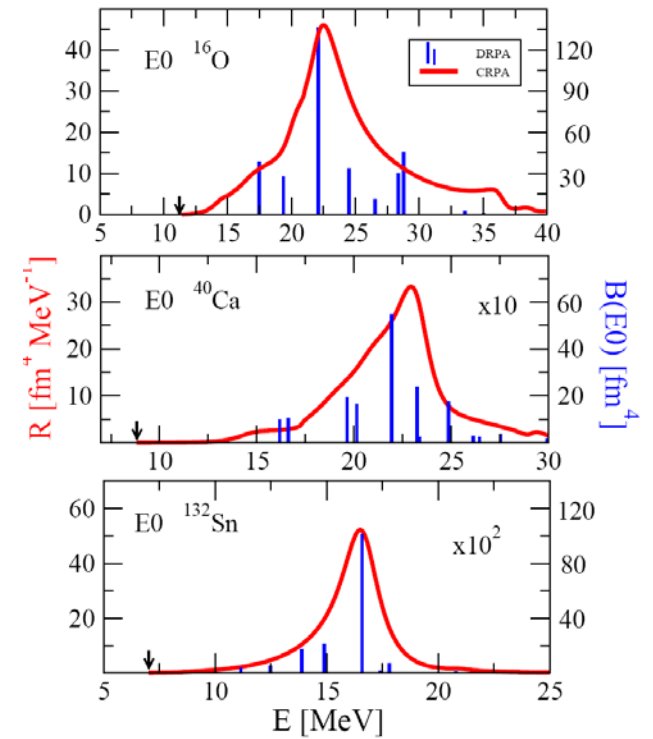
# Applications: E0

ISGMR

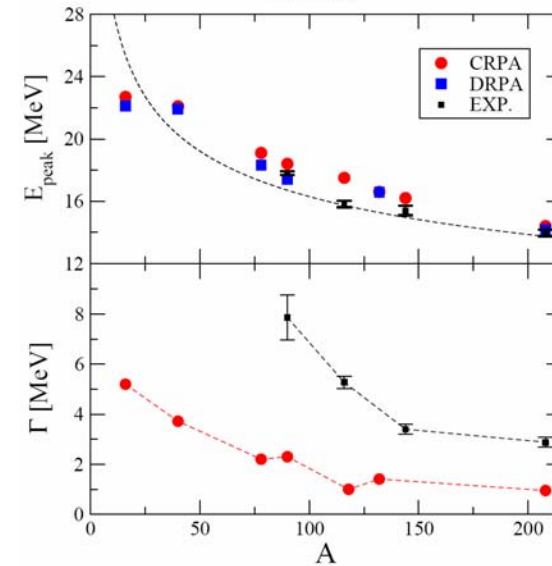


peak energy:

escape width:

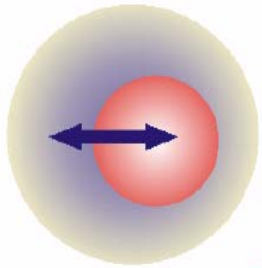


ISGMR





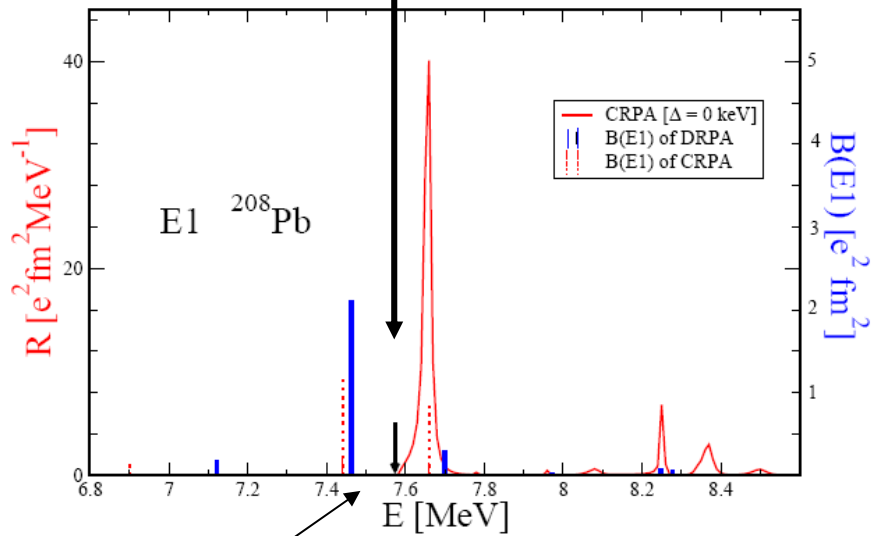
# IV-dipole strength



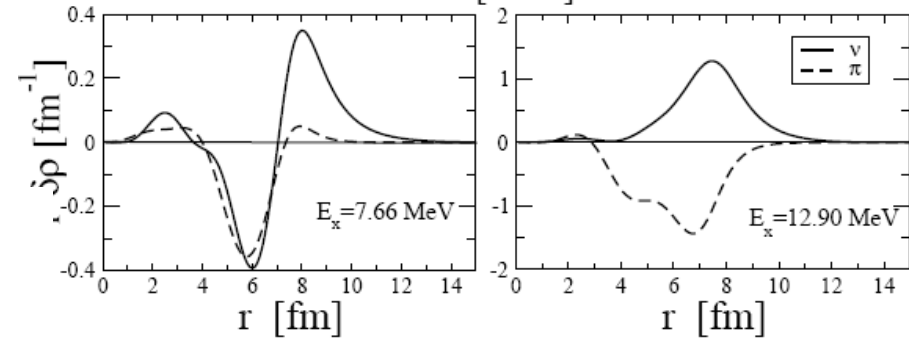
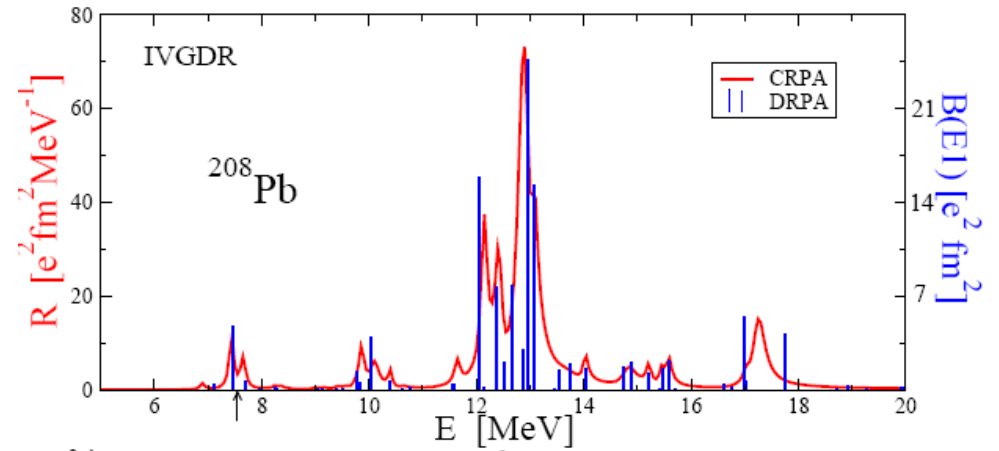
$^{208}\text{Pb}$

pygmy-mode

neutron threshold:



pygmy-region



| No. | CRPA |       | DRPA |       |
|-----|------|-------|------|-------|
|     | E    | B(E1) | E    | B(E1) |
| 1   | 6.90 | 0.19  | 7.12 | 0.23  |
| 2   | 7.44 | 1.45  | 7.46 | 2.82  |
| 3   | 7.66 | 1.11  | 7.69 | 0.40  |
| Σ   |      | 2.75  |      | 3.45  |

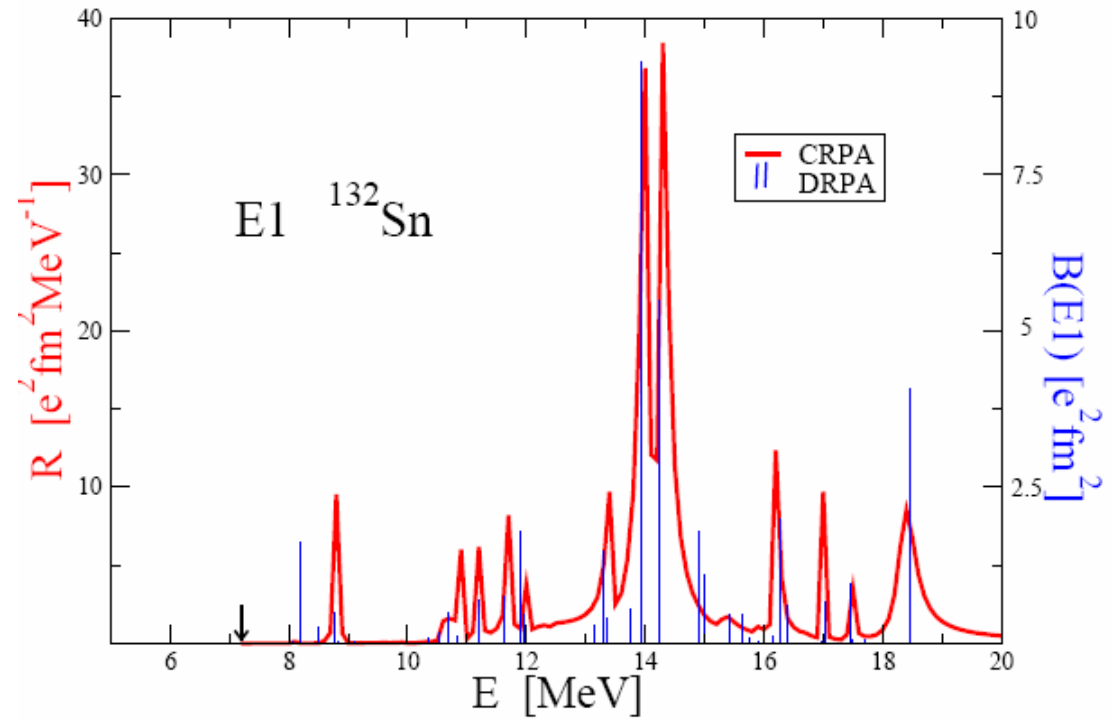
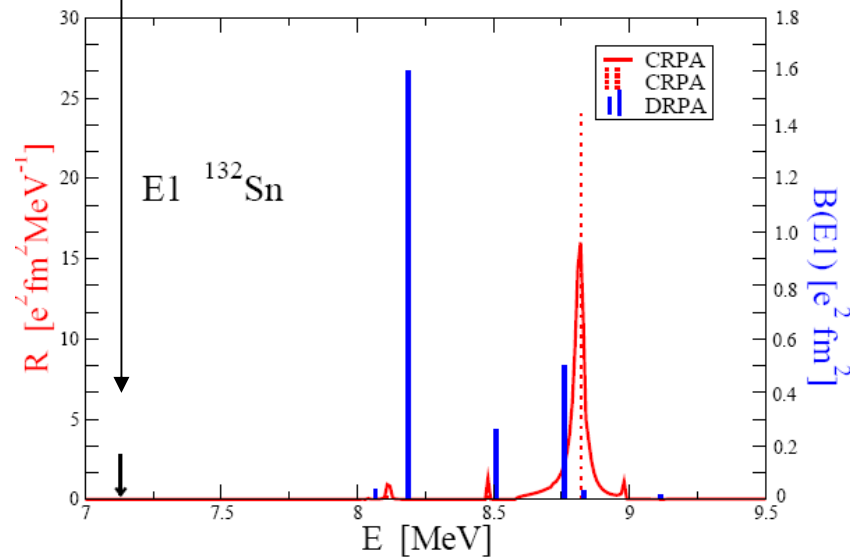
1.9 %

2.8 % of EWSR

# IV-dipole strength

$^{132}\text{Sn}$

neutron threshold:

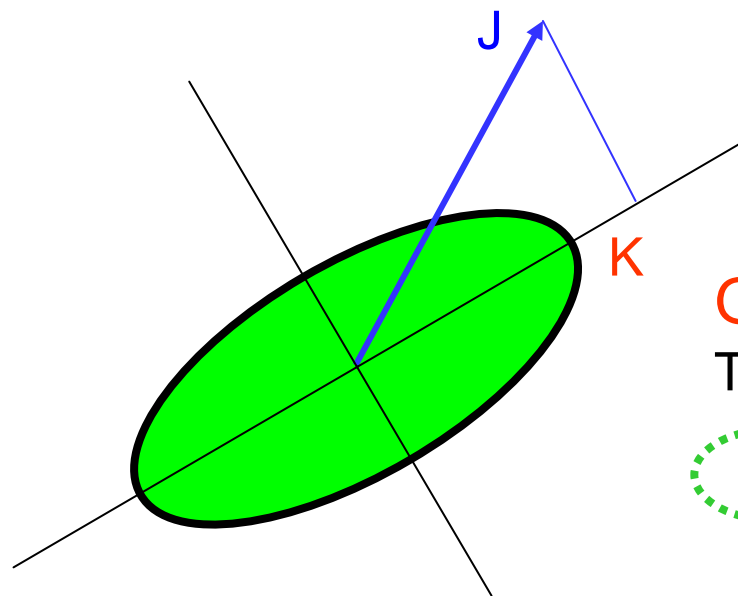


| No.      | CRPA |       | DRPA  |       |
|----------|------|-------|-------|-------|
|          | E    | B(E1) | E     | B(E1) |
| 1        | 8.11 | 0.03  | 8.067 | 0.037 |
| 2        | 8.48 | 0.02  | 8.186 | 1.601 |
| 3        | 8.82 | 1.44  | 8.511 | 0.260 |
| $\Sigma$ |      | 1.490 |       | 1.898 |

2.4 %

3.4 % of EWSR

# Vibrations in deformed nuclei

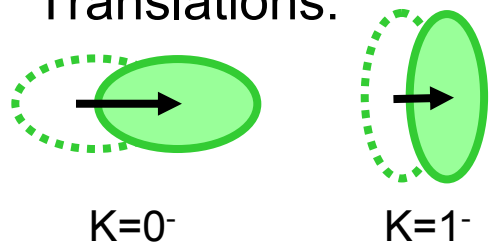


T=0

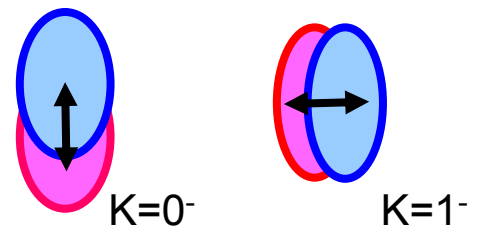
T=1

## Goldstone modes

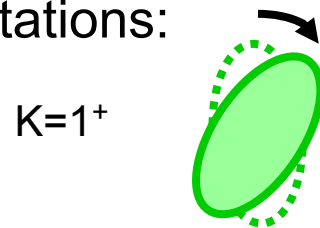
Translations:



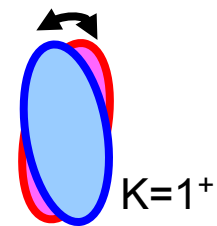
Giant dipole modes:



Rotations:

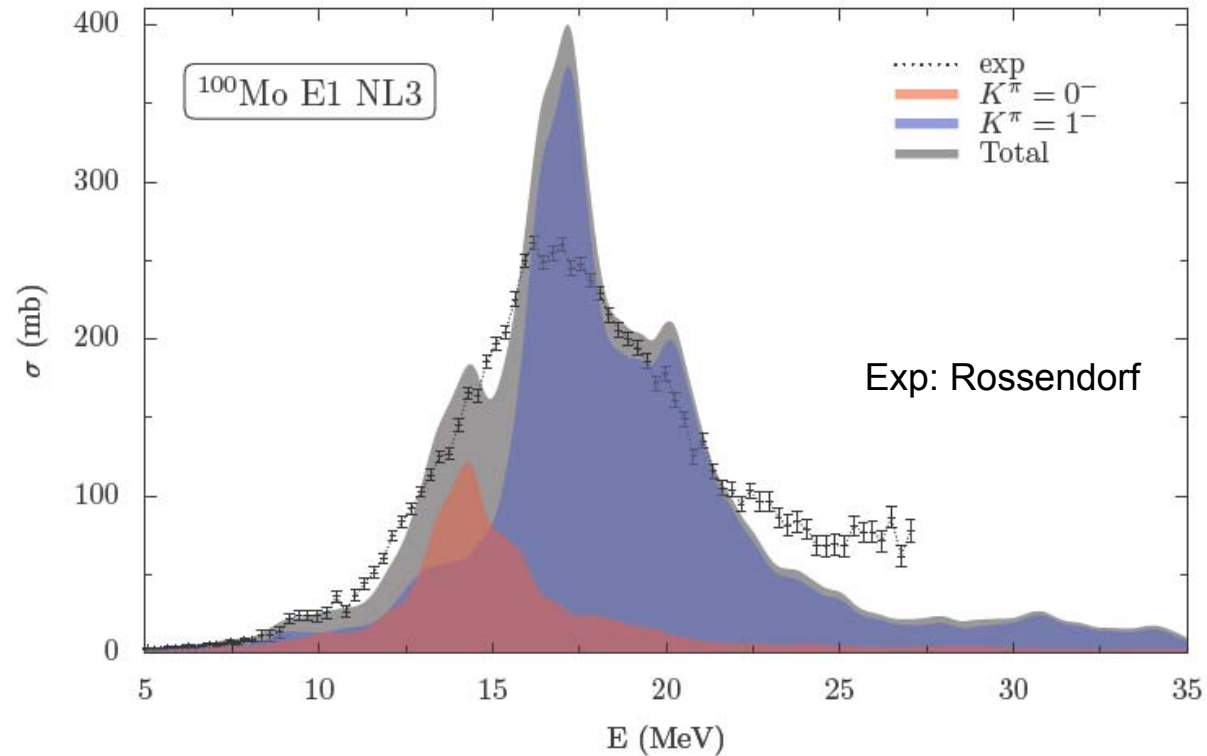


Scissor modes:



Gauge rotations: K=0+

# isovector-dipole response in $^{100}\text{Mo}$



|        | $m_1/m_0$ | $\sqrt{\frac{m_3}{m_1}} - \frac{m_1}{m_0}$ | $\bar{E}_{E1}$ | $\Gamma_{E1}$ |
|--------|-----------|--|----------------|---------------|
| NL3    | 17.4      | 1.4  | 17.2           | 2.4           |
| DD-ME2 | 18.3      | 1.2  | 18.5           | 3.3           |
| Exp    | 18.0      | 1.4  | 17.4           | 3.6           |

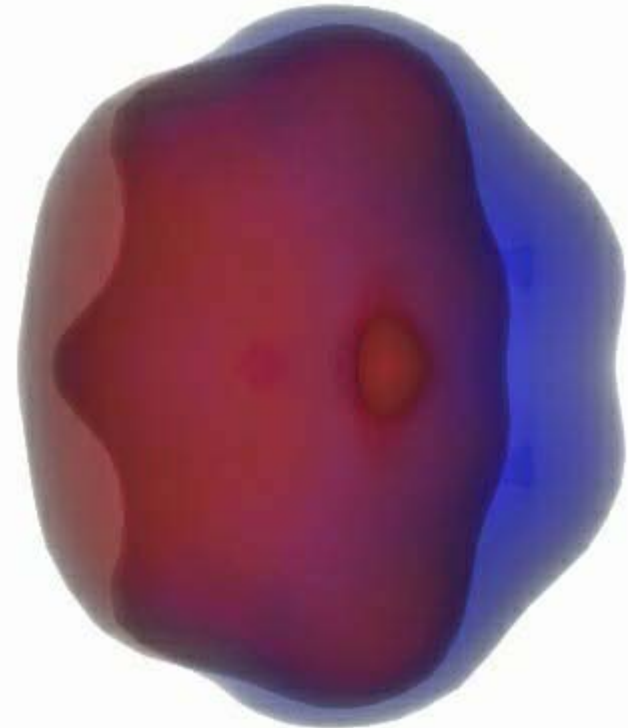
in MeV

# IV-GDR in $^{100}\text{Mo}$

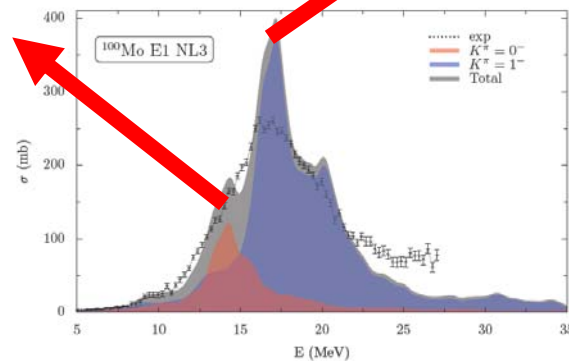


IV-GDR

$$\rho_0 + \delta\rho(t)$$

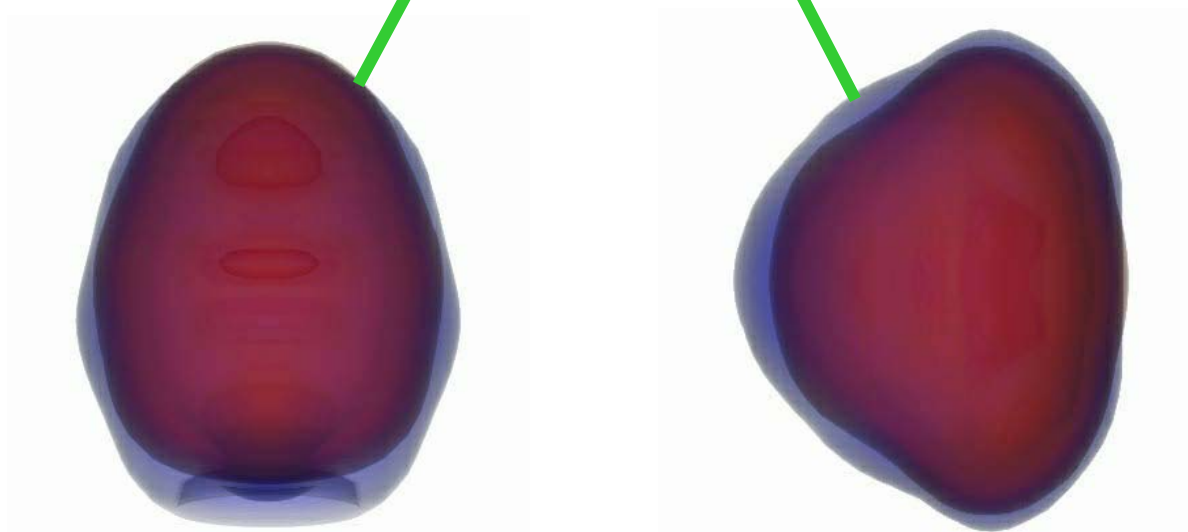
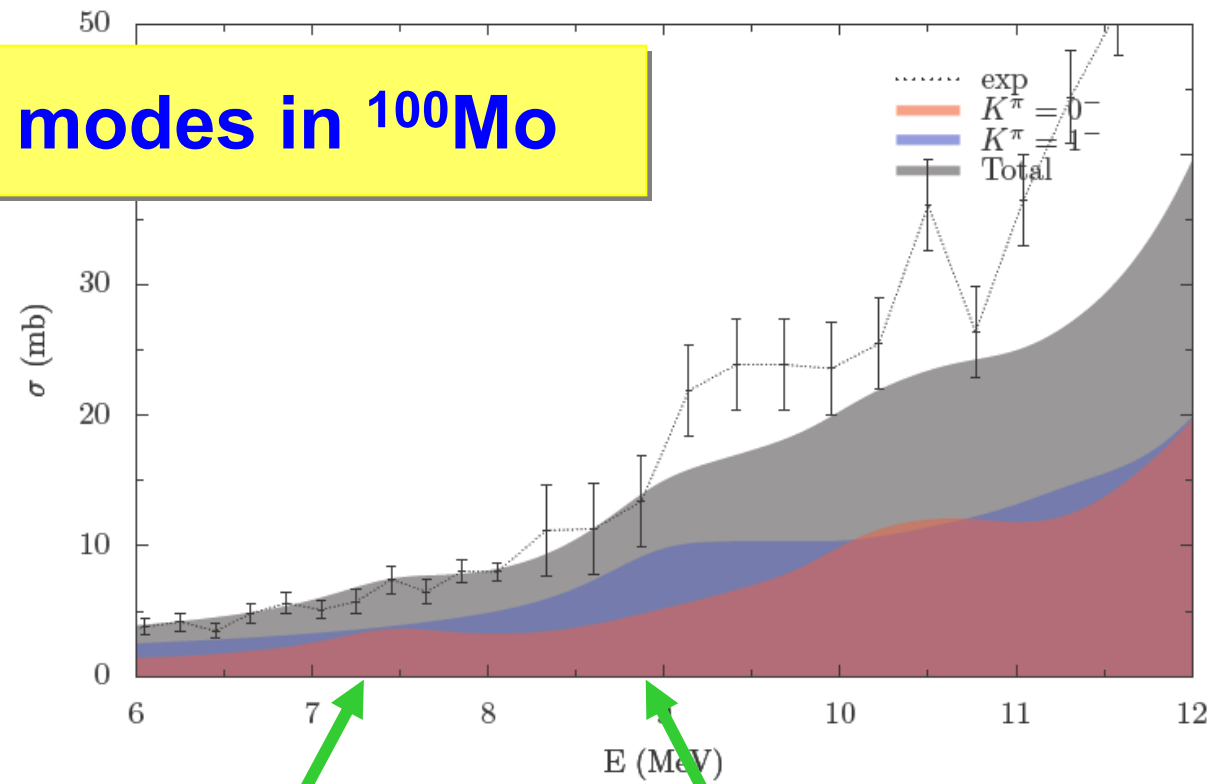


$K=0^-$

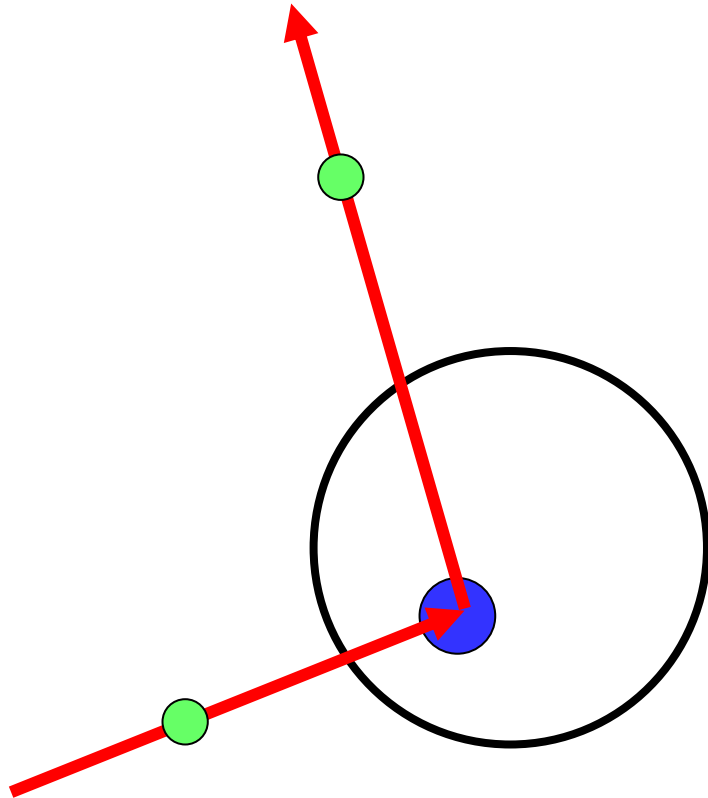


$K=1^-$

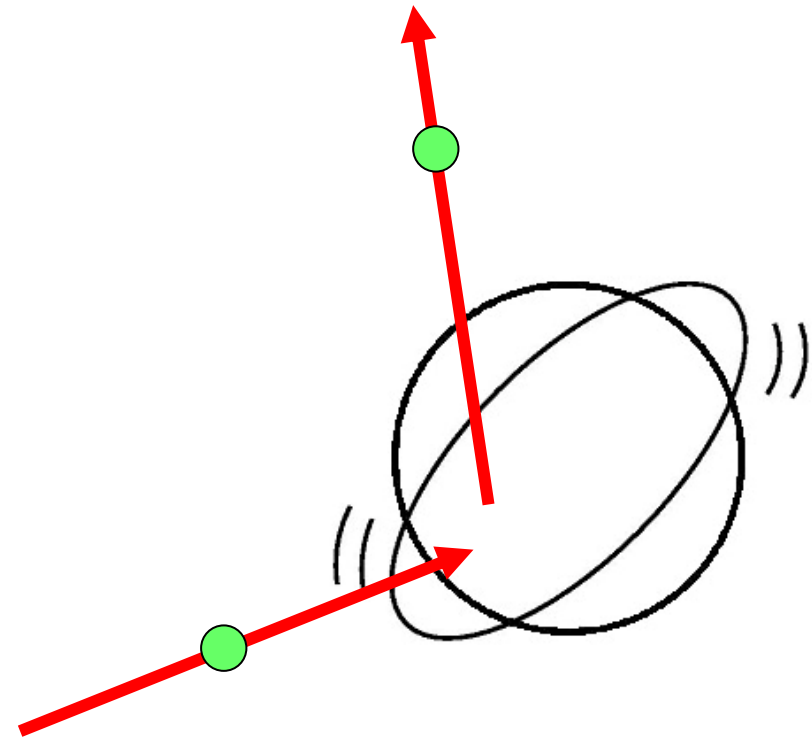
# pygmy modes in $^{100}\text{Mo}$



# response of the nucleus to an incoming particle

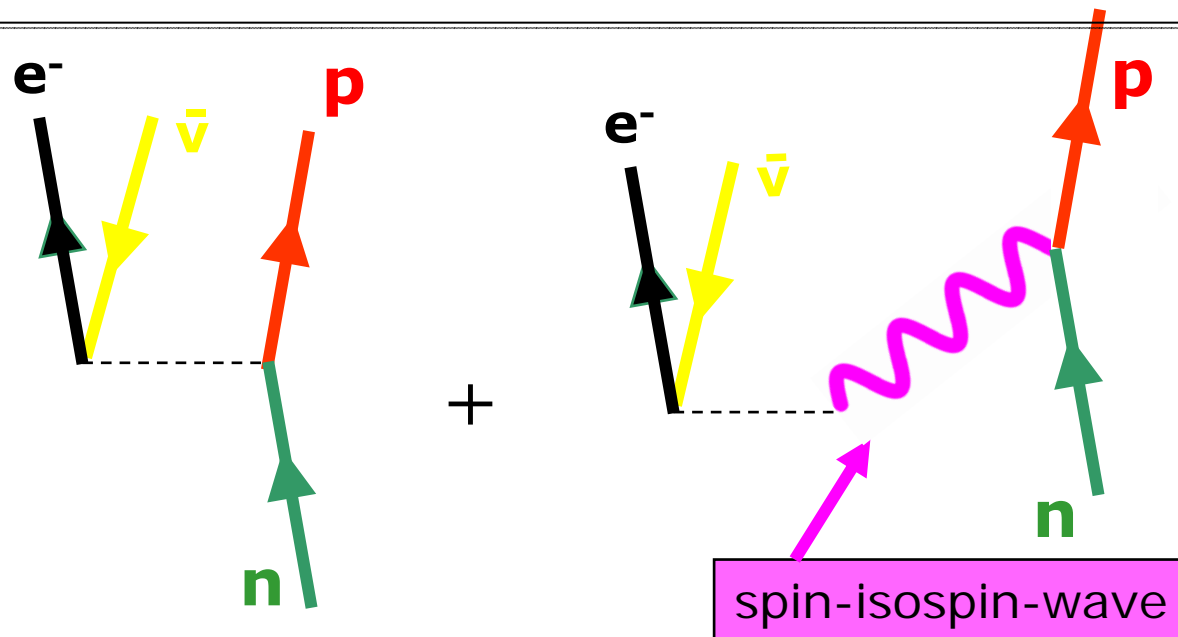
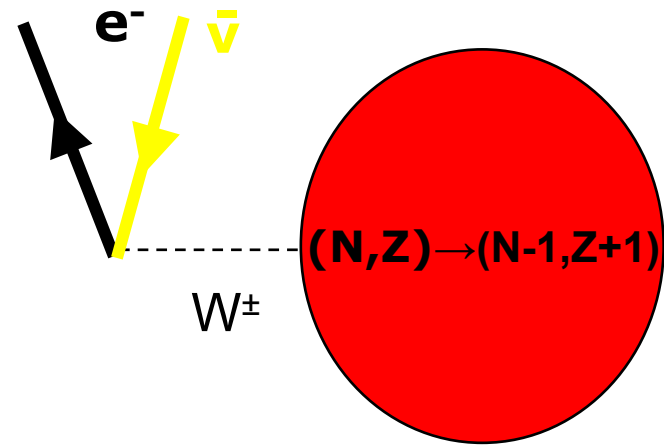


scattering at a single nucleon



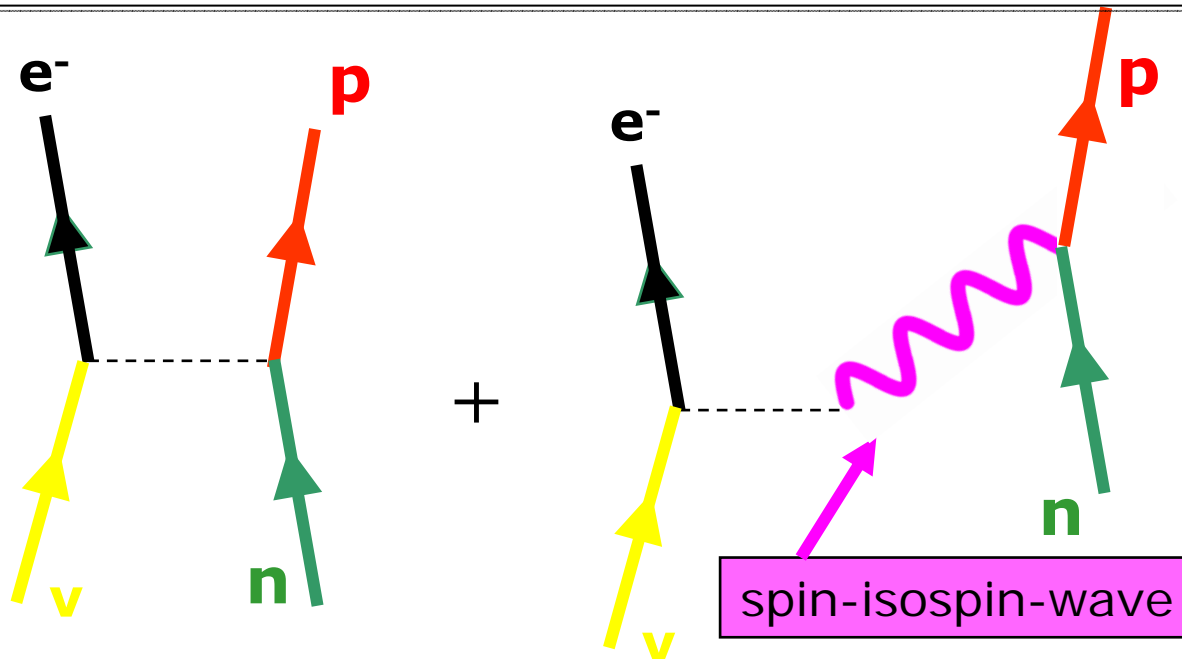
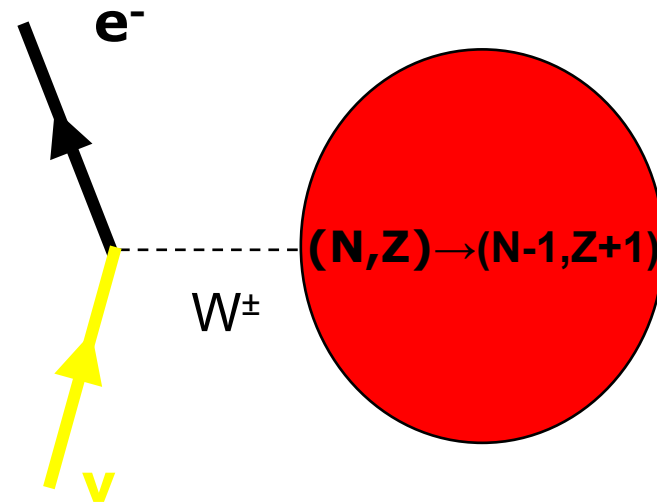
excitation of the entire nucleus  
we need the nuclear spectrum

# beta-decay





# neutrino scattering

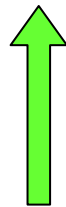


# Spin-Isospin Resonances: IAR - GTR

$Z, N$

$Z+1, N-1$

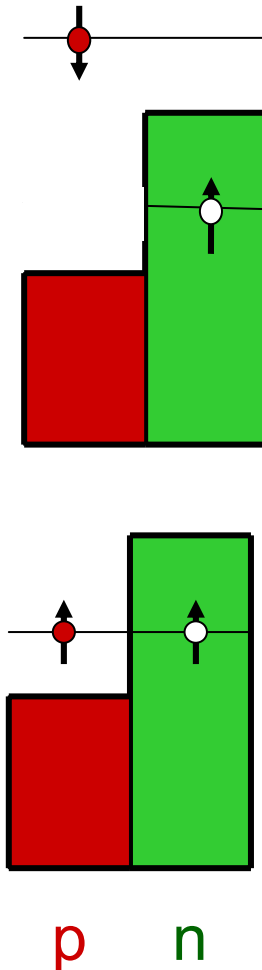
$$|GTR\rangle = S_- T_+ |Z, N\rangle$$



spin flip  $\sigma$

$$|Z, N\rangle \longrightarrow |IAR\rangle = T_+ |Z, N\rangle$$

isospin flip  $\tau$



$$E_{GTR} - E_{IAR} \sim \Delta(l \cdot s) \sim \frac{dV}{dr} \sim \text{neutron skin} = r_n - r_p$$

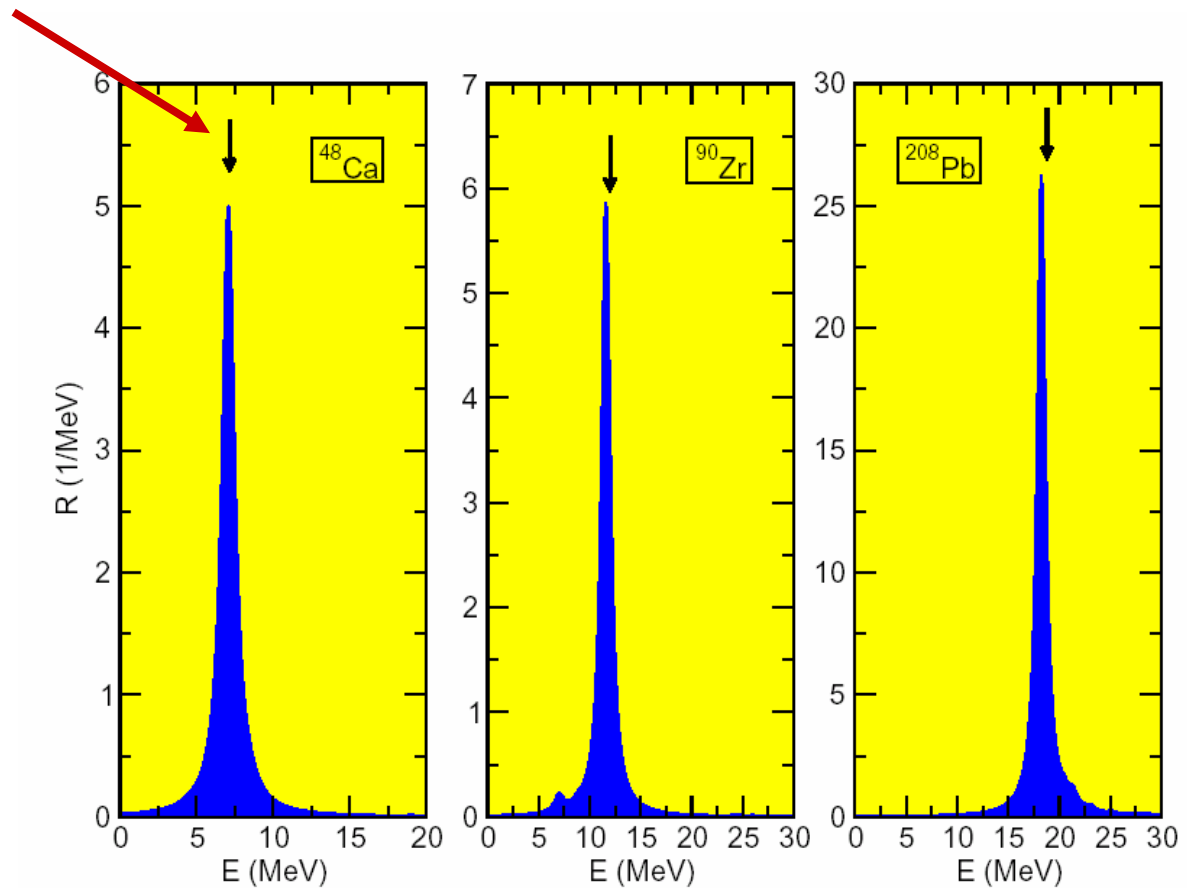
# Isobaric Analog Resonance: IAR

PR C69, 054303 (2004)

experiment

Isospin-flip  
excitations

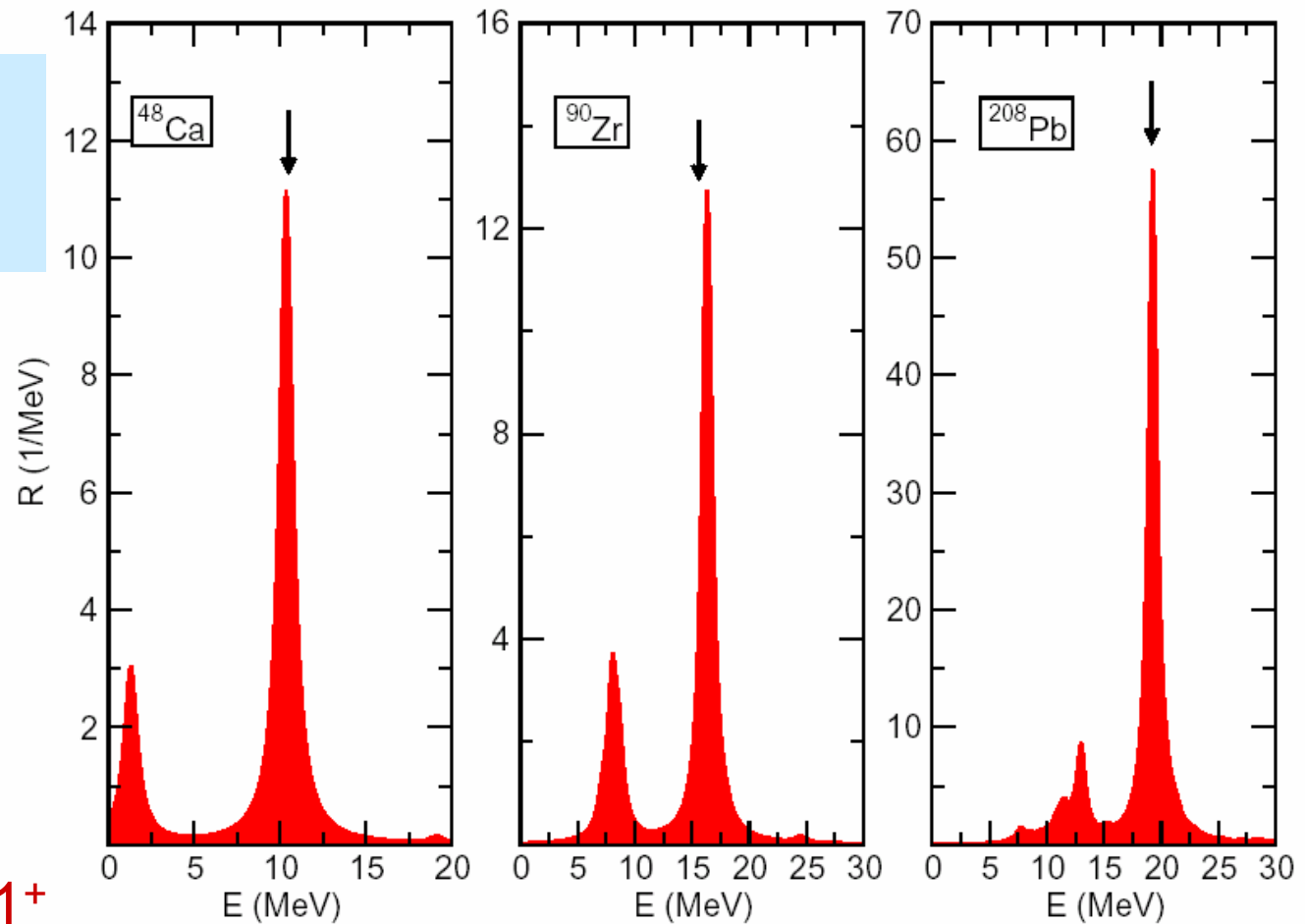
$S=0$   $T=1$   
 $J^\pi = 0^+$



# GT-Resonances

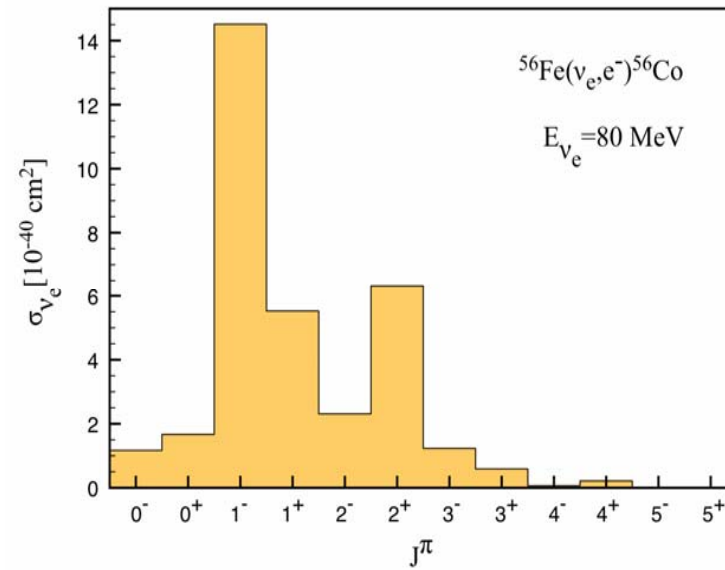
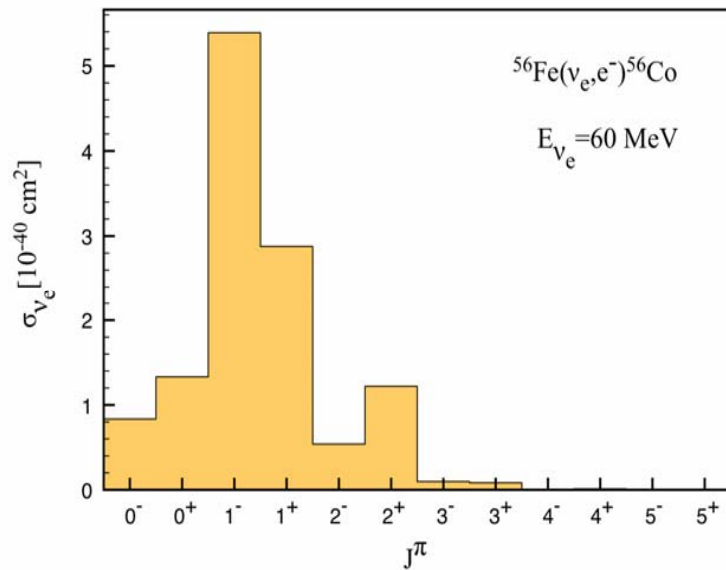
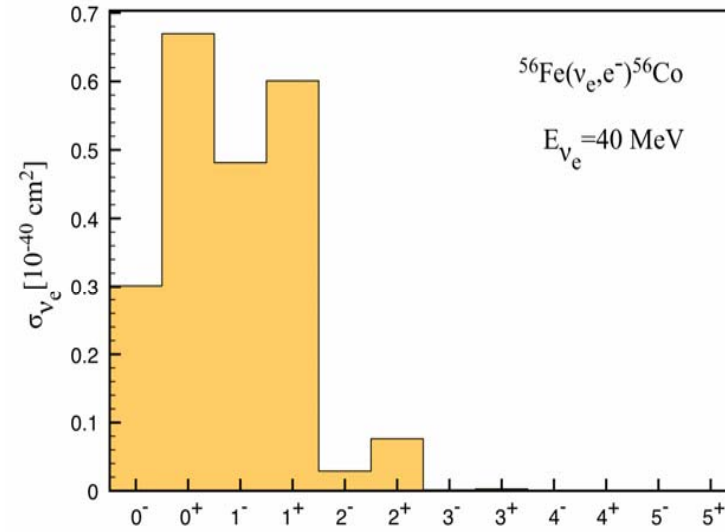
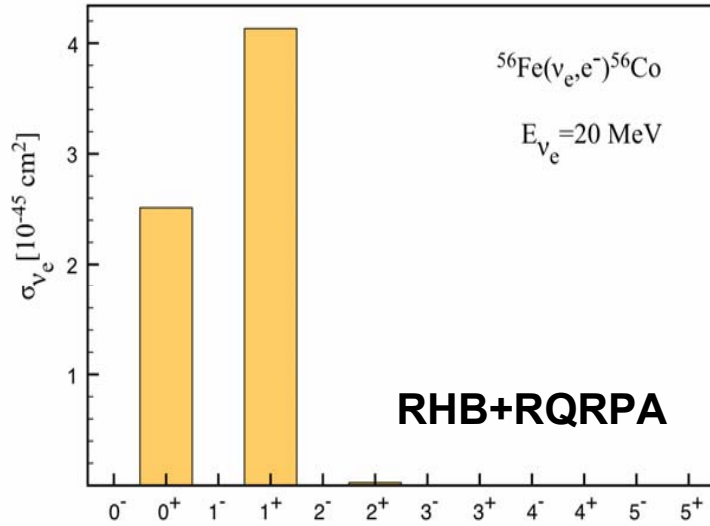
PR C69, 054303 (2004)

Spin-flip  
isospin-flip  
excitations



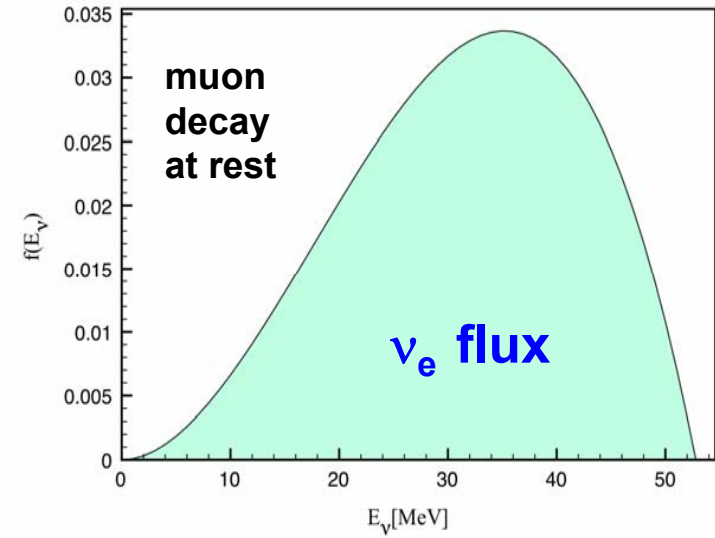
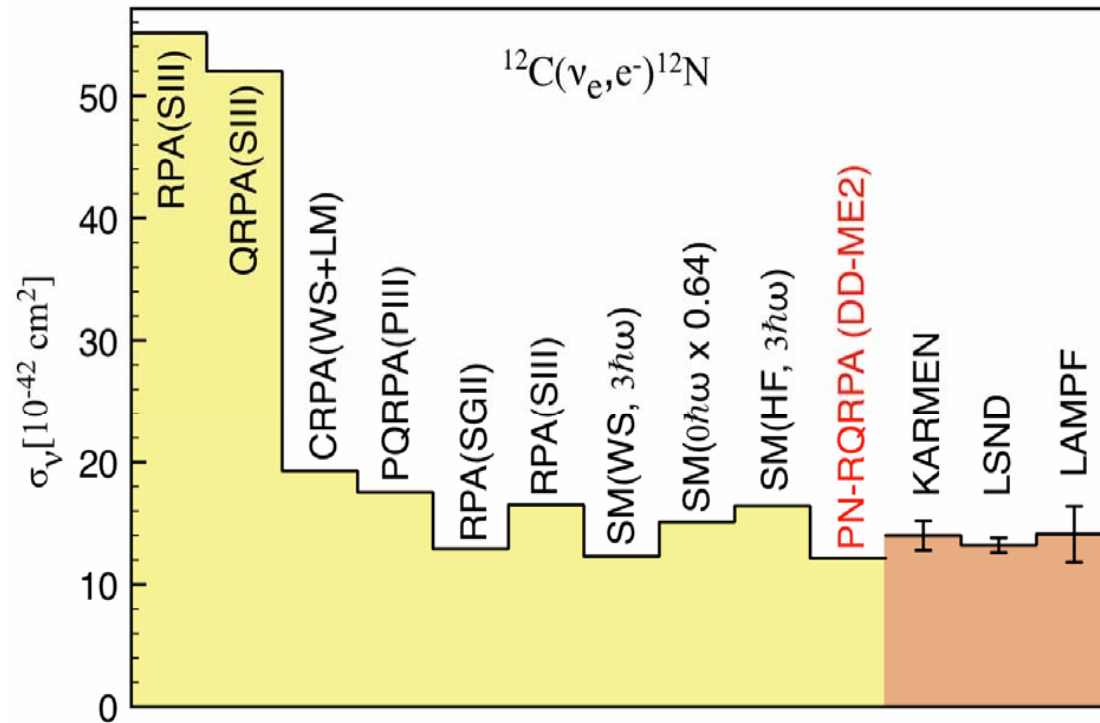
$S=1$   $T=1$   $J^\pi = 1^+$

# Distribution of cross section over multipolarities



# Cross section ( $\nu_e, e^-$ ) averaged over supernova neutrino flux

$$\langle \sigma_\nu \rangle = \frac{\int dE_\nu \sigma_\nu(E_\nu) f(E_\nu)}{\int dE'_\nu f(E'_\nu)}$$



- **Static density functionals in nuclei**
- **Time dependent density functionals in nuclei**
- **Quasiparticle RPA**
- **Applications:**
  - relativistic RPA
  - continuum RPA
  - deformed RPA
  - pn - RPA
- **Extensions**
  - energy dependent KS-fields
  - Mixing of deformed configurations
  - Symmetry restoration before variation
- **Conclusions and outlook**

## Problems with mean field:

- **no fluctuation** in transitional nuclei
- **no energy dependence** of the self energy
- **symmetry violations** have to be restored
- **no spectroscopy**
- .....

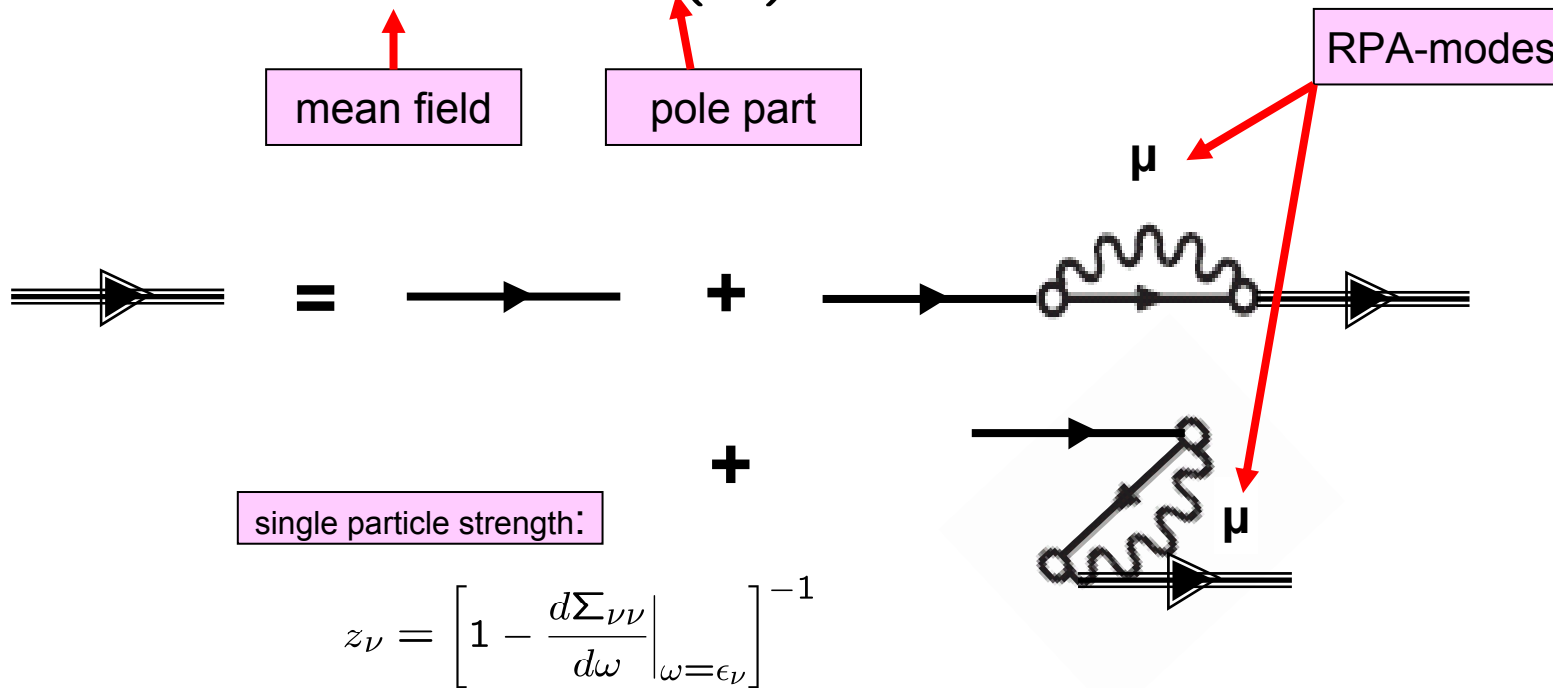


# Particle-vibrational coupling: energy dependent self-energy

$$\Sigma = S + V + \Sigma(\omega)$$

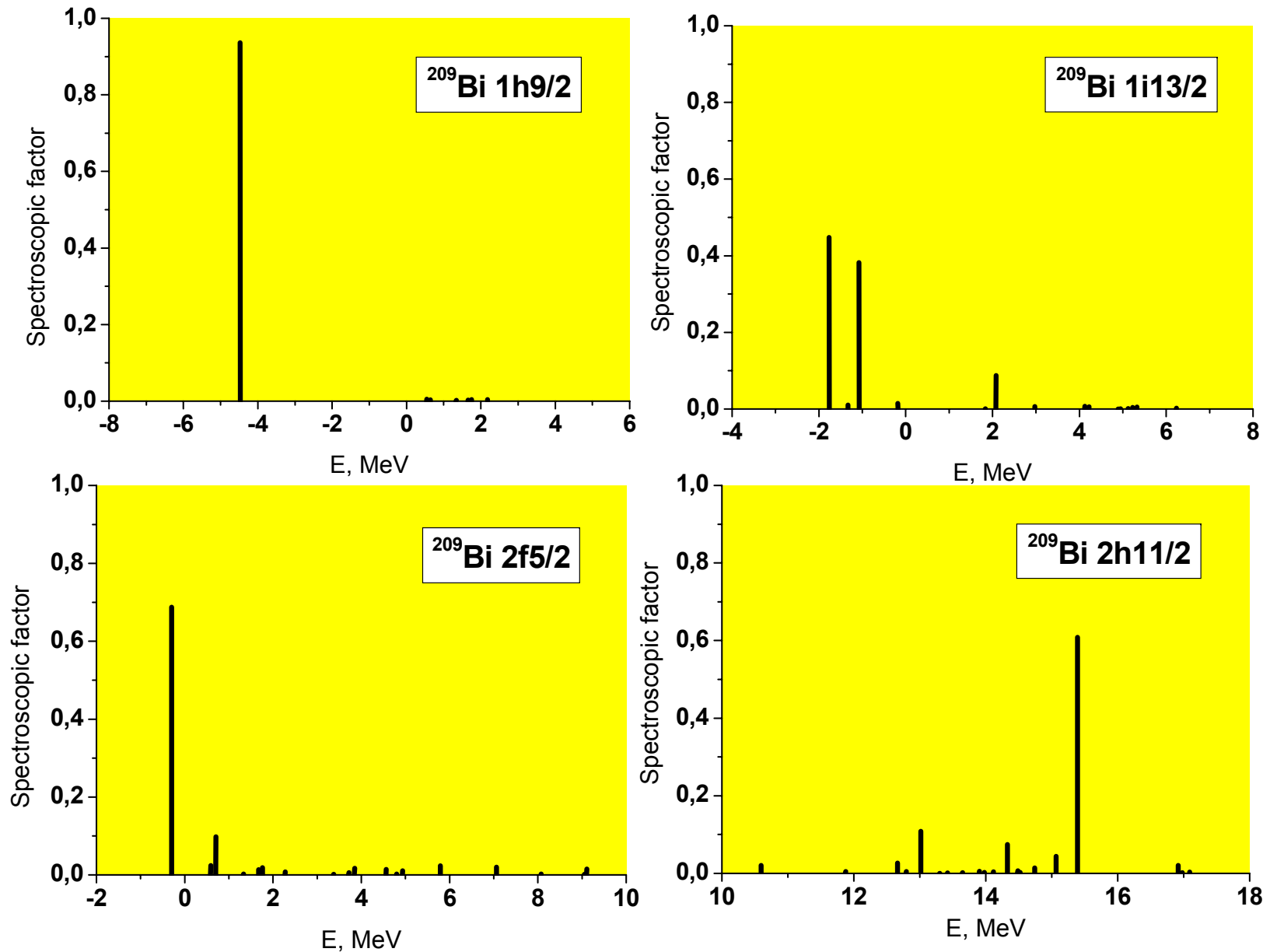
mean field

pole part

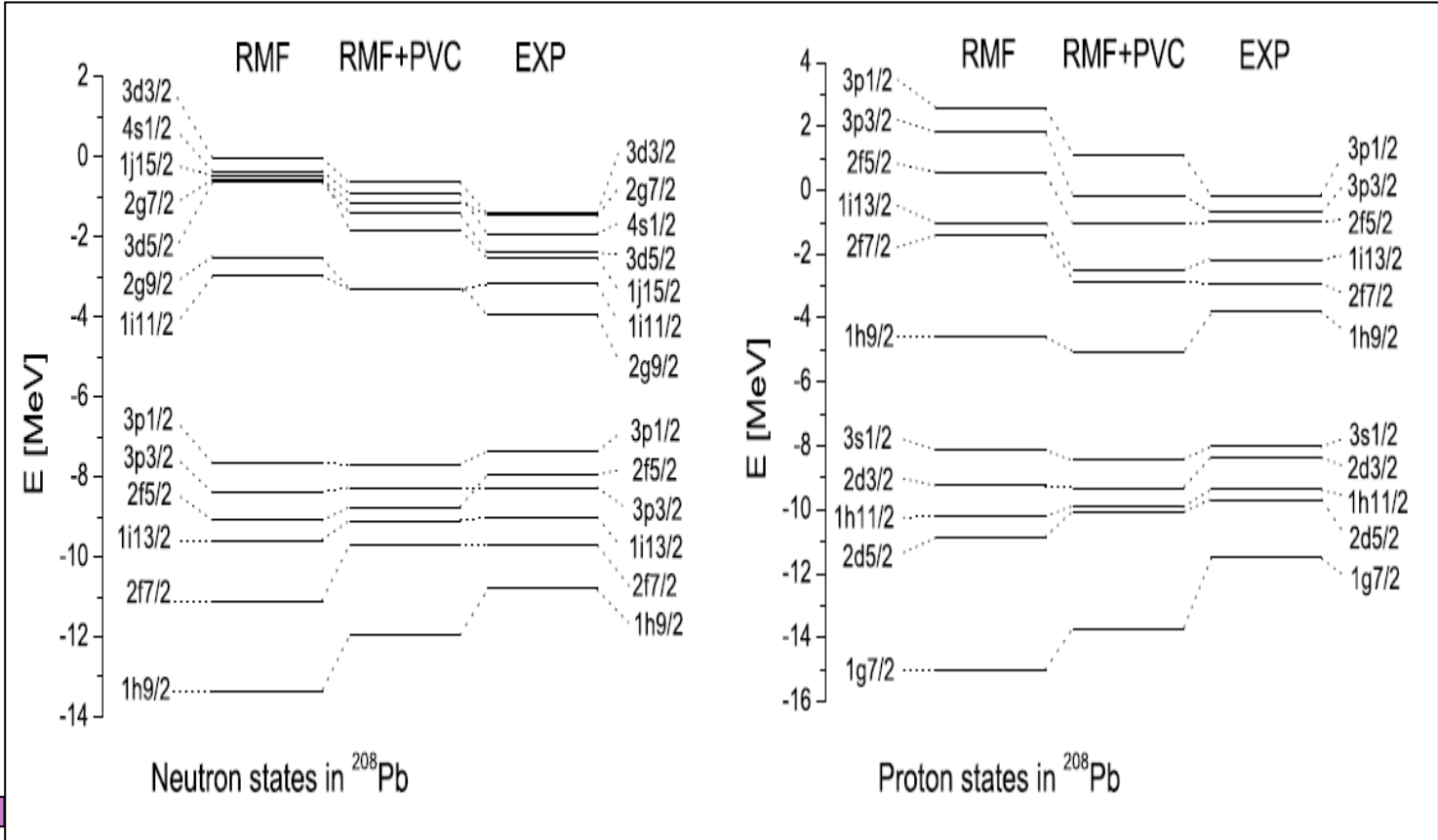



Density functional theory - Landau-Migdal theory

# Distribution of single-particle strength in $^{209}\text{Bi}$



# Single particle spectrum in the Pb region




 **$m_{\text{eff}}$     0.76    0.92    1.0                      0.71    0.85    1.0**

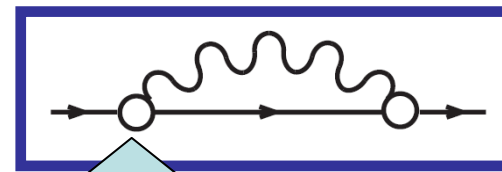
E. Litvinova, P.R., PRC 73, 44328 (2006)

# Two-body problem: from self-energy to effective interaction

$$U(14, 23) = i \frac{\delta \Sigma(3, 4)}{\delta G(1, 2)}$$

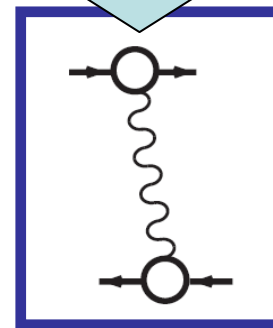
$$\Sigma_{12}^e(\varepsilon) = \sum_{3, \eta, m} \frac{\delta_{\eta, \eta_3} g_{13}^{m(\eta)*} g_{23}^{m(\eta)}}{\varepsilon - E_3 - \eta (\omega_m - i \cdot 0)}$$

$$\Sigma = \tilde{\Sigma} + \Sigma^e$$

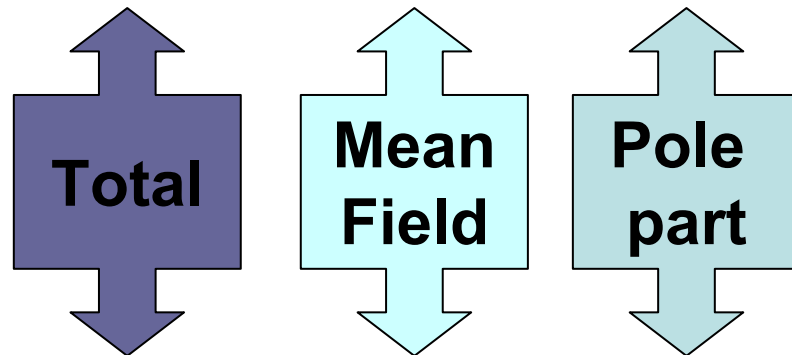


**Mass operator**

$g$  – phonon amplitudes (QRPA)

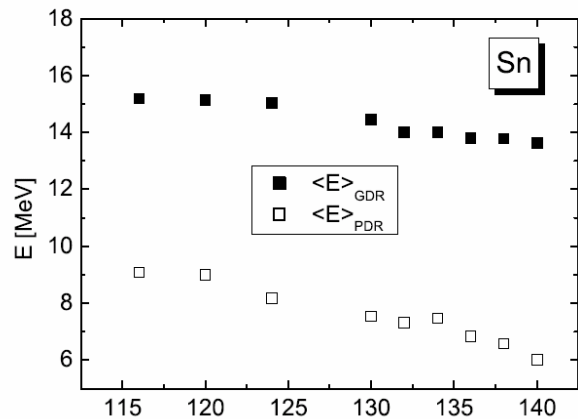
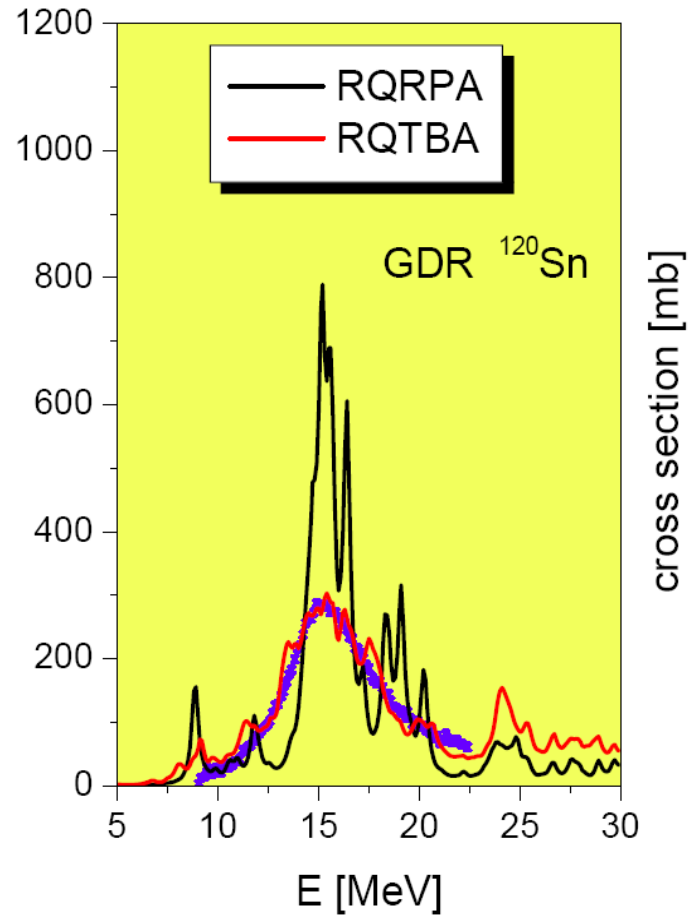
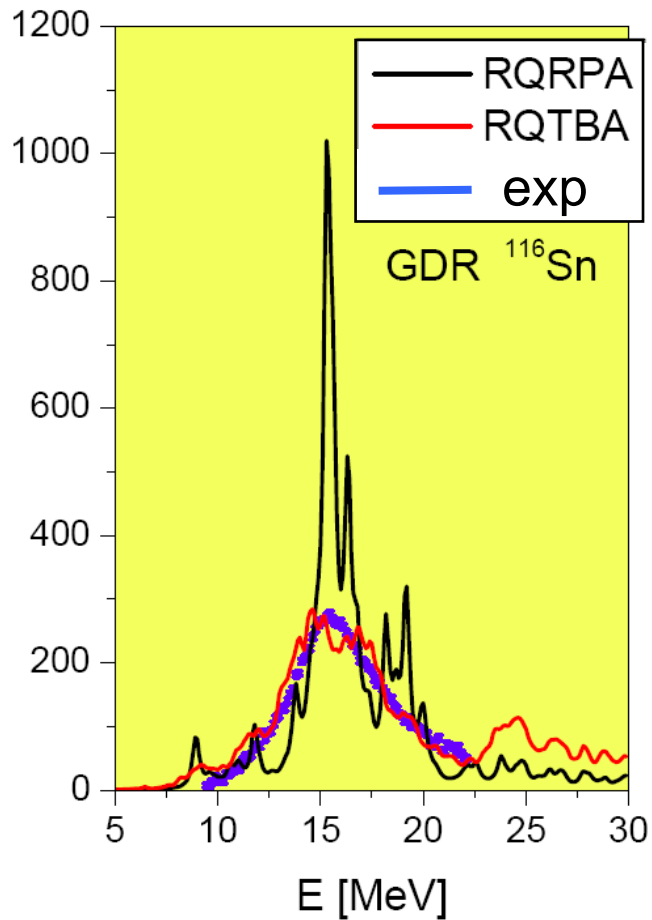


**Induced p-h interaction amplitude**



$$\mathcal{U} = \tilde{\mathcal{U}} + \mathcal{U}^e$$

$$\mathcal{U}_{12,34}^e(\omega, \varepsilon, \varepsilon') = \sum_{\eta, m} \frac{\eta g_{31}^{m(\eta)*} g_{42}^{m(\eta)}}{\varepsilon - \varepsilon' + \eta (\omega_m - i \cdot 0)}$$



centroid energies for  
GDR and PDR

Litvinova, P.R. Tselyaev, PRC 78, 14312 (2008)

Litvinova, P.R. Tselyaev, Langanke, PRC 79, 054312 (2009)

## Parameters of Lorentz distribution\* (GDR)

|                                       |          | $\langle E \rangle$ (MeV) | $\Gamma$ (MeV) | EWSR (%) |
|---------------------------------------|----------|---------------------------|----------------|----------|
| <b><math>^{208}\text{Pb}</math></b> → | RRPA     | 12.9                      | 2.0            | 128      |
|                                       | RRPA-PC  | 13.7                      | 4.3            | 134      |
|                                       | Exp. [1] | 13.4                      | 4.1            |          |
| <b><math>^{132}\text{Sn}</math></b> → | RRPA     | 14.5                      | 2.6            | 126      |
|                                       | RRPA-PC  | 15.1                      | 4.4            | 131      |
|                                       | Exp. [2] | 16.1(7)                   | 4.7(2.1)       |          |
| <b><math>^{48}\text{Ni}</math></b> →  | RRPA     | 17.9                      | 3.1            | 119      |
|                                       | RRPA-PC  | 18.6                      | 5.1            | 125      |
| <b><math>^{46}\text{Fe}</math></b> →  | RRPA     | 17.9                      | 3.2            | 122      |
|                                       | RRPA-PC  | 18.7                      | 5.5            | 128      |

\*Averaging interval: 0-30 MeV

[1] Reference Input Parameter Library, Version 2

[2] Adrich et al., PRL **95**, 132501 (2005).

;

## Generator Coordinate Method (GCM)

$$\langle \delta\Phi | \hat{H} - q\hat{Q} | \Phi \rangle = 0$$



$$|q\rangle = |\Phi(q)\rangle$$

Constraint Hartree Fock produces wave functions depending on a **generator coordinate  $q$**

$$|\Psi\rangle = \int dq f(q) |q\rangle$$

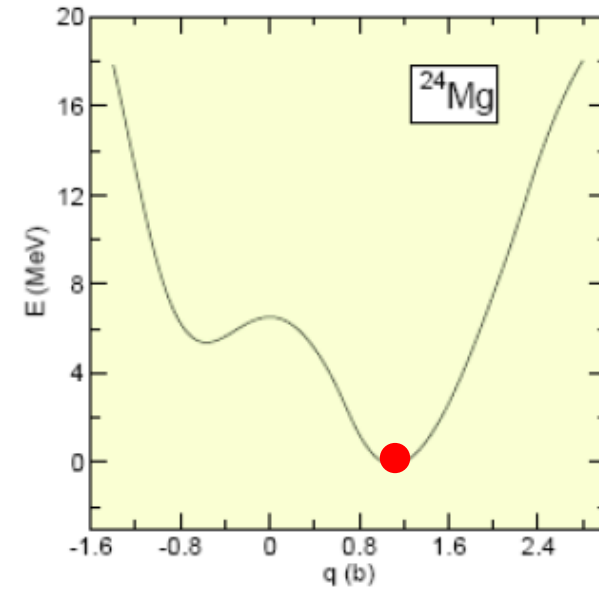
GCM wave function is a **superposition of Slater determinants**

$$\int dq' [\langle q | H | q' \rangle - E \langle q | q' \rangle] f(q') = 0$$

Hill-Wheeler equation:

with projection:

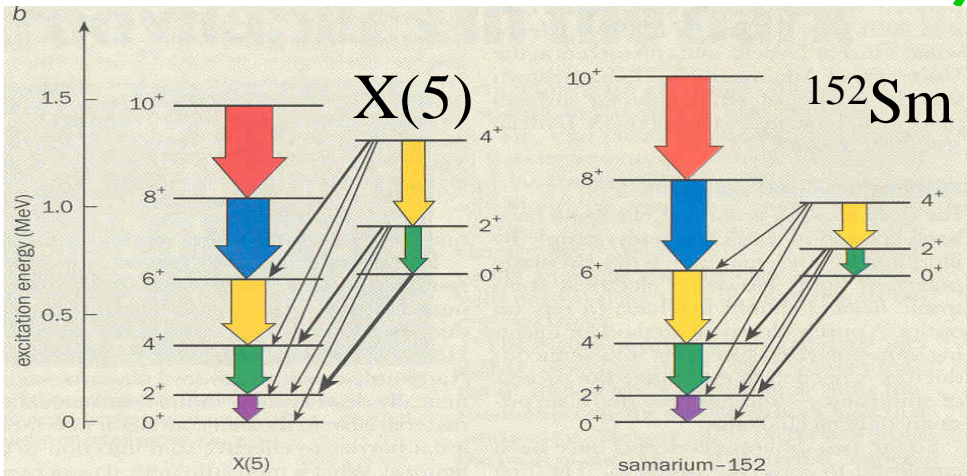
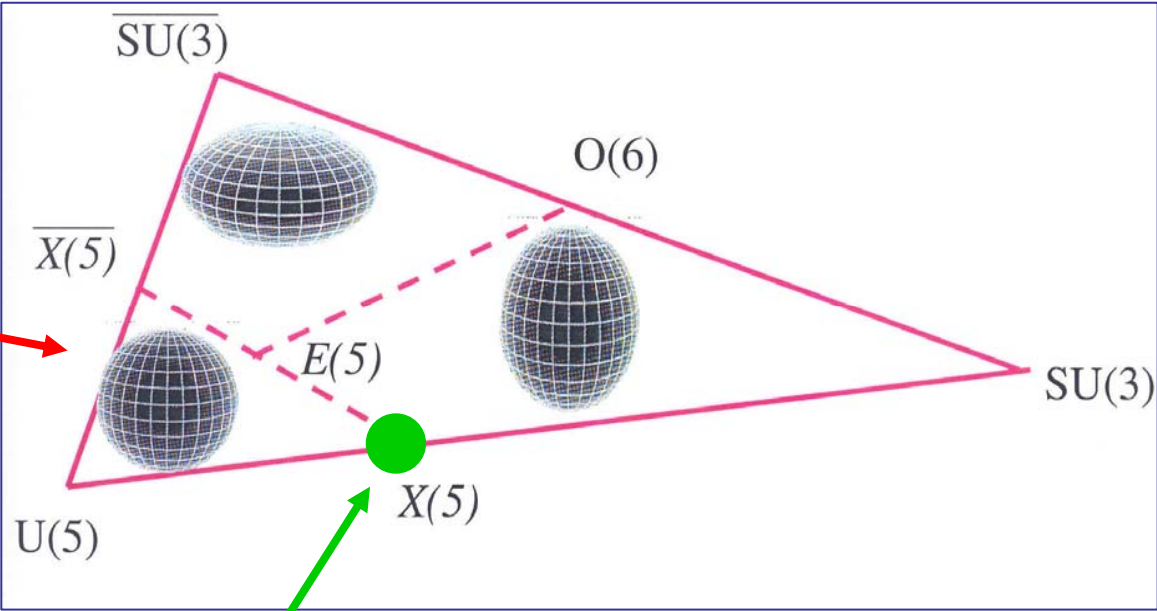
$$|\Psi\rangle = \int dq f(q) \hat{P}^N \hat{P}^I |q\rangle$$



# Quantum phase transitions and critical symmetries

**Interacting Boson Model**

**Casten Triangle**



E(5): F. Iachello, PRL 85, 3580 (2000)  
 X(5): F. Iachello, PRL 87, 52502 (2001)

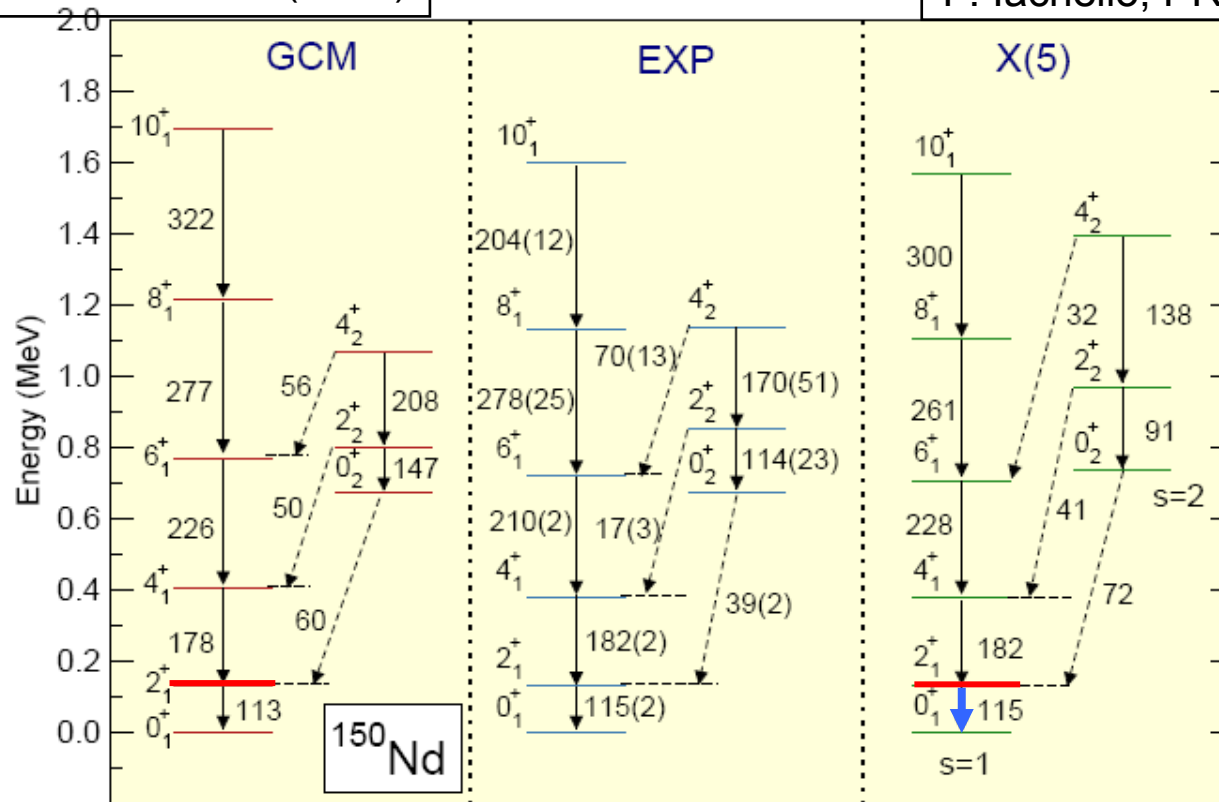
R.F. Casten, V. Zamfir, PRL 85 3584, (2000)



R. Krücken *et al*, PRL 88, 232501 (2002)

Niksic *et al* PRL 99, 92502 (2007)

F. Iachello, PRL 87, 52502 (2001)



**GCM: only one scale parameter:**

$E(2_1)$

**X(5): two scale parameters:**

$E(2_1)$ ,  $BE2(2_2 \rightarrow 0_1)$

**Problem of GCM at this level:**

**restricted to  $\gamma=0$**

## AGP and number projected HFB:

AGP is equivalent to number projected HFB  
(variation after projection: VAP)

$$|AGP\rangle = (C^\dagger)^{\frac{N}{2}}|0\rangle = \hat{P}^N e^{C^\dagger}|0\rangle = \hat{P}^N|\Phi\rangle$$

with

$$C^\dagger = \sum_{ik} C_{ik} a_i^\dagger a_k^\dagger$$

The coefficients  $C_{ik}$  are variational parameters and  $\Phi$  is a generalized Slater determinant.

The AGP-energy is the number projected HFB-energy.

$$E_{AGP}[\mathcal{R}] = E^N[\mathcal{R}] = \frac{\langle \Phi | H \hat{P}^N | \Phi \rangle}{\langle \Phi | \hat{P}^N | \Phi \rangle}$$

where  $\mathcal{R}$  is the Valatin density. This is an intrinsic density. The corresponding projected HFB-Field is

$$\mathcal{H} = \frac{\delta E^N[\mathcal{R}]}{\delta \mathcal{R}}$$

i.e. we need the analytic dependence of  $E^N[\mathcal{R}]$  on  $\mathcal{R}$

**projected  
density  
functionals:**

$$E^I[\rho] = \langle \Phi | H \hat{P}^I | \Phi \rangle$$

where  $H$  is an effective Hamiltonian.

The projected HF (or KS) field is given by

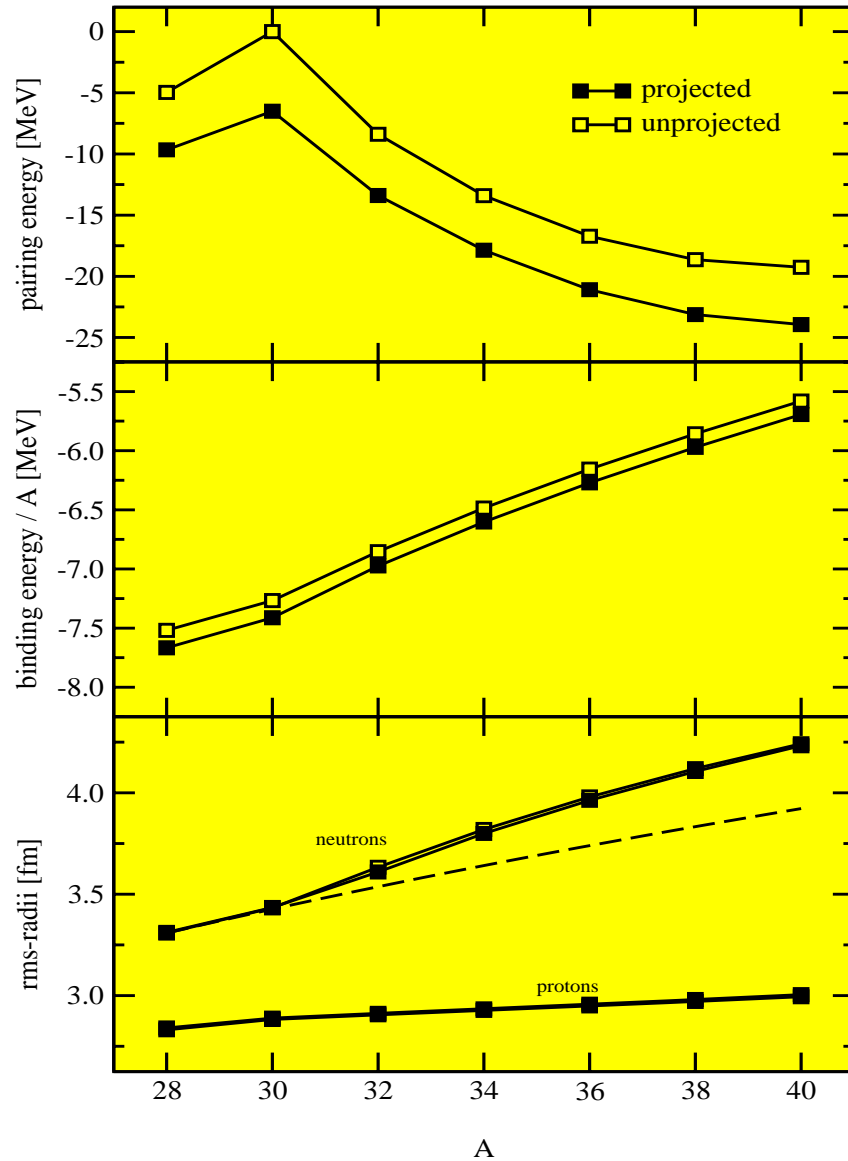
$$h_{KS}^I = \frac{\delta E^I}{\delta \rho}$$

The generalized Wick theorem shows, that  $E^I[\rho]$  is an integral over the rotational angle  $\alpha$  and it depends on the mixed density

$$\rho(\alpha) = \langle \Phi | a^\dagger a e^{i\alpha \hat{J}} | \Phi \rangle.$$

It can be shown that the mixed density can be expressed analytically by the intrinsic density

$$\rho(\alpha) = e^{i\alpha J} \rho (1 + (e^{i\alpha J} - 1) \rho)^{-1}$$



pairing energies

binding energies

rms-radii

L. Lopes, PhD Thesis, TUM, 2002

## Conclusions:

### Phenomenological DFT in nuclei produces excellent results

- static: binding energies, radii, deformations, ....
- quasistatic: treatment of rotational excitations in the rotating frame
- dynamic: QRPA reproduces positions collective excitations and the response of the nuclear systems to external fields

### Beyond mean field:

- energy dependence of the self energy reduces the shell gap
- and allows to calculate the width of giant resonances
- configuration mixing of def. states describes fluctuations and phase transitions
- restoration of symmetries allows to calculate spectroscopic properties

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G. A. Lalazissis (Thessaloniki)

E. Litvinova (GSI)  
V. Tselyayev (St. Petersburg)  
J. Meng (Beijing)